Tutorial letter 201/1/2017

Distribution Theory I

STA1503

Semester 1

Department of Statistics

Solutions to Assignment 01, Solutions to Assignment 02, Trial Examination Paper and Solutions, Questionnare to Assignment 03

Define tomorrow.

UNISA university of south africa

Solutions to Assignment 01

QUESTION 1

(a)
$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.3 + 0.6 - 0.7 \\ &= 0.2 \end{aligned}$$

(b)
$$\begin{aligned} P(\overline{A} \cup \overline{B}) &= P(\overline{A}) + P(\overline{B}) - P(\overline{A} \cap \overline{B}) \\ &= 1 - P(A \cap B) \\ &= 1 - 0.2 \\ &= 0.8 \end{aligned}$$

(c)
$$\begin{aligned} P(\overline{A} \cap B) &= P(A) - P(A \cap B) \\ &= 0.6 - 0.2 \\ &= 0.4 \end{aligned}$$

(d)
$$\begin{aligned} P(B/A) &= \frac{P(A \cap B)}{P(A)} \\ &= \frac{0.2}{0.3} \\ &= 2/3 = 0.6 \end{aligned}$$

(e) If A and B are independent then $P(B/A) = 2/3 = 0.6$

$P(B) = 0.6$

$P(B/A) \neq P(B)$

A and B are not independent.

QUESTION 2

$P(E) = \frac{1}{2};\ P(S) = \frac{1}{4};\ P(L) = \frac{1}{4};\ P(R/E) = \frac{1}{8};\ P(S/L) = \frac{1}{4};\ P(R/L) = \frac{1}{2};$

$\frac{1}{4} = P(R) = P(R \cap E) + P(R \cap S) + P(R \cap L)$

$P(1^{st}/E) = P(1^{st}/E) = P(1^{st}/L) = P(1^{st}/E) = P(1^{st}/L) = \frac{3}{4}$

(a)
$$
\begin{aligned}
P(\text{pass}) &= P(\text{pass} \cap 1^{st} \cup \text{pass} \cap 2^{nd}) \\
&= P(\text{pass} \cap 1^{st}) + P(\text{pass} \cap 2^{nd}) \\
&= P(\text{pass } 1^{st} \cap E) + P(\text{pass } 1^{st} \cap S) P(\text{pass } 1^{st} \cap L) + P(\text{pass } 2^{nd} \cap E) \\
&\quad + P(\text{pass } 2^{nd} \cap S) + P(\text{pass } 2^{nd} \cap L) \\
&= P(\text{pass } 1^{st}/E)P(E) + P(\text{pass } 1^{st}/S)P(S) + P(\text{pass } 1^{st}/L)P(L) \\
&\quad + P(\text{pass } 2^{nd}/E)P(E) + P(\text{pass } 2^{nd}/S)P(S) + P(\text{pass } 2^{nd}/L)P(L) \\
&= \frac{1}{2} \times \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \\
&= \frac{16}{16} \\
&= 1
\end{aligned}
$$

(b)
$$
\begin{aligned}
P(\text{Law}/\text{pass}) &= P(\text{Law} \cap \text{pass})/P(\text{pass}) \\
&= \frac{P(\text{Law } \cap \text{ pass} 1^{st}) \text{ or Law } \cap \text{ pass } 2^{nd}}{P(\text{pass})} \\
&= \frac{\frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}}{P(\text{pass})} \\
&= \frac{5}{16}/1 \\
& \frac{5}{16}
\end{aligned}
$$

(c)
$$
\begin{aligned}
P(2^{nd}S/\text{pass}) &= \frac{P(2^{nd}S \cap pass}{P(\text{pass})} \\
&= \frac{P(2^{nd}pass/S)P(S)}{P(\text{pass})} \\
&= \frac{\frac{1}{4} \times \frac{1}{4}}{P(\text{pass})} \\
&= \frac{1}{16}/1 \\
& \frac{1}{16}
\end{aligned}
$$

QUESTION 3

5 white balls, 4 blue balls, and 4 red balls = 13 balls

(a) 4 balls selected without replacement.

$$\begin{aligned} P(2 \text{ blue balls} \cap 4 \text{ other }) &= P(2 \text{ blue balls}) \times P(\text{ 2 other }) \\ &= P(2 \text{ blue balls}) \left(P(\text{ 2 other }) + P(1\text{White and } 1 \text{ red }) + P(2 \text{ red})\right) \\ &= \left(\frac{4}{13} \times \frac{3}{12}\right)\left(\frac{5}{13} \times \frac{4}{12} + \frac{5}{13} \times \frac{4}{12} + \frac{4}{13} \times \frac{3}{12}\right) \end{aligned}$$

(b) 4 balls selected with replacement.

$$\begin{aligned} P(2 \text{ blue balls}) &= P(BBWR) + P(BBBB) + P(BBWR) + P(BBWW) \\ &= \left(\frac{4}{13}\right)^2 \left(\frac{5}{13}\right)\left(\frac{4}{13}\right) + \left(\frac{4}{13}\right)^2 \left(\frac{4}{13}\right)^2 + \left(\frac{4}{13}\right)^2 \left(\frac{5}{13}\right)^2 \\ &= 0.0112 + 0.00896 + 0.01401 \\ &= 0.0342 \end{aligned}$$

(c) Here Y, the number of tries until I get a blue balls,Geometric disribution, 2 tries $p = \frac{2}{3}$

therefore $P\ (Y = k) = p\ (1 - p)^{k-1}$ for $k = 1, 2, ..$ and in particular

$$\begin{aligned} P\ (Y \geq 2) &= 1 - P(Y \leq 1) = 1 - (P(0) + P(1)) \\ &= 1 - \frac{2}{13}\left(\frac{11}{13}\right) - \frac{2}{3} \cdot \left(\frac{11}{13}\right) = 1 - \frac{2}{13} - \left(\frac{22}{39}\right) = 0.2821 \end{aligned}$$

(d) 6 balls selected

$$\begin{aligned} P(2 \text{ white balls,} 2 \text{ blue balls,} 2 \text{ red balls}) &= P(2 \text{ white balls}) + P(2 \text{ blue balls}) + P(2 \text{ red balls}) \\ &= \left(\frac{5}{13}\right)^2 + \left(\frac{4}{13}\right)^2 + \left(\frac{4}{13}\right)^2 \\ &= 0.03373 \end{aligned}$$

QUESTION 4

$\lambda = 2$ calls per hour

(a) Poisson distribution
$$P(X=1) = \frac{e^{-2}.2^x}{x!} = \frac{e^{-2}.2^1}{1!} = 2e^{-2} = 0.271$$

(b) $X = 3$ $P(X = 2\backslash X = 3)$ 2 calls per hour in 3 hours $\lambda = 2 \times 3 = 6$

(c) $E(X) = \lambda t$ means of calls in 8 hours
$$= 2 \times 8$$
$$= 16$$

Var$(X) = 16$

QUESTION 5

(a)
$$\begin{aligned} P(Y \le 0) &= P(Y=-1) + P(Y=0) \\ &= 0.2 + 0.3 \\ &= 0.5 \end{aligned}$$

(b)
$$\begin{aligned} P(Y \ge 0) &= P(Y=1) + P(Y=0) \\ &= 0.2 + 0.3 \\ &= 0.5 \end{aligned}$$

(c)
$$\begin{aligned} E(y) &= \textstyle\sum_{y=1}^{4} yP(y) \\ &= -1(0.2) + 0(0.3) + 1(0.2) + 3(0.3) \\ &= 0.9 \end{aligned}$$

(d) $V(Y) = E(Y^2) - (E(Y))^2$
$$\begin{aligned} E(y^2)p(y) &= -1^2(0.2) + 0^2(0.3) + 1^2(0.2) + 3^2(0.3) \\ &= 0.2 + 0.2 + 2.7 \\ &= 3.1 \end{aligned}$$

$$
\begin{aligned}
V(Y) &= E(Y^2) - (E(Y))^2 \\
&= 3.1 - 0.9^2 \\
&= 2.29
\end{aligned}
$$

(e)
$$
\begin{aligned}
M(t) &= E[e^{ty}] \\
&= \sum_{y=-1}^{3} e^{ty} p(y) \\
&= 0.2e^{-t} + 0.3e^0 + 0.2e^t + 0.3e^{3t} \\
&= 1 + 0.2e^{-t} + +0.2e^t + 0.3e^{3t}
\end{aligned}
$$

QUESTION 6

(a) $\int_{-\infty}^{\infty} f(y)dy = 1$

$$
\begin{aligned}
\int_{-1}^{1} c(y+1)dy &= c\int_{-1}^{1} y + 1dy \\
&= c\left[\frac{y^2}{2} + y\right]_{-1}^{1} \\
&= c\left[\frac{1}{2} + 1 - \left(\frac{1}{2} + 1\right)\right] \\
&= c\left[1.5 - \left(-\frac{1}{2}\right)\right] \\
&= c(2) \\
2c &= 1 \\
c &= \frac{1}{2}
\end{aligned}
$$

Therefore $f(y) = \begin{cases} \frac{1}{2}(y+1) & \text{for } -1 \le y \le 1 \\ 0 & \text{Otherwise} \end{cases}$

(b) $F(y) = \begin{cases} 0 & \text{if } y \le -1 \\ \frac{1}{2}\int_{-1}^{y}(t+1)dt = \frac{1}{2}\left(\frac{y^2}{2} + y - \frac{1}{2} + 1\right) = \frac{1}{2}\left(\frac{y^2}{2} + y + \frac{1}{2}\right) & \text{if } -1 \le y \le 1 \\ 1 & \text{if } y \ge 1 \end{cases}$

(c) $$\begin{aligned} P(Y \leq 0) &= F(0) \\ &= \frac{1}{2}\left(0 + 0 + \frac{1}{2}\right) \\ &= \frac{1}{4} \end{aligned}$$

(d) $$\begin{aligned} P\left(Y > \frac{1}{2}/Y > 0\right) &= \frac{P\left(Y > \frac{1}{2} \cap Y > 0\right)}{P(Y > 0)} \\ &= \frac{P\left(Y > \frac{1}{2}\right)}{P(Y > 0)} \\ &= \frac{1 - P\left(Y \leq \frac{1}{2}\right)}{1 - P(Y \leq 0)} \\ &= \frac{1 - F\left(\frac{1}{2}\right)}{1 - P(Y \leq 0)} \\ &= \frac{1 - \frac{1}{2}\left(\frac{1}{8} + \frac{1}{2} + \frac{1}{2}\right)}{1 - \frac{1}{4}} \end{aligned}$$

(e)
$$
\begin{aligned}
E(Y) &= \frac{1}{2}\int_{-1}^{1} y(y+1)dy \\
&= \frac{1}{2}\int_{-1}^{1} (y^2+1)dy \\
&= \frac{1}{2}\left(\frac{y^3}{3}+\frac{y^2}{2}\right)_{-1}^{1} \\
&= \frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{3}-1\right) \\
&= \frac{1}{2}\left(\frac{2}{3}-1\right) \\
&= \frac{1}{2}\left(\frac{4-3}{6}\right) \\
&= \frac{1}{12}
\end{aligned}
$$

$$
\begin{aligned}
E(Y) &= \frac{1}{2}\int_{-1}^{1} y^2(y+1)dy \\
&= \frac{1}{2}\int_{-1}^{1} (y^3+y^2)dy \\
&= \frac{1}{2}\left(\frac{y4}{4}+\frac{y^3}{3}\right)_{-1}^{1} \\
&= \frac{1}{2}\left(\frac{1}{4}+\frac{1}{3}-\frac{1}{4}+\frac{1}{3}\right) \\
&= \frac{1}{3}
\end{aligned}
$$

$$\Rightarrow V(Y) = \frac{1}{3} - \frac{1}{144} = \frac{47}{144}$$

(f)
$$\begin{aligned} m(t) &= \frac{1}{2}\int_{-1}^{1} e^{ty}(y+1)dy \\ &= \frac{1}{2}\int_{-1}^{1} ye^{ty} + e^{ty})dy \\ &= \frac{1}{2}(\frac{y}{t}e^{ty} - \frac{y}{t^2}e^{ty} + \frac{1}{t}e^{ty})/_{-1}^{1} \\ &= \frac{1}{2}(\frac{1}{t}e^{ty} - \frac{1}{t^2}e^{ty} + \frac{1}{t}e^{t}) - (-\frac{1}{t}e^{-t} - \frac{y}{t^2}e^{-t} + \frac{1}{t}e^{-t}) \\ &= \frac{1}{2}(\frac{2}{t} - \frac{1}{t^2})e^{ty} + \frac{1}{t^2}e{-}^{t} \qquad t \neq 0 \end{aligned}$$

QUESTION 7

(a) Uniform Distribution

Let y be the income

$f(y) = \dfrac{1}{b-a}$ where $a \leq y \leq b$

$$f(y) = \begin{cases} \dfrac{1}{10000-5000}, \;\; = \dfrac{1}{5000} \;\; ; & 5000 \leq y 10000 \\ 0 & \text{otherwise} \end{cases}$$

(i)
$$\begin{aligned} P(6000 \leq Y \leq 8000) &= \int_{6000}^{8000} f(y)dy \\ &= \frac{y}{5000}/_{6000}^{8000} \\ &= (8000-6000) \times \frac{1}{5000} \\ &= \frac{2000}{5000} \\ & \quad 0.4 \end{aligned}$$

(ii) Expected salary per months

$$\begin{aligned}E(Y) &= \int_{5000}^{10000} y f(y) dy \\ &= \frac{y^2}{10000} / _{6000}^{10000} \\ &= (10000 - 6000) \times \frac{1}{10000} \\ &= \frac{4000}{10000} \\ & 0.4\end{aligned}$$

(b) (i)

$$\begin{aligned}0.95 &= P(Y \geq Y_0) \\ &= P\left(Z \geq \frac{Y_0 - 750}{\sigma}\right) = \frac{Y_0 - 750}{\sigma} = -1.645 \\ &= P(Z \geq ?)\end{aligned}$$

(ii)

$$\begin{aligned}Z_{0.99} &= P(745 \leq Y \leq 755) \\ &= P\left(\frac{745 - 750}{\sigma} \leq Z \leq \frac{755 - 750}{\sigma}\right) \\ &= P\left(\frac{-5}{\sigma} \leq Z \leq \frac{5}{\sigma}\right)\end{aligned}$$

Solutions to Assignment 02

QUESTION 1

(a) $1 = c\int\limits_0^2\int\limits_0^1 (y_1 + y_2)dy_1dy_2$

$$= c\int\limits_0^2 \left(\frac{y_1^2}{2} + y_1y_2^2\Big|_0^1 \right) dy_2$$

$$= c\int\limits_0^2 \left(\frac{1}{2} + y_2^2 \right) dy_2$$

$$= c\left(\frac{1}{2}y_2 + \frac{1}{3}y_2^3\Big|_0^2 \right)$$

$$= c\left(1 + \frac{8}{3}\right)$$

$$= c(\frac{11}{3})$$

$$\Rightarrow c = \frac{3}{11}$$

(b) $f_1(y_1) = \frac{3}{11}\int\limits_0^2 \left(y_1 + y_2^2\right) dy_2$

$$= \frac{3}{11}\left(y_1y_2 + \frac{1}{3}y_2^3\Big|_0^2 \right) dy_2$$

$$= \frac{3}{11}(2y_1 + \frac{8}{3})$$

$$= \begin{cases} \frac{6}{11}y_1 + \frac{8}{11}, & 0 < y_1 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

$f_2(y_2) = \frac{3}{11}\int\limits_0^1 \left(y_1 + y_2^2\right) dy_1$

$$= \frac{3}{11}\left(\frac{1}{2}y_1^2 + y_1y_2^2\Big|_0^1 \right)$$

$$= \frac{3}{11}(\frac{1}{2} + y_2^2)$$

$$= \begin{cases} \frac{3}{11}y_2^2 + \frac{3}{22}, & 0 < y_1 < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(c) $P(Y_1 < \frac{1}{2}, Y_2 < 1)$

$$= \frac{3}{11} \int_0^1 \int_0^{\frac{1}{2}} \left(y_1 + y_2^2\right) dy_1 dy_2$$

$$= \frac{3}{11} \int_0^1 \left(\frac{y_1^2}{2} + y_1 y_2^2 \Big|_0^{\frac{1}{2}} \right) dy_2$$

$$= \frac{3}{11} \int_0^1 \left(\frac{1}{8} + \frac{1}{2} y_2^2 \right) dy_2$$

$$= \frac{3}{11} \left(\frac{1}{8} y_2 + \frac{1}{6} y_2^3 \Big|_0^1 \right)$$

$$= \frac{3}{11} \left(\frac{1}{8} + \frac{1}{6} \right)$$

$$= \frac{3}{11} \times \frac{7}{24}$$

$$= \frac{7}{88}$$

(d) $P(Y_1 < \frac{1}{2} | Y_2 < 1)$

$$= \frac{P(Y_1 < \frac{1}{2}, Y_2 < 1)}{P(Y_2 < 1)}$$

$$= \frac{7}{88} / \int_0^1 (\frac{3}{11} y_2^2 + \frac{3}{22}) dy_2$$

$$= \frac{7}{88} / (\frac{1}{11} y_2^3 + \frac{3}{22} y_2)|_0^1$$

$$= \frac{7}{88} / (\frac{1}{11} + \frac{3}{22}) = \frac{7}{88} \frac{22}{5} = \frac{7}{20}.$$

(e) The conditional probability density function of Y_1given that $Y_2 = 1$ is given by

$$f_1(y_1 | y_2 = 1) = \frac{f(y_1, 1)}{f_2(1)}$$

$$= \frac{3}{11} \frac{(y_1+1)}{(\frac{3}{11} + \frac{3}{22})}$$

$$= \begin{cases} \frac{1}{3}(y_1 + 1), & 0 < y_1 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

$P(Y_1 < \frac{1}{2} | Y_2 = 1)$

$$= \int_0^{\frac{1}{2}} f_1(y_1 | y_2 = 1) dy_1 = \frac{1}{3} \int_0^1 (y_1 + 1) dy_1$$

$$= \frac{1}{3} (\frac{1}{2} y_1^2 + y_1)|_0^{\frac{1}{2}}$$

$$= \frac{1}{3} (\frac{1}{8} + \frac{1}{2}) = \frac{1}{3} \frac{5}{8} = \frac{5}{24}$$

(f) $E(Y_1Y_2)$

$= \frac{3}{11}\int_0^2\int_0^1 y_1y_2(y_1+y_2^2)dy_1dy_2$
$= \frac{3}{11}\int_0^2\int_0^1 (y_1^2y_2+y_1y_2^3)dy_1dy_2$
$= \frac{3}{11}\int_0^2 (\frac{y_1^3}{3}y_2+\frac{y_1^2}{2}y_2^3)|_0^1dy_2$
$= \frac{3}{11}\int_0^2 (\frac{1}{3}y_2+\frac{1}{2}y_2^3)dy_2$
$= \frac{3}{11}(\frac{1}{6}y_2^2+\frac{1}{8}y_2^4)|_0^2 = \frac{3}{11}(\frac{4}{6}+\frac{16}{8}) = \frac{8}{11}$

$E(Y_1)$

$= \int_0^1 y_1(\frac{6}{11}y_1+\frac{8}{11})dy_1$
$= \int_0^1 (\frac{6}{11}y_1^2+\frac{8}{11}y_1)dy_1$
$= (\frac{6}{33}y_1^3+\frac{8}{22}y_1^2)|_0^1$
$= \frac{6}{11}.$

$E(Y_2)$

$= \int_0^2 y_2(\frac{3}{11}y_2^2+\frac{3}{22})dy_2$
$= \int_0^2 (\frac{3}{11}y_2^3+\frac{3}{22}y_2)dy_2$
$= (\frac{3}{44}y_2^4+\frac{3}{44}y_2^2)|_0^2$
$= \frac{12}{11}+\frac{3}{11} = \frac{15}{11}.$

$Cov(Y_1,Y_2)$

$= E[Y_1Y_2]-E[Y_1]E[Y_2]$
$= \frac{8}{11}-\frac{6}{11}\frac{15}{11} = -\frac{2}{121}$

(g) $Cov(Y_1,Y_2)\neq 0$ implies that Y_1 and Y_2 are not indpendent.

QUESTION 2

$f_1(y_1) = 2e^{-2y_1}$if $y_2 > 0$ and 0 elsewhere. $f_2(y_2) = 3e^{-3y_2}$if $y_2 > 0$ and 0 elsewhere.

(a) Independence implies that $f(y_1,y_2)$

$$= f_1(y_1) \times f_2(y_2)$$
$$= \begin{cases} 6e^{-(2y_1+3y_2)}, & y_1 > 0, y_2 > 0 \\ 0, & \text{elsewhere} \end{cases}$$

(b) Indepndence implies that $P(Y_1 > 1, Y_2 > 1)$

$$= P(Y_1 > 1) \times P(Y_2 > 1)$$
$$= (\int_1^\infty 2e^{-2y_1} dy_1) \times (\int_1^\infty 3e^{-3y2} dy_2)$$
$$= (-e^{-2y_1})|_0^\infty \times (-e^{-3y_2})|_0^\infty = e^{-2} \times e^{-3} = e^{-5} =$$

(c) Indpendence implies that $E(Y_1Y_2) = E(Y_1)E(Y_2) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

QUESTION 3

(a) $u = 2 - 3y$ implies that when $y = -1$ then $u = 2 - 3(-1) = 5$, and when $y = 1$ then $u = 2 - 3(1) = -1$.Hence $-1 \leq u \leq 5$.

The cumulative distribution function of Y is $F_y(y)$

$$= P(Y \leq y) = \int_{-\infty}^{y} f(t)dt$$
$$= \begin{cases} 0, & y < -1 \\ \int_{-1}^{y} \frac{3}{2}t^2 dt = \frac{1}{2}t^3|_{-1}^{y} = \frac{1}{2}(y^3 + 1) & -1 \leq y \leq 1 \\ 1, & y \geq 1 \end{cases}$$

The cumulative distribution function of U is $F_u(u)$

$$= P(U \leq u) = P(2 - 3Y \leq u) = P[-Y \leq \frac{1}{3}(u - 2)]$$
$$= P[Y > \frac{1}{3}(2 - u)] = 1 - P[Y \leq \frac{1}{3}(2 - u)]$$
$$= 1 - F_y(\frac{1}{3}(2 - u))$$
$$= \begin{cases} 1 - F_y(1) = 1 - 1 = 0, & u < -1 \\ 1 - F_y(\frac{1}{3}(2 - u)) = 1 - \frac{1}{2}[\frac{1}{27}(2 - u)^3 + 1)], & -1 \leq u \leq 5 \\ 1 - F_y(-1) = 1 - 0 = 1 & u \geq 5 \end{cases}$$

The probability density function of U is given by $f_u(u) = \frac{\partial F_u(u)}{\partial u}$

$$= \begin{cases} -\frac{1}{2}[-\frac{3}{27}(2 - u)] = \frac{1}{18}(2 - u)^2, & -1 \leq u \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

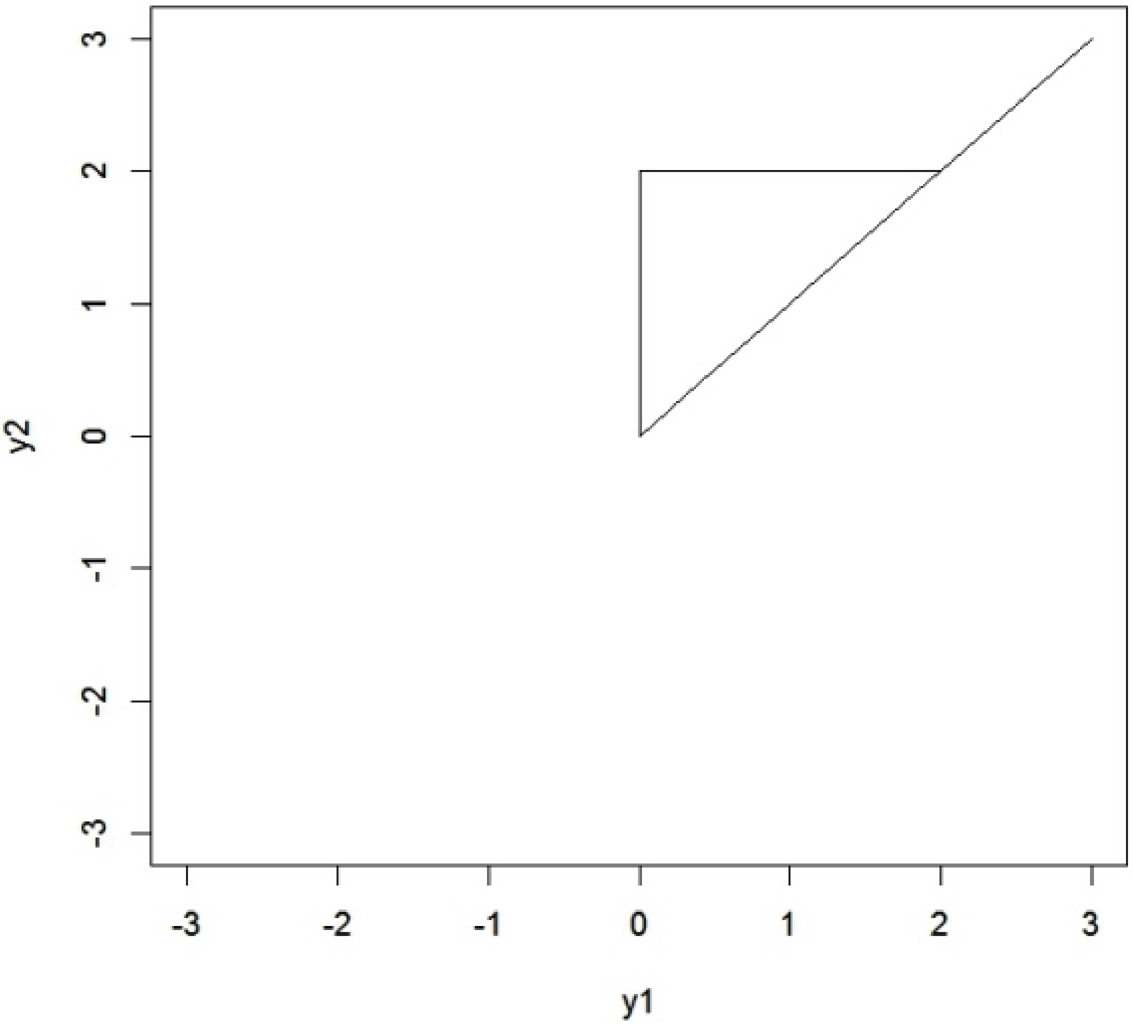
(b) The probability density function of U is given by $f_u(u) = f(\frac{1}{3}(2-u)) \times |\frac{\partial y}{\partial u}|$

$$= \begin{cases} \frac{3}{2}[\frac{1}{9}(2-u)^2] \times (\frac{1}{3}), & -1 \leq u \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$= \begin{matrix} \frac{1}{18}(2-u)^2, & -1 \leq u \leq 5 \\ 0 & \text{elsewhere} \end{matrix}$$

QUESTION 4

(a)



Area **outside** the triangle with coordinates $(0,0)$, $(0,2)$ and $(2,2)$ in the figure above has $f(y_1, y_2) = 0$.

(b) $1 = c\int_0^2 \int_0^{y_2} dy_1 dy_2$

$= c\int_0^2 y_1|_0^{y_2} dy_2$

$= c\int_0^2 y_2 dy_2$

$= c\frac{1}{2}y_2^2|_0^2$

$= c\frac{4}{2} = 2c$

$\Rightarrow c = \frac{1}{2}.$

(c) $f_1(y_1) = \frac{1}{2}\int_{y_1}^2 dy_2$

$= \frac{1}{2}y_2|_{y_1}^2$

$= \begin{cases} 1 - \frac{1}{2}y_1 & 0 < y_1 < 2 \\ 0, & \text{elsewhere} \end{cases}$

$f_2(y_2) = \frac{1}{2}\int_0^{y_2} dy_1$

$= \frac{1}{2}y_1|_0^{y_2}$

$= \begin{cases} \frac{1}{2}y_2 & 0 < y_2 < 2 \\ 0, & \text{elsewhere} \end{cases}$

(d) $P(Y_2 > 1)$

$= \int_1^2 f_2(|y_2)dy_2 = \frac{1}{2}\int_1^2 y_2 dy_2$

$= \frac{1}{4}y_2^2|_1^2 = 1 - \frac{1}{4} = \frac{3}{4}.$

(e) $E(Y_1Y_2)$

$= \frac{1}{2}\int_0^2 \int_0^{y_2} y_1 y_2 dy_1 dy_2$

$= \int_0^2 \frac{1}{4}y_1^2 y_2|_0^{y_2} dy_2$

$= \int_0^2 \frac{1}{4}y_2^3 dy_2 = \frac{1}{16}y_2^4|_0^2 = 1$

1 Trial Examination Paper

QUESTION 1 **[Total 15 marks]**

If A and B are *independent* events with $P(A) = 0.50$, and $P(B) = 0.20$, find

(a) $P(A \cap B)$ (3)

(b) $P(\overline{A} \cup \overline{B})$ (3)

(c) $P(\overline{A} \cap \overline{B})$ (4)

(d) $P(\overline{A}/B)$ (5)

QUESTION 2 **[Total 15 marks]**

Let Y be a binomial random variable with $n = 10$ and $p = 2$.

(a) Use Table 1, Appendix 3, to obtain $P(2 < Y < 5)$ and $P(2 \leq Y < 5)$. Are the probabilities that Y falls in the interval $(2, 5)$ and $[2, 5)$ equal? Why or why not? (5)

(b) Use Table 1, Appendix 3, to obtain $P(2 < Y < 5)$ and $P(2 \leq Y \leq 5)$. Are these two probabilities equal? Why or why not? (5)

(c) If Y is continuous and $a < b$, then $P(a < Y < b) = P(a \leq Y < b)$. Does the result in part (a) contradict this claim? Why? (5)

QUESTION 3 **[Total 10 marks]**

These are all questions on chapter 3: **Discrete random variables and their probability distributions.**
To verify the accuracy of their accounting entries, a company uses auditors for verification on a regular basis. The company's employees make erroneous entries 5% of the time. Suppose that an auditor randomly checks three entries.

(a) Find the probability distribution for Y, the number of errors detected by the auditor. (5)

(b) Find the probability that the auditor will detect more than one error. (5)

QUESTION 4 **[Total 20 marks]**

Let Y be a random variable with $p(y)$ given in the accompanying table.

y	1	2	3	4
$p(y)$	0.4	0.3	0.2	0.1

Calculate the following:

(a) $E(Y)$ (5)

(b) $E\left(\frac{1}{Y}\right)$ (5)

(c) $E(Y^2 - 1)$ (5)

(d) $V(Y)$ (5)

QUESTION 5 **[Total 20 marks]**

Let X be random variable with probability density function given by

$$f_X(x) = \begin{cases} c, & 0 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find c (5)

(b) Find $E(X)$ and $Var(X)$ (8)

(c) Find $E(4X+5)$ and $Var(6X+2)$ (7)

QUESTION 6 **[Total 20 marks]**

Let Y be a random variable with density function

$$f(y) = \begin{cases} 2y, & 0 \leq y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the cumulative distribution function of Y (6)

(b) Find the density function of $4Y - 1$ (8)

(c) Find $E(Y)$ (6)

QUESTION 7 **[Total 20 marks]**

Suppose that $Y_1, Y_2, \ldots Y_5$ denotes a random sample from a uniform distribution defined on the interval $(0,1)$. That is,

$$f(y) = \begin{cases} 1, & 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the density function for the second-order statistics. (10)

(b) Give the joint density function for the second- and fourth - order statistics (10)

Total marks [120]

2 Trial Examination Solutions

QUESTION 1

A and B are independent, $P(A) = 0.5$, $P(B) = 0.2$:

(a) $P(A \cap B) = P(A) \cdot P(B)$ (because of independency) $= 0.5.0.2 = 0.1$ (3)

(b) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cup B)$ (3)

(c)

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= P(\overline{A \cup B}) = 1 - P(A \cup B) \\ &= 1 - (P(A) + P(B) - P(A \cup B)) \\ &= 1 - (0.5 + 0.2 - 0.1) = 0.4 \end{aligned}$$

(4)

(d)

$$\begin{aligned} P(\bar{A}|B) &= \frac{P(\bar{A} \cap \bar{B})}{P(B)} = \frac{P(\bar{A}) \cdot P(B)}{P(B)} \\ &= \frac{(1 - P(A)) \cdot P(B)}{P(B)} \\ &= \frac{0.5 \cdot 0.2}{0.2} = 0.5 \end{aligned}$$

or directly,

$$P(\bar{A}|B) = P(\bar{A}) = 1 - P(A) = 0.5$$

In both solutions we use the fact that if A and B are independent, then of course A^c and B must also be independent. (5)

[Total marks: 15]

QUESTION 2

$$Y \sim Bin(n = 10, \;\; p = 0.2) :$$

The table to use is Table 1, Appendix 3 of the textbook; and for $n = 10$ we choose table (b) on page 839. For $p = 0.2$, we look at the fourth column of values.
Note that the table given probabilities of the type:

$$P(Y \leq a)$$

Therefore we must express the gives probabilities we wish to evaluate in terms of these types of probabilities, keeping in mind that:
$P(Y < a)$ and $P(Y \leq a)$ are not necessarily the same thing. Note that of cause $P(a < Y \leq b) = P(Y \leq b) - P(Y \leq b)$, this therefore will also be easy to evaluate.

(a)

$$\begin{aligned}
P(Z < Y < 5) &= P(Z < Y \leq 4) \\
&= P(Y \leq 4) - P(Y \leq 2) \\
&= 0.967 - 0.678 = 0.289 \qquad \text{(from table)} \\
P(2 \leq Y < 5) &= P(1 < Y \leq 4) \\
&= P(Y \leq 4) - P(Y \leq 1) \\
&= 0.967 - 0.376 \\
&= 0.591
\end{aligned}$$

The values are not equal because $Y = 2$ is included in the second interval but in the first one; the difference between the probabilities is:

$$0.591 - 0.289 = 0.302$$

Which is $P(Y = 2)$ (5)

(b)

$$\begin{aligned}
P(2 < Y < 5) &= P(2 < Y \leq 4) = 0.289 \\
P(2 \leq Y \leq 5) &= P(1 < Y \leq 5) = P(Y \leq 5) - P(Y \leq 1) \\
&= 0.994 - 0.376 = 0.618
\end{aligned}$$

The values are not the same and the reason is because the second probability includes the values $Y = 2$ and $Y = 5$, while the first probability does not. The difference between the probabilities is equal to $P(Y = 2) + P(Y = 5)$. (5)

(c) No, there is no contradiction; the Binomial distribution is not a continuous distribution but rather discrete and therefore $P(a < Y < b) = 0(a \leq Y < b)$ also not have to hold. (5)

[Total marks: 15]

QUESTION 3

(a) Assuming the three chosen entries are independent of each other, each with an error with the probability 0.05, then Y, the number of errors detected by the auditor will have Binomial distribution with parameters $n = 3$ and $p = 0.05$ (5)

(b)

$$\begin{aligned}
P(Y > 1) &= 1 - P(Y \leq 0) - P(Y = 1) \\
&= 1 - \begin{pmatrix} 3 \\ 0 \end{pmatrix} P^0(1-P)^3 - \begin{pmatrix} 3 \\ 1 \end{pmatrix} P^1(1-P)^2 \\
&= 1 - 1(0.05)^0(0.95)^3 - 3(0.05)(0.95)^2
\end{aligned}$$

or alternatively,

$$\begin{aligned} P(Y > 1) &= P(Y = 2) + P(Y = 3) \\ &= \binom{3}{2} P^2(1-P)^1 + \binom{3}{3} P^3(1-P)^0 \\ &= 3(0.05)^2(0.95)^1 + 1(0.05)^3(0.95)^0 \end{aligned}$$

(5)

[Total marks: 10]

QUESTION 4

The probability function $P(y) = P(Y = y)$ is as follows:

y	1	2	3	4
$p(y)$	0.4	0.3	0.2	0.1

(a)

$$\begin{aligned} E(Y) &= \sum_{y=1}^{4} yP(y) = 1 \cdot 0.4 + 2 \cdot 0.3 + 3 \cdot 0.2 + 4 \cdot 0.1 \\ &= 2.0 \end{aligned}$$

(5)

(b)

$$\begin{aligned} E\left(\frac{1}{Y}\right) &= \sum_{y=1}^{4} \frac{1}{y} P(y) = \frac{1}{1} \cdot 0.4 + \frac{1}{2} \cdot 0.3 + \frac{1}{3} \cdot 0.2 + \frac{1}{4} \cdot 0.1 \\ &= \frac{77}{120} \end{aligned}$$

(5)

(c)

$$\begin{aligned} E(Y^2 - 1) &= E(Y^2) - 1 = \left(\sum_{y=1}^{4} y^2 p(y)\right) - 1 \\ &= (1^2 \cdot 0.4 + 2^2 \cdot 0.3 + 3^2 \cdot 0.2 + 4^2 \cdot 0.1) - 1 \\ &= 5 - 1 = 4 \end{aligned}$$

(5)

(d)

$$V(Y) = E(Y^2) - (E(Y))^2$$

where

$$E(Y) = 2, \qquad E(Y^2) = 5$$

As calculated in (a) and (c). Therefore:

$$V(Y) = 5 - (2)^2 = 1$$

Alternatively,

$$\begin{aligned} V(Y) &= E(Y - E(Y)^2) \\ &= E((Y-2)^2) \\ &= \sum_{i=1}^{4} (y-2)^2 P(y) \\ &= (1-2)^2 \cdot 0.4 + (2-2)^2 \cdot 0.3 + (3-2)^2 \cdot 0.2 + (4-2)^2 \cdot 0.1 \\ &= 1 \end{aligned}$$

(5)

[Total marks: 20]

QUESTION 5

$$f_X(x) = \begin{cases} C, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(a) The value of c can be determined from the fact that we know that for f_X to be a density function, it must integrate to the value 1 when integrated over all possible x-values. Here, we therefore get:

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} f_X(x)dx \\ &= \int_0^2 c dx = 2c \\ \therefore \quad C &= \frac{1}{2} \end{aligned}$$

(5)

(b)

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f_X(x)dx = \int_0^2 x \cdot \frac{1}{2}dx \\ &= \left(\frac{1}{4}x^2\right)\Big|_0^2 = \frac{1}{4}2^2 - \frac{1}{4}0^2 = 1 \end{aligned}$$

$$\begin{aligned} E(X^2) &= \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^2 x^2 \cdot \frac{1}{2}dx \\ &= \left(\frac{1}{6}x^3\right)\Big|_0^2 = \frac{1}{6}2^3 - \frac{1}{6}\odot^3 = \frac{8}{6} = \frac{4}{3} \\ Var(X) &= E(X^2) - (E(X))^2 = \frac{4}{3} - 1^2 = \frac{1}{3} \end{aligned}$$

Alternative, we can recognize the distribution as being the uniform distribution on the interval $(0,2)$ for which $E(X) = \frac{0+2}{2} + 1$ and $Var(X) = \frac{(2-0)^2}{12} - \frac{4}{12} - \frac{1}{3}$. (8)

(c)

$$\begin{aligned} E(4X+5) &= 4E(X)+5 = 4\cdot 1+5 = 9 \\ Var(6X+2) &= 6^2 \cdot Var(X) - 36 \cdot \frac{1}{3} = 12 \end{aligned}$$

(7)

[Total mark 20]

QUESTION 6

$$f_Y(y) = \begin{cases} 2y, & 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) The cumulative distribution function is found by integrating the density function.

$$F_Y(y) = \int_{-\infty}^{y} f_Y(a)da$$

Since the density function is defined piecewise, we will also find the distribution function piecewise.

If $y < 0$,

$$F_Y(y) = \int_{-\infty}^{y} f_Y(a)da = \int_{-\infty}^{y} 0da = 0$$

If $0 \le y \le 1$,

$$\begin{aligned} F_Y(y) &= \int_{-\infty}^{y} f_Y(a)da = \int_{-\infty}^{0} 0da + \int_0^y 2ada \\ &= (a^2)\Big|_0^y = y^2 \end{aligned}$$

If $y > 1$

$$\begin{aligned} F_X(y) &= \int_{-\infty}^{y} f_Y(a)da = \int_{-\infty}^{0} 0da + \int_{0}^{1} 2ada + \int_{1}^{y} 0da \\ &= (a^2)|_0^1 = 1 \end{aligned}$$

That is:

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ y^2, & 0 \le y \le 1 \\ 1, & y > 1 \end{cases} \tag{6}$$

(b) Let $U = 4Y - 1$. The mapping $u = h(y) = uy - 1$ is increasing, so we can use the transformation method. The inverse mapping is:

$$y = \frac{1}{4}(u+1) = h^{-1}(u)$$

with derivative:

$$\frac{d}{du}(h^{-1}(u)) = \frac{1}{4}$$

Therefore the density function of U is:

$$\begin{aligned} f_U(u) &= f_Y(h^{-1}(u))|\frac{d}{du}h^{-1}(u)| \\ &= \begin{cases} 2\left(\frac{1}{4}\right)(u+1))\frac{1}{4}, & 0 \le \frac{1}{4}(u+1) \le 1 \\ 0. & \text{elsewhere} \end{cases} \\ &= \begin{cases} \frac{1}{8}(u+1), & 1 \le u \le 3 \\ 0. & \text{elsewhere} \end{cases} \end{aligned}$$

Alternatively we can use the distribution function method:

$$F_U(u) = P(U \le u) = P(4Y - 1 \le u) = P\left(Y \le \frac{u+1}{4}\right) = F_Y\left(\frac{u+1}{4}\right)$$

Since we already have the distribution function F_Y, there is no need to integrate f_Y again. We get:

$$\begin{aligned} F_U(u) = F_Y\left(\frac{u+1}{4}\right) &= \begin{cases} 0, & \frac{u+1}{4} < 0 \\ \left(\frac{u+1}{4}\right)^2, & 0 \le \frac{u+1}{4} \le 1 \\ 1, & \frac{u+1}{4} > 1. \end{cases} \\ &= \begin{cases} 0, & u < -1 \\ \left(\frac{u+1}{4}\right)^2, & -1 \le u \le 3 \\ 1, & u > 3. \end{cases} \end{aligned}$$

Next, we differentiate this with respect to u to find the corresponding density function.

$$f_u(u) = \frac{d}{du}F_y(u) = \begin{cases} 0, & u < -1 \\ \frac{1}{4}\left(\frac{u+1}{4}\right)^2, & -1 \leq u \leq 3 \\ 0, & u > 3 \end{cases}$$

$$= \begin{cases} \frac{1}{8}(u+1), & -1 \leq u \leq 3 \\ 0, & \text{elsewhere.} \end{cases}$$

(8)

(c)

$$\begin{aligned} E(Y) &= \int_{-\infty}^{\infty} y f_Y(y) dy \\ &= \int_{e}^{1} y \cdot 2y dy \\ &= \int_{0}^{1} 2y^2 dy \\ &= \frac{2}{3}(y^3)|_0^1 = \frac{2}{3} \end{aligned}$$

(6)

[Total marks: 20]

QUESTION 7

(a) The distribution associated with each of the $Y's$ is

$$F(y) = \begin{cases} 0 & \text{for} \quad y < 0 \\ y & \text{for} \quad 0 \leq y \leq 0 \\ 1 & \text{for} \quad y > 1 \end{cases}$$

The density function of the second-order statistics, $Y_{(2),}$ can be obtained directly from Theorem 6.5 with $n = 5, k = 2$. Thus, with $f(y)$ and $F(y)$ as noted,

$$\begin{aligned} g_{(2)}(y_2) &= \frac{5!}{(2-1)!(5-2)!}\left[F((y_2)\right]^{2-1}\left[1 - f(y)^{5-2} f(y_2)\right] \\ &= \begin{cases} 20y_2(1-y_2)^2, & -\infty < y_2 < \infty \\ 0, & \text{elsewhere} \end{cases} \end{aligned}$$

(10)

(b) The joint density of the second- and fourth- order statistics is ready obtained from the second result . With $f(y)$ and $F(y)$ as before, $j = 2, k = 4$, and $n = 5$

$$g_{(2)(4)}(y_2 y_4) = \frac{5!}{(2-1)!(4-1-2)(5-4)!}\left[F(y_2)\right]^{2-1}\left[F(y_4) - F(y_2)^{4-1-2}\right] \times \left[1 - F(y_4)\right]^{5-4} f(y_2) f(y_4), \qquad -\infty < y_2 < y_4 < \infty$$

$$= \begin{cases} 5! y_2(y_4 - y_2)(1 - y_4), & -\infty < y_2 < y_4 < \infty \\ 0 & \text{elsewhere} \end{cases}$$

(10)

[Total marks: 20]

STA1503 Assignment 3
QUESTIONNAIRE FOR STUDENTS

Dear student,

We need your help to evaluate this module and its study guide. We try to do our best to make this an excellent module but would like to use your experience of it to further improve on it where possible.

We are aware that we are asking for some time and effort from you in filling out this questionnaire. To make it worthwhile to you, as explained in Tutorial Letter 101, we have decided to make this questionnaire into the third assignment of this module.

Any student who fills in and submits this questionnaire as Assignment 3 will get the full 10% that this assignment contributes to the semester mark. This means 2 percentage points in the final mark, absolutely free!

The due date for this assignment is 28 April 2017. Please make sure you do submit it by this date, since if you submit it later, I cannot guarantee that you will get the promised semester mark contribution – if you miss this due date, please do contact your lecturer for advice!

Please send the filled-in questionnaire to the Assignments Department in the usual assignment covers, numbered as Assignment number 03 for STA1503. Your replies to the questions will be most valuable for us, and ultimately yourself, as they will help us improve the way we teach this and other modules. The questionnaire should only take you 5 minutes to complete. If you cannot answer a particular question, for instance you have not completed that section yet, or do not have an opinion on it, please feel free to leave that question unanswered!

The questionnaires will be treated as confidential so please do feel free to be totally honest with your responses! We want your opinions **-** there is no right or wrong answer to any of the questions.

Thank you very much.

Ms M A Managa

Module lecturer, STA1503

INSTRUCTIONS: Please answer each question by ticking the appropriate box Or by filling in the information asked for.

SECTION A: GENERAL QUESTIONS

A. 1. Are you studying: Full-time ☐ Part-time ☐

A. 2. How many UNISA modules are you taking this year? ………….. modules

A. 3. How are you mostly studying for this module?

- On your own ☐
- In an informal study group ☐
- In a tutorial group at a learning centre or a college ☐

A. 4. Is English your:

- First language (home language) ☐
- Second language ☐
- Third language ☐
- Fourth or further language ☐

SECTION B: STA1503 EXPECTATIONS

B.1. What was your expectation about the level of difficulty of this module? Compared to other Statistics modules, did you expect it to be

Much easier	☐	More difficult	☐
Easier	☐	Much more difficult	☐
About the same	☐	I did not have an opinion	☐

B.2. What was this expectation based on? (Tick all that apply):

- The description of the module in the calendar ☐
- Previous experiences with similar modules ☐
- Discussions with students who had taken this module ☐
- Discussions with tutors or lecturers, ☐
- Not applicable (no prior expectations) ☐
- Other, please specify ………………………………. ☐

SECTION C: THE CONTENTS OF THE MODULE

C.1. What is your own opinion about the level of difficulty of the content of the module so far?

- Very difficult ☐
- Difficult ☐
- Acceptable ☐
- Easy ☐

C.2. What is your opinion on the total workload of the module so far:

- Too demanding ☐
- Demanding ☐
- Manageable ☐
- Insufficient ☐

C.3. How difficult or easy did you find it to understand the following concepts? (leave empty if you have not yet come across the concept)

	Very difficult	Difficult	Unsure	Easy	Very easy
Knowing which distribution to use	☐	☐	☐	☐	☐
Discrete distribution	☐	☐	☐	☐	☐
Continuous distributions	☐	☐	☐	☐	☐
Joint distributions of two random variables	☐	☐	☐	☐	☐
Applying the transformation method to find joint distributions of functions of random variables	☐	☐	☐	☐	☐
Differentiating distribution functions to find densities	☐	☐	☐	☐	☐
Integrating joint density functions to find joint distributions	☐	☐	☐	☐	☐
Integrating joint density functions to find joint probabilities	☐	☐	☐	☐	☐
Determining the integration limits in one-dimensional distributions	☐	☐	☐	☐	☐
Determining the integration limits in two-dimensional distributions	☐	☐	☐	☐	☐
Determining whether given random variables are independent	☐	☐	☐	☐	☐
Working with conditional distributions	☐	☐	☐	☐	☐
Working with marginal distributions	☐	☐	☐	☐	☐
Calculating expected values of functions of random variables	☐	☐	☐	☐	☐
Finding moment generating functions	☐	☐	☐	☐	☐
Using moment generating functions to find moments	☐	☐	☐	☐	☐
Appplying the normal limit theorem	☐	☐	☐	☐	☐

C.4. How much difference do you think the following changes in the presentation of the contents would make in helping you understand the subject matter?

	Would help: a lot	a bit	not very much	not at all
There should be more examples	☐	☐	☐	☐
Each section should start with very elementary examples	☐	☐	☐	☐
There should be clearly stated strategies on how to apply the methods	☐	☐	☐	☐
Examples should be worked out in more detail	☐	☐	☐	☐

The following question tries to help us understand the strategies that you, the students, prefer to use in solving problems in this module. Please tick the options which apply to <u>you</u>, rather than the ones you think you should be choosing!

C.5. How would you rate the usefulness of the following strategies when you solve assignment problems? Use a scale of 1 (not useful) to 5 (extremely useful).

	1 Not useful	2	3	4	5 Extremely useful
Look for an example which is similar to the one I am trying to solve	☐	☐	☐	☐	☐
Read through all the examples	☐	☐	☐	☐	☐
Go through all the definitions and results of the section	☐	☐	☐	☐	☐
Try to solve the problem in two different ways to eliminate errors	☐	☐	☐	☐	☐

SECTION D: THE TEACHING OF THE MODULE

The assignments

D.1. How far do you agree/disagree with the following statements?

	Strongly agree	Agree	Unsure	Disagree	Strongly disagree
The lecturers' comments in my marked assignments were helpful	☐	☐	☐	☐	☐
The assignments involve much work	☐	☐	☐	☐	☐
I am not sure what lecturers expect from me in the assignments	☐	☐	☐	☐	☐
The assignments are marked fairly	☐	☐	☐	☐	☐
The standard expected of me in the assignments is high	☐	☐	☐	☐	☐
The lecturers' comments in my marked assignments were demotivating	☐	☐	☐	☐	☐

Teaching of the module

D.2. How much difference do you think the following changes in the way we teach the module would make in helping you pass the module:

	Would help: a lot	a bit	not very much	not at all
Detailed explanations on what I must be able to do to pass the exam	☐	☐	☐	☐
A workbook with more examples with their solutions	☐	☐	☐	☐
Regular feedback on how the other students taking this module are doing	☐	☐	☐	☐
Better availability of the lecturers	☐	☐	☐	☐
More detailed comments in the marked assignments	☐	☐	☐	☐
Old examination papers	☐	☐	☐	☐
Old examination papers with their solutions	☐	☐	☐	☐
More assignments	☐	☐	☐	☐
No fixed due dates for the assignments	☐	☐	☐	☐
Automatic examination admission (no compulsory assignment 1)	☐	☐	☐	☐
Getting my marked assignments back sooner	☐	☐	☐	☐
More tutorial letters	☐	☐	☐	☐
More hints on how to solve the questions in the assignments	☐	☐	☐	☐

SECTION E: THE STUDY GUIDE

E.1. Please assess the following aspects of the study guide:

	Very poor	Poor	Average	Good	Excellent
Appearance	☐	☐	☐	☐	☐
Clarity of sketches	☐	☐	☐	☐	☐
Logical ordering of the material	☐	☐	☐	☐	☐
Ease of finding things.	☐	☐	☐	☐	☐
Identification of key concepts and results	☐	☐	☐	☐	☐
Layout of text on pages (spacing, margins)	☐	☐	☐	☐	☐
Design of covers	☐	☐	☐	☐	☐
Page size	☐	☐	☐	☐	☐

E.2. How would you rate the level of difficulty of the language used in the study guide?

Very difficult to understand	☐
Difficult to understand	☐
Easy to understand	☐
Very easy to understand	☐

Finally, please comment on any aspect of STA1503 that you feel strongly about. We are particularly interested in your suggestions for improving the module!

………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………
………………………………………………………………………………………………

Thank you again for the time and trouble you have taken to answer this questionnaire!