**SECTION A: MULTIPLE CHOICE QUESTIONS**

**30 MARKS**

*There may be minor editorial differences between the questions in this memo and those on the final question paper. These do not affect the answers.*

**The correct option is shaded.**

1. Other things equal, for option-free bonds:

   1. a bond’s value is more sensitive to yield increases than to yield decreases.
   2. the value of a long-term bond is more sensitive to interest rate changes than the value of a short-term bond.
   3. the value of a low-coupon bond is less sensitive to interest rate changes than the value of a high-coupon bond.
   4. the duration of a zero-coupon bond rises as yields rise.

*Long-term, low-coupon bonds are more sensitive than short-term and high-coupon bonds. Prices are more sensitive to rate decreases than to rate increases (duration rises as yields fall).*

2. An investor has a 1-year, semiannual, 10% coupon bond which is priced at R 1,025. If the 6-month spot rate on a bond-equivalent basis is 8%, the 1-year theoretical spot rate as a BEY is:

   1. 6.4%.
   2. 7.3%.
   3. 8.0%.
   4. 9.6%

*A BEY of 8% is equivalent to a 6-month discount rate of 8/2 = 4%.

\[
1,025 = \left( \frac{50}{1.04} \right) + \frac{1,050}{(1 + r)^2}
\]

\[
(1 + r)^2 = \frac{1,050}{976.92} = 1.0748
\]

\[
r = (1.0748)^{0.5} - 1 = 0.0367 \text{ or } 3.67\% \text{ on a bond-equivalent basis}
\]

3. The 3-year annual spot rate is 7%, the 4-year annual spot rate is 7.5%, and the 5-year annual spot rate is 8%. Based on the pure expectations theory of interest rates, the 1-year implied forward rate in four years is closest to:

   1. 7%.
   2. 8%.
   3. 9%.
   4. 10%.

\[
4r_1 = \frac{(1 + R_4)^5}{(1 + R_4)^4} - 1 = \frac{(1.08)^5}{(1.075)^4} - 1 = \frac{1.47}{1.335} - 1 = 0.10
\]
4. A bond priced at par (R 1,000) has a modified duration of 8 and a convexity of 50. If interest rates fall 50 basis points, the new price will be:

1. R 875.00.
2. R 958.75.
3. R 1,041.25.
4. R 1,059.55.

\[
\Delta P \over P = -D_{mod} \Delta r + C(\Delta r)^2 = -8(-0.005) + 50(-0.005)^2 = 0.0400 + 0.00125 = +4.125\%
\]

The price would thus be 1000 \times 1.04125 = 1.041.25

5. Which of the following statements about embedded options is least likely correct?

1. An investor benefits when a floating rate bond has an interest rate floor.
2. The prepayment right granted with a mortgage favours the issuer/borrower.
3. If the issuer/borrower prepays, the holder of the bond has reinvestment risk.
4. If the market value of a putable bond falls below the par value, the issuer will likely exercise the option.

A put option may be exercised by the holder or buyer of the bond, not the issuer. The other statements are true. Interest rate floors benefit the holder by providing a lower limit (or minimum) on the interest rate the bondholder will receive. To understand why the prepayment right granted to a mortgage holder benefits the issuer, remember that the issuer here is the homeowner. The bank acts as a passthrough from the homeowner to the owner of a bond collateralised by the loan. The right to repay if interest rates decrease is a benefit to the homeowner. If the homeowner repays, the holder of the bond has reinvestment risk.

6. A R 1,000 par, semiannual-pay bond is trading for 89.14, has a coupon rate of 8.75%, and accrued interest of R 43.72. The clean price of the bond is:

1. R 847.69.
2. R 869.17.
4. R 935.12.

The clean price of the bond is the quoted price, 89.14% of par value, which is R 891.40.

7. Which of the following scenarios would be most beneficial to a domestic investor purchasing foreign bonds?

1. Appreciation of both the asset and the foreign currency.
2. Depreciation of both the asset and the foreign currency.
3. Appreciation of the asset and depreciation of the foreign currency.
4. Depreciation of the asset and appreciation of the foreign currency.

When the foreign currency appreciates, each foreign currency-denominated cash flow buys more domestic currency units – increasing the domestic currency return from the investment. The appreciation of the foreign asset benefits the investor as well.
8. Consider the following Treasury spot rates expressed as bond-equivalent yields:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
<td>3.0%</td>
</tr>
<tr>
<td>1 year</td>
<td>3.5%</td>
</tr>
<tr>
<td>1.5 years</td>
<td>4.0%</td>
</tr>
<tr>
<td>2 years</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

If a Treasury note with two years remaining to maturity has a 5% semiannual coupon and is priced at R 1,008. The Treasury note is:

1. overpriced.
2. underpriced.
3. correctly priced.
4. impossible to value with the given information.

The market value of the Treasury note is the present value of the remaining coupons plus the present value of the principal, discounted at the semiannual rates available from dividing each annual spot rate in the table by two.

\[
Value\ of\ T\text{-note} = \frac{25}{1.015} + \frac{25}{(1.0175)^2} + \frac{25}{(1.02)^3} + \frac{1,025}{(1.0225)^4} = 1,010.05
\]

At R 1,008, the T-note is priced below the present value of its cash flows (R 1,010) and is therefore underpriced.

9. An investor is considering the purchase of Security X, which matures in ten years and has a par value of R 1,000. During the first five years, X has a 6% coupon with quarterly payments. During the remaining five years, X has an 8% coupon with quarterly payments. The face value is paid at maturity. A second 10-year security, Security Z, has a 6% semiannual coupon and is selling at par. Assuming that X has the same (bond-equivalent) yield as Z, the price of Security X is closest to:

1. R 943.
2. R 1,036.
3. R 1,067.
4. R 1,074.

The bond equivalent yield rate on the par bond (Z) is 6% or a 3% semiannual rate. The equivalent quarterly rate, \(\sqrt[4]{1.03} - 1 = 0.014889\).

Security X makes 20 quarterly payments of R 15 and 20 quarterly payments of R 20. We need to use the cash flow function as follows:

\[
\begin{align*}
0 & \quad CFJ \\
15 & \quad CFJ \\
20 & \quad Nj \\
20 & \quad CFJ \\
19 & \quad Nj \\
1020 & \quad CFJ \\
1,4889 & \quad I/YR \\
\end{align*}
\]

Compute NPV

Note that the final CFj contains the final quarterly payment of R 20 along with the R 1,000 face value payment.
10. Consider a 25-year, R 1,000 par semiannual-pay bond with a 7.5% coupon and 9.25% YTM. Based on a yield change of 50 basis points, the effective duration of the bond is closest to:

1. 8.73.
2. 10.03.
3. 12.50.

Calculate the new bond prices at the 50 basis point change in rates both up or down and then plug into the effective duration equation:

Current price: N=50; FV=1000; PMT=(0.075/2)x1000=37.50; I/YR=4.625 → PV=830.54

+50 Basis Pts: N=50; FV=1000; PMT=(0.075/2)x1000=37.50; I/YR=4.875 → PV=790.59

- 50 Basis Pts: N=50; FV=1000; PMT=(0.075/2)x1000=37.50; I/YR=4.375 → PV=873.93

\[ D_{\text{effective}} = \frac{V_+ - V_-}{2V_0 \Delta y} = \frac{873.93 - 790.59}{2(830.54)(0.005)} = 10.03 \]

11. The minimum data required to calculate the implied forward rate for three years beginning three years from now are:

1. the 3-year and 6-year spot rates.
2. the 1-year, 2-year, and 3-year spot rates.
3. the 3-year, 4-year, 5-year and 6-year spot rates.
4. spot rates at 1-year intervals for the 6-year period.

If we want the 3-year forward rate in three years, the appropriate formula is

\[ 3f_3 = \left[ \frac{(1 + z_6)^6}{(1 + z_3)^3} \right]^{1/3} - 1 \]

\[ z_6 = 6\text{-year spot rate and } z_3 = 3\text{-year spot rate.} \]
12. Consider four bonds that are identical in all features except those shown in the following table:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Embedded Option</th>
<th>Amount Outstanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Call</td>
<td>R 20 million</td>
</tr>
<tr>
<td>B</td>
<td>Call</td>
<td>R 80 million</td>
</tr>
<tr>
<td>C</td>
<td>Put</td>
<td>R 20 million</td>
</tr>
<tr>
<td>D</td>
<td>Put</td>
<td>R 80 million</td>
</tr>
</tbody>
</table>

The bond most likely to have the largest spread to a comparable Treasury security is:

1. Bond A.
2. Bond B.
3. Bond C.
4. Bond D.

The call benefits the issuer, so a bondholder will require a higher yield on a callable bond, while the put option has value to the bondholder. Therefore a callable bond will have a higher yield spread to Treasuries than an otherwise identical putable bond. Other things equal, a bond with less liquidity will have a higher spread to Treasuries than a bond with more liquidity. Smaller issue size is associated with less liquidity.

13. A 3-year, 6% coupon, semiannual-pay note has a yield to maturity of 5.5%. If an investor holds this note to maturity and earns a 4.5% return on reinvested coupon income, his realized yield on the note is closest to:

1. 5.46%.
2. 5.50%.
3. 5.57%.
4. 5.68%.

This question does not require calculations. Because the return on reinvested coupon interest is less than the note’s yield to maturity, the investor’s realized yield on the note must be less than the YTM. Only choice 1 can be correct.

14. Treasury spot rates (expressed as semiannual-pay yields to maturity) are as follows:

6 months = 4%; 1 year = 5%; 1.5 years = 6%.

A 1.5-year, 4% Treasury note is trading at R 965. The arbitrage trade and arbitrage profit are:

1. sell the bond, buy the pieces, earn R 7.91 per bond.
2. buy the bond, sell the pieces, earn R 7.91 per bond.
3. sell the bond, buy the pieces, earn R 7.09 per bond.
4. buy the bond, sell the pieces, earn R 7.09 per bond.

\[
\text{Arbitrage-free value} = \frac{20}{1.02} + \frac{20}{1.02^2} + \frac{1020}{1.03^3} = 972.09
\]

Since the bond price (R 965) is less, buy the bond and sell the pieces for an arbitrage profit of R 7.09 per bond.
15. Given two bonds that are equivalent in all respects except tax status, the marginal tax rate that will make an investor indifferent between an 8.2% taxable bond and a 6.2% tax-exempt bond is closest to:

1. 24.39%.
2. 31.37%.
3. 37.04%.
4. 43.47%.

The tax rate that makes investors indifferent between two otherwise equivalent bonds is determined by solving for the tax rate in the equation: tax-exempt yield = (1 – tax rate) x taxable yield. Rearranging this equation, we have:

\[
\text{marginal tax rate} = 1 - \frac{\text{tax-exempt rate}}{\text{taxable rate}} = 1 - \frac{6.2}{8.2} = 24.39\%
\]
SECTION B: LONG QUESTIONS                    (20 MARKS)

Question 1 [11 marks]

a) How does the liquidity preference theory differ from the pure expectations theory? (2)

Prescribed Workbook pg 197; Chapter 4; Question 3:

The pure expectations theory asserts that the only factor affecting the shape of the yield curve is expectations about future interest rates. √ The liquidity preference theory asserts that there are two factors that affect the shape of the yield curve: expectations about future interest rates and a yield premium to compensate for interest rate risk. √

b) Explain the difference between a mortgage pass-through security and a collateralised mortgage obligation. Also explain why a collateralised mortgage obligation is created. (3)

Prescribed Workbook pg 193; Chapter 3; Question 10a & b:

In a mortgage pass-through security, the monthly cash flow from the underlying pool of mortgages is distributed on a pro rata basis to all the certificate holders. √ In contrast, for a collateralised mortgage obligation, there are rules for the distribution of the interest (net interest) and the principal (scheduled and prepaid) to different tranches. √

The rules for the distribution of interest and rules for the distribution of principal to the different tranches in a CMO structure effectively redistributes prepayment risk among the tranches. √
c) Suppose that a bond is purchased between coupon periods. The days between the settlement date and the next coupon period is 115. There are 183 days in the coupon. Suppose that the bond purchased has a coupon rate of 8.7% and there are 10 semi-annual coupon payments remaining.

(i) What is the dirty price for this bond if a 6.5% discount rate is used? (4)
(ii) What is the accrued interest for this bond? (1)
(iii) What is the clean price? (1)

\[
\begin{align*}
\text{(i) } w & \text{ must be calculated: } \\
& = \frac{\text{days between settlement date and next coupon payment date}}{\text{days in coupon period}} \\
& = \frac{115}{183} = 0.6284 \\

\text{The present value of each cash flow must then be calculated using the following formula:}

\[
PV_t = \frac{\text{expected cash flow}}{(1 + i)^{t-1+w}}
\]

<table>
<thead>
<tr>
<th>Period (t in the formula)</th>
<th>Cash Flow</th>
<th>PV, at 3.25% (using the above formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.35</td>
<td>4.2634</td>
</tr>
<tr>
<td>2</td>
<td>4.35</td>
<td>4.1292</td>
</tr>
<tr>
<td>3</td>
<td>4.35</td>
<td>3.9993</td>
</tr>
<tr>
<td>4</td>
<td>4.35</td>
<td>3.8734</td>
</tr>
<tr>
<td>5</td>
<td>4.35</td>
<td>3.7515</td>
</tr>
<tr>
<td>6</td>
<td>4.35</td>
<td>3.6334</td>
</tr>
<tr>
<td>7</td>
<td>4.35</td>
<td>3.5190</td>
</tr>
<tr>
<td>8</td>
<td>4.35</td>
<td>3.4082</td>
</tr>
<tr>
<td>9</td>
<td>4.35</td>
<td>3.3010</td>
</tr>
<tr>
<td>10</td>
<td>104.35</td>
<td>76.6926</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>110.5710</strong></td>
<td><strong>√√√√</strong></td>
</tr>
</tbody>
</table>

(ii) **Accrued Interest** = semiannual coupon payment \(\times (1 - w)\) = 4.35 \(\times (1 - 0.6284)\) = 1.6164 \(\sqrt{\text{√}}\)

(iii) **Clean Price** = dirty price - accrued interest = 110.5710 - 1.6164 = 108.9546 \(\sqrt{\text{√}}\)

Reference: Prescribed Textbook; Chapter 5; page 107
Question 2 [9 marks]

a) Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5%. You also observe a 2-year T-note with a 6% coupon trading at a yield to maturity of 5.5%. And, finally, you observe a 3-year T-note with a 7% coupon trading at a yield to maturity of 6.0%. Assume annual coupon payments. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

<table>
<thead>
<tr>
<th>1-year security</th>
<th>2-year security</th>
<th>3-year security</th>
<th>HP10bII Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>100</td>
<td>FV</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>7</td>
<td>PMT</td>
</tr>
<tr>
<td>5</td>
<td>5.5</td>
<td>6</td>
<td>I/YR</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>- 95.2381</td>
<td>- 100.9232</td>
<td>- 102.6730</td>
<td>PV</td>
</tr>
</tbody>
</table>

Cash flows associated with the three bonds:

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year</td>
<td>-95.2381</td>
<td>+100</td>
<td></td>
</tr>
<tr>
<td>2-year</td>
<td>-100.9232√</td>
<td>+6</td>
<td>+106</td>
</tr>
<tr>
<td>3-year</td>
<td>-102.6730√</td>
<td>+7</td>
<td>+7</td>
</tr>
</tbody>
</table>

To find the 2-year spot rate ($Z_2$): $100.9232 = \frac{6}{1.05} + \frac{106}{(1+Z_2)^2} \Rightarrow Z_2 = 5.51\% \sqrt{\sqrt{}}$

To find the 3-year spot rate ($Z_3$): $102.6730 = \frac{7}{1.05} + \frac{7}{1.0551^2} + \frac{107}{(1+Z_3)^3} \Rightarrow Z_3 = 6.05\% \sqrt{\sqrt{}}$

b) Explain the reinvestment risk and interest rate risk associated with the yield to maturity measure.

The reinvestment risk is that to realise the computed yield, it is necessary to reinvest the interim cash flows (i.e., coupon payment in the case of non amortising security and principal plus coupon in the case of an amortising security) at the YTM yield. $\sqrt{1/2}$

The price risk comes into play because it is assumed the security will be held to maturity date. If it is not, the yield no longer applies because there is risk of having to sell the security below its purchase price. $\sqrt{1/2}$

Work book page 37 + 209 (Q 8)

**Note that we found this question to have an error. Reinvestment risk is a type of interest rate risk. The questions should actually read:**

“Explain the reinvestment risk and price risk associated with the yield to maturity measure.”

During marking, students who discussed either reinvestment risk or interest rate risk received the full 3 marks.