Bonus Questions for MAT2611

Partha Pratim Ghosh

This document provides **Bonus Questions** for each *Video Lecture*. You should answer these questions and submit them by the deadline suggested in the document. No submission of answers to questions after their deadline shall be accepted.

If A_1, A_2, A_3 be the percentage marks obtained for the first, second and third assignments, respectively, and B be the percentage mark obtained from all the questions in this document then the year mark Y shall be:

$$Y = \frac{3A_1 + 2A_2 + 3A_3 + 12B}{20}.$$

1. Video Lecture 1

This video lecture conveys the definition of a real vector space and provides some examples. Material relevant to this lecture shall be obtained in [**Ant, 2005**, Chapter 5.1].

The following questions are to be answered and answers submitted to ghoshpp.unisa.ac.za on or before **February 26, 2018**.

No submission to the questions in this section shall be entertained after this date.

PROBLEM 1. For each of the following show that they are indeed examples of a vector space:

(a) Given any $n \geq 1$ the set $\mathbb{R}^n$ with the addition and scalar multiplication defined by:

$$(x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n) = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n),$$

and

$$\lambda(x_1, x_2, \ldots, x_n) = (\lambda x_1, \lambda x_2, \ldots, \lambda x_n).$$

(b) Given any set X the set $\mathbb{R}^X$ of all real valued functions on X with vector addition and scalar multiplication defined by:

$$(f + g)(x) = f(x) + g(x), \quad \text{for all } x \in X,$$

and

$$(\lambda f)(x) = \lambda f(x),$$

where $X \overset{f}{\underset{g}{\rightrightarrows}} \mathbb{R}$ are two functions and $\lambda \in \mathbb{R}$.

(c) The set $C([0,1])$ of all real valued continuous functions on $[0,1]$ with usual addition and scalar multiplication of functions.

(d) The set $C_1([0,1])$ of all real valued functions on $[0,1]$ which are differentiable with usual addition and scalar multiplication of functions.

(e) The set $\mathbb{R}[x]$ of all polynomial functions with real coefficients with usual addition and scalar multiplication of polynomials.

(f) Given a natural number $n \geq 1$, the set $\mathbb{R}[x]_{\deg \leq n}$ of polynomial functions of degree at most n with usual addition and scalar multiplication of polynomials.

[$6 \times 10 = 60$ marks]

PROBLEM 2. It is always important, in studying a mathematical structure, to be able to recognise an example as a proper one, and to recognise a pretender asone that fails in some respects. In each of the following, state with reasons which one is an example and which is a pretender.

(a) The set $\mathbb{R}$ of real numbers with usual addition of real numbers as vector addition and scalar multiplication defined by $(\lambda, x) \mapsto \lambda^2 x$. [1]
(b) The set $\mathbb{R}^2$ with vector addition being usual coordinate-wise addition and the scalar multiplication being defined by $(\lambda, (x, y)) \mapsto (\lambda x, 0)$.
(c) The set $\mathbb{Q}$ of rational numbers with usual addition of rational numbers as vector addition and the scalar multiplication being given by $(\lambda, x) \mapsto \lambda x$.
(d) The set $\mathbb{R}_{>0}$ of positive real numbers with usual multiplication of positive reals as the vector addition and the scalar multiplication being given by $(\lambda, x) \mapsto x^\lambda$.

[$4 \times 10 = 40$ marks]

[1]This means that the scalar multiplication $\mathbb{R} \times V \xrightarrow{s} V$ on V is given by $s(\lambda, x) = \lambda^2 x$.

References

[Ant, 2005] (2005). *Elementary Linear Algebra with Applications*. John Wiley & Sons, 9th edition.

Department of Mathematical Sciences, Unisa Science Campus, corner of Christiaan de Wet & Pioneer Avenue, Florida 1709, Johannesburg, Gauteng, South Africa

E-mail address: ghoshpp@unisa.ac.za