Bonus Questions for MAT2611

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Bonus Questions? What is it?

A series of *Video Lecture*s shall be uploaded to the **MAT2611-18-S1** web site. The purpose of these *Video Lectures* would be to aid understanding material of the course at your own pace in your own pre-defined sets of piecemeals.

The *Video Lecture*s would go unnoticed unless it is added with some reward system. This is augmented by the **Bonus Questions**. Each *Video Lecture* shall come paired with a **Bonus Questions** section which would test you on the material of the *Video Lecture*. The posting and submission dates, as well as the weight in the calculation of the *Year Mark*, of the **Bonus Questions** is given in the table below. Each **Bonus Question**, with the exception of the last one, has a period of fourteen days from the date of posting for submission.

The last one, i.e., **Bonus Questions 8**, has a submission time of eleven days and has lesser weight. This shall not be linked to any particular *Video Lecture*, but instead would be connected to all of them. This set of questions would depend on the full material of the course and would give you a judgement of the level of examination.

Now regarding the computation of the *Year Mark*. If A_1, A_2, A_3 be the percentage marks obtained for the first, second and third assignments, respectively, and B be the percentage mark obtained from all the questions in this document then the *Year Mark* Y shall be:

$$Y = \frac{3A_1 + 2A_2 + 3A_3 + 12B}{20}.$$

Hence the **Bonus Questions** contribute to 60% of the *Year Mark* and the assignments given do the rest.

Note: The number B shall be attained as follows:

Bonus Questions No.	Posting Date	Submission Date	Percentage Weight
Bonus Questions 1	18-Feb-2018	26-Feb-2018	8%
Bonus Questions 2	05-Mar-2018	19-Mar-2018	8%
Bonus Questions 3	12-Mar-2018	26-Mar-2018	8%
Bonus Questions 4	19-Mar-2018	02-Apr-2018	8%
Bonus Questions 5	26-Mar-2018	09-Apr-2018	8%
Bonus Questions 6	02-Apr-2018	16-Apr-2018	8%
Bonus Questions 7	09-Apr-2018	23-Apr-2018	8%
Bonus Questions 8	16-Apr-2018	27-Apr-2018	4%
Total			60%

For instance, if the marks obtained by a candidate in the eight sections of Bonus Questions have percentages $11, 44, 77, 78, 99, 33, 50, 47$ then for this candidate the calculation of B is:

$$B = \frac{8 \times (11 + 44 + 77 + 78 + 99 + 33 + 50) + 4 \times 47}{60} = \frac{3324}{60} = 55.4\%.$$

Hence, his contribution from Bonus Questions to year mark is $0.6 \times 55.4 = 33.24$.

If, further, his percentage marks for the three assignments are $40, 60, 90$, then his year mark would be

$$Y = \frac{3 \times 40 + 2 \times 60 + 3 \times 90 + 12 \times 55.4}{20} = \frac{1174.8}{20} = 58.74\%$$

I hope that the calculation of the year mark should be clear and your tensions regarding when to expect the **Bonus Questions** and their deadlines should be clear. The *Video Lectures* do augment the syllabus, although **not** in the same order as the text book. However, the sections of the textbook which correspond to a *Video Lecture* shall be stated in the respective section.

Please feel free to send your queries, suggestions, opinions..., directly to me at my email address ghoshpp@unisa.ac.za. Please **do not** send your emails through the *myUnisa* website or the Microsoft Office 365 service, since there are complaints of students finding me unreachable from any of the above two services; use an independent email client instead.

3. Video Lecture 3

This video lecture conveys the following notions:

(a) The notion of a vector subspace.
(b) While giving examples of vector subspaces one looks into the set $\mathbf{Sub}(W)$ of all vector subspaces of a vector space, and shows that there is the smallest vector subspace, naley $\{0\}$, the largest vector subspace, namely W itself. These are the *trivial subspaces*. If there are other non-trivial subspaces, then so there is their intersection, their *sum* which is the smallest subspace containing them.
(c) The investigation on the subspaces lead to the notion of the *span* of a subset $S \subseteq W$ as the smallest vector subspace $\text{span}[S]$ of W which contains S.
(d) The idea of *span* gives rise to the notion of a vector $v \in W$ being *linearly dependent* or *linearly independent* on a set $S \subseteq W$ of vectors and hence the notion of *linearly dependent* or *linearly independent* sets of vectors.
(e) This naturally leads to the notion of a *finite dimensional vector space* and its *dimension* and *basis*.
(f) The lecture ends with the notion of *coordinatisation with respect to a basis B* of a vector space as a linear bijective transformation.

Material relevant to this lecture shall be obtained in [**Ant, 2005**, Chapter 5.2-5.4].

The following questions are to be answered and answers submitted to ghoshpp.unisa.ac.za on or before **March 26, 2018**.

Please use any email client, except either of the Microsoft Office 365 or *myUnisa* services — it seems that mails from these sites are not properly forwarded to the Unisa email server.

No submission to the questions in this section shall be entertained after this date.

Problem 5. (a) Show that $S \subseteq W$ is a vector subspace of the vector space W, if and only if, for all $\lambda, \mu \in \mathbb{R}$ and every $s, t \in S$, $\lambda s + \mu t \in S$.

(b) Given the vector space $\mathbb{R}^3$ of triples of real numbers with the usual addition and scalar multiplication, which of the following subsets make a vector subspace:
(i) $S = \{(0, y, z) : y, z \in \mathbb{R}\}$.
(ii) $S = \{(x, y, z) : x = 0 \text{ or } y = 0\}$.
(iii) $S = \{(x, y, z) : x + y = 0\}$.
(iv) $S = \{(x, y, z) : x + y = 1\}$.

(c) Given the vector space $\mathbb{R}[x]$ of all polynomial functions with their usual addition and scalar multiplication which of the following subsets make a vector subspace of $\mathbb{R}[x]$.
(i) $S = \{p(x) : \deg p(x) = 3\}$.
(ii) $S = \{p(x) : 2p(0) = p(1)\}$.
(iii) $S = \{p(x) : 0 \leq x \leq 1 \Rightarrow p(x) \geq 0\}$.
(iv) $S = \{p(x) : p(x) = p(1-x)\}$.

In each of the cases also evaluate span$[S]$.

[$10 + 8 \times (2 + 3) = 50$ marks]

PROBLEM 6. (a) Let $S = \{(1,0,0),(0,1,0),(0,0,1),(1,1,1)\} \subseteq \mathbb{R}^3$.

Show that S is a linearly dependent set, but any set of three vectors from S is linearly independent.

What can you say about two element subsets or one element subsets of S — are they linearly independent or linearly dependent subsets?

(b) True or false: if $S = \{x, y, z\}$ be a linearly independent set of vectors in a vector space W then so also is the set $T = \{x+y, y+z, z+x\}$?

If it is true then prove it, else give an example to disprove it.

(c) Show that a set of two vectors (x_1, x_2, x_3) and (y_1, y_2, y_3) from $\mathbb{R}^3$ is linearly dependent, if and only if, the coordinates are in a constant ratio.

Can you provide a simpler condition (as above, in terms of the coordinates only) which equivalently states that three vectors (x_1, x_2, x_3), (y_1, y_2, y_3) and (z_1, z_2, z_3) of $\mathbb{R}^3$ are linearly dependent?

What can you say about a set of four vectors or more from $\mathbb{R}^3$?

(d) Find two bases B and C of $\mathbb{R}^4$ such that no vector is common to both B and C, while B contains the vectors $(1,0,0,0)$ and $(1,1,0,0)$ while C contains $(1,1,1,0)$ and $(1,1,1,1)$.

[$10 + 10 + 10 + 10 = 40$ marks]

PROBLEM 7. Let $V \xrightarrow{f} W$ be a linear transformation from the vector space V to the vector space W.

(a) Show that the set $\operatorname{Ker} f = \{v \in V : f(v) = 0\}$ is a vector subspace of V.[2]

(b) Show that the set $f[V] = \{w \in W : w = f(v) \text{ for some } v \in V\}$ is a vector subspace of W.[3]

[5 + 5 = 10 marks]

[2] $\operatorname{Ker} f$ is said to be the *kernel* of f.
[3] $f[V]$ is said to be the *image* of f.

References

[Ant, 2005] (2005). *Elementary Linear Algebra with Applications*. John Wiley & Sons, 9th edition.

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