UNIVERSITY EXAMINATIONS — UNIVERSITEITSEKSAMENS

MAT2611
LINEAR ALGEBRA
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Duration: 2 hours **Marks:** 100

ANSWER ALL QUESTIONS

QUESTION 1: 25 Marks

Let V be a vector space and $B = \{b_1, b_2, \ldots, b_n\} \subseteq V$.

(1.1) (a) B is linearly independent if for any scalars $c_1, c_2, \ldots, c_n$, the vector equation

$$c_1 b_1 + c_2 b_2 + \cdots + c_n b_n = 0,$$

has only the trivial solution i.e. we have $c_1 = c_2 = \cdots = c_n = 0$. (2)

(b) B *spans* V if every vector in V can be expressed as a linear combination of vectors from B i.e. for each $v \in V$ there are scalars $k_1, k_2, \ldots, k_n$ such that $v = k_1 b_1 + k_2 b_2 + \cdots + k_n b_n$. (2)

(1.2) V is finite-dimensional if it contains a finite set of vectors $\{v_1, v_2, \ldots, v_n\}$ that forms a basis for V. (1)

(1.3) B is a subspace of V if

(a) B is non-empty,

(b) B is closed under addition i.e. if $u, v \in B$ then $u + v \in B$ and

(c) B is closed under scalar multiplication i.e. if k is any scalar and $v \in B$ then $kv \in B$.

(3)

(1.4) (a) Let a, c and c be any scalars. Then

$$
\begin{aligned}
& ap(x) + bq(x) + cr(x) = 0 \qquad (*)\\
\Rightarrow\ & a(1-x) + b(5+3x-2x^2) + c(1+3x-x^2) = 0\\
\Rightarrow\ & (a+5b+c) + (-a+3b+3c)x + (-2b-c)x^2 = 0
\end{aligned}
$$

$$
\Rightarrow \begin{array}{rcrcrcl} a & + & 5b & + & c & = & 0\\ -a & + & 3b & + & 3c & = & 0\\ & & -2b & - & c & = & 0 \end{array}
$$

$$
\Rightarrow \begin{bmatrix} 1 & 5 & 1\\ -1 & 3 & 3\\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} a\\ b\\ c \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}.
$$

We may then solve the augmented matrix to get

$$
\begin{bmatrix} 1 & 5 & 1 & : & 0\\ -1 & 3 & 3 & : & 0\\ 0 & -2 & -1 & : & 0 \end{bmatrix} \underset{R_2+R_1}{\sim} \begin{bmatrix} 1 & 5 & 1 & : & 0\\ 0 & 8 & 4 & : & 0\\ 0 & -2 & -1 & : & 0 \end{bmatrix} \underset{\frac{1}{4}R_2}{\sim} \begin{bmatrix} 1 & 5 & 1 & : & 0\\ 0 & 2 & 1 & : & 0\\ 0 & -2 & -1 & : & 0 \end{bmatrix} \underset{R_3+R_2}{\sim} \begin{bmatrix} 1 & 5 & 1 & : & 0\\ 0 & 2 & 1 & : & 0\\ 0 & 0 & 0 & : & 0 \end{bmatrix}
$$

from which we obtain non-trivial solutions for a, b and c as

$$
\begin{bmatrix} a\\ b\\ c \end{bmatrix} = t \begin{bmatrix} -3\\ 1\\ -2 \end{bmatrix}, \quad t \in \mathbb{R}.
$$

Thus, the equation $(*)$ has non-trivial solutions.

Alternatively, we may use Theorem 2.3.8. Since

$$
\det \begin{bmatrix} 1 & 5 & 1\\ -1 & 3 & 3\\ 0 & -2 & -1 \end{bmatrix} = -(-2)\det \begin{bmatrix} 5 & 1\\ 3 & 3 \end{bmatrix} + (-1)\det \begin{bmatrix} 1 & 5\\ -1 & 3 \end{bmatrix} = 2(12) - 8 = 16 \neq 0,
$$

the equation $(*)$ has non-trivial solutions.

Hence, B is a linearly dependent subset of P_2. (6)

(b) A basis is a linearly independent spanning set. Since B is not linearly independent, B cannot be a basis for P_2. (1)

(1.5) (a) (i) Clearly $0_{\mathbb{R}^3} = (0,0,0) \in S$ since it is of the defining form describing membership in S (with $x = 0 \in \mathbb{R}$). Thus S is a nonempty subset of $\mathbb{R}^3$.

(ii) Let $u = (x, -x, 0)$ and $v = (y, -y, 0)$ be members of S. Then

$$
u + v = (x, -x, 0) + (y, -y, 0) = (x+y, -x-y, 0+0) = (x+y, -(x+y), 0).
$$

Since $x + y \in \mathbb{R}$, we see that $u + v \in S$. Thus S is closed under vector addition.

(iii) Let $k \in \mathbb{R}$ and $u = (x, -x, 0) \in S$. Then

$$ku = k(x, -x, 0) = (kx, k(-x), k\,0) = (kx, -kx, 0).$$

Since $kx \in \mathbb{R}$, we have that $ku \in S$. Thus S is closed under scalar multiplication.

Hence, S is a subspace of $\mathbb{R}^3$. (6)

(b) For any $u = (x, -x, 0) \in S$, $u = x(1, -1, 0)$. Thus, $\{(1, -1, 0)\}$ spans S. Of course, by Theorem 4.3.2(c), $\{(1, -1, 0)\}$ is linearly independent. Hence, $\{(1, -1, 0)\}$ is a basis for S. (2)

(c) The dimension of a vector space V is the number of vectors in a basis for V. Thus, $\dim(S) = 1$ since the basis for S consists of just one vector as shown in (b). (2)

[25]

QUESTION 2: 25 Marks

(2.1) (a) The nullspace of A is the solution space of the homogeneous system $A\mathbf{x} = 0$ i.e.

$$\text{Nullspace}(A) = \{\mathbf{x} \in \mathbb{R}^6 \,:\, A\mathbf{x} = 0.\}.$$

For $\mathbf{x} \in \text{Nullspace}(A)$, the augmented matrix for the system $A\mathbf{x} = 0$ is solved accordingly.

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & : & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & : & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & : & 0 \\ 2 & 6 & 0 & 8 & 4 & 18 & : & 0 \end{bmatrix} \begin{matrix} \sim \\ R_2 - 2R_1 \\ \frac{1}{5}R_3 \\ R_4 - 2R_1 \end{matrix} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & : & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & : & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & : & 0 \\ 0 & 0 & 4 & 8 & 0 & 18 & : & 0 \end{bmatrix} \begin{matrix} \\ \sim \\ R_3 + R_2 \\ R_4 + 4R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & : & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & : & 0 \end{bmatrix} \begin{matrix} \\ -R_2 \\ \sim \\ \frac{1}{6}R_4 \end{matrix} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & : & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & : & 0 \end{bmatrix} \begin{matrix} \\ R_2 - 3R_4 \\ \sim \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & : & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & : & 0 \end{bmatrix} \begin{matrix} R_1 + 2R_2 \\ \sim \\ R_3 \leftrightarrow R_4 \\ \\ \end{matrix} \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & : & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & : & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}.$$

From the above we get

$$\begin{aligned} x_1 &= -3r - 4s - 2t \\ x_2 &= r \\ x_3 &= -2s \\ x_4 &= s \\ x_5 &= t \\ x_6 &= 0. \end{aligned}$$

Thus

$$\mathbf{x} = r\begin{bmatrix}-3\\1\\0\\0\\0\\0\end{bmatrix} + s\begin{bmatrix}-4\\0\\-2\\1\\0\\0\end{bmatrix} + t\begin{bmatrix}-2\\0\\0\\0\\1\\0\end{bmatrix}$$

so that a basis for the nullspace of A is

$$\left\{\begin{bmatrix}-3\\1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-4\\0\\-2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-2\\0\\0\\0\\1\\0\end{bmatrix}\right\}.$$

(10)

(b) From (a), nullity$(A) = 3$. Since A is a 4×6 matrix ($n = 6$), by the Dimension Theorem for Matrices (Theorem 4.8.2),

$$\text{rank}(A) + \text{nullity}(A) = 6$$

so that rank$(A) = 3$. (3)

(2.2) Let $u_1 = \begin{bmatrix}1\\-3\end{bmatrix}$ and $u_2 = \begin{bmatrix}2\\2\end{bmatrix}$. Then $\|u_1\| = \sqrt{10}$. By the Gram-Schmidt process (including normalization):

Step 1: Let $\mathbf{v_1} = u_1 = \begin{bmatrix}1\\-3\end{bmatrix}$ and $\|\mathbf{v_1}\| = \sqrt{10}$.

Step 2: Let

$$\begin{aligned}
\mathbf{v_2} = \mathbf{u_2} - \text{proj}_{\mathbf{v_1}}\mathbf{u_2} &= \mathbf{u_2} - \frac{\langle \mathbf{u_2}, \mathbf{v_1}\rangle}{\|\mathbf{v_1}\|^2}\mathbf{v_1} \\
&= \mathbf{u_2} - \frac{\mathbf{u_2}\cdot\mathbf{v_1}}{\|\mathbf{v_1}\|^2}\mathbf{v_1} \\
&= \mathbf{u_2} + \frac{2}{5}\mathbf{v_1} \\
&= \begin{bmatrix}2\\2\end{bmatrix} + \frac{2}{5}\begin{bmatrix}1\\-3\end{bmatrix} \\
\therefore \mathbf{v_2} &= \begin{bmatrix}\frac{12}{5}\\\frac{4}{5}\end{bmatrix}
\end{aligned}$$

Then $\{\mathbf{q_1}, \mathbf{q_2}\}$ is an orthonormal basis where

$$\mathbf{q_1} = \frac{\mathbf{v_1}}{\|\mathbf{v_1}\|} = \frac{1}{\sqrt{10}}\begin{bmatrix}1\\-3\end{bmatrix} \quad \text{and} \quad \mathbf{q_2} = \frac{\mathbf{v_2}}{\|\mathbf{v_2}\|} = \frac{1}{\sqrt{10}}\begin{bmatrix}3\\1\end{bmatrix}.$$

(8)

(2.3) P is orthogonal if and only if $PP^T = I$. Since

$$PP^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 1 \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 0 & \frac{3}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix} \neq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

P is not orthogonal. (4)

[25]

QUESTION 3: 22 Marks

(3.1) (a) The *algebraic multiplicity* of λ is the number of factors $x - \lambda$ in $\det(\lambda I - A)$.(2)

(b) The *geometric multiplicity* of λ is the dimension of the eigenspace associated with λ i.e. $\dim(E_\lambda)$. (2)

(3.2) The geometric multiplicity. (1)

(3.3)

$$\begin{aligned} \lambda = 0 \text{ is an eigenvlaue of } A &\Leftrightarrow \det(0 \cdot I - A) = 0 \\ &\Leftrightarrow (-1)^n \det(A) = 0 \\ &\Leftrightarrow \det(A) = 0 \\ &\Leftrightarrow A \text{ is not invertible.} \end{aligned}$$

(6)

(3.4) We find the eigenvalues of $A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$:

$$\begin{aligned} \det(A - \lambda I) = 0 &\Rightarrow \det \begin{bmatrix} 3-\lambda & -1 \\ 2 & -\lambda \end{bmatrix} = 0 \\ &\Rightarrow (-\lambda)(3-\lambda) - (-1)(2) = 0 \\ &\Rightarrow \lambda^2 - 3\lambda + 2 = 0 \\ &\Rightarrow (\lambda - 1)(\lambda - 2) = 0 \\ &\Rightarrow \lambda = 1 \quad \text{or} \quad \lambda = 2. \end{aligned}$$

The eigenvalues are then $\lambda_1 = 1$ and $\lambda_2 = 2$.

We next find the bases for the eigenspaces E_{λ_1} and E_{λ_2} corresponding to the eigenvalues λ_1 and λ_2.

$\underline{E_{\lambda_1}}$: We solve the associated homogeneous system

$$(A - \lambda_1 I)\mathbf{x} = \mathbf{0}$$

for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and get the augmented system

$$\begin{bmatrix} 2 & -1 & : & 0 \\ 2 & -1 & : & 0 \end{bmatrix} \underset{R_2 - R_1}{\sim} \begin{bmatrix} 2 & -1 & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}$$

This gives

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

so that

$$E_{\lambda_1} = \left\{ x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} : x_1 \in \mathbb{R} \right\} = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

A basis for E_{λ_1} is thus $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$.

$\underline{E_{\lambda_2}}$: We solve the associated homogeneous system

$$(A - \lambda_2 I)\mathbf{x} = \mathbf{0}$$

for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and get the augmented system

$$\begin{bmatrix} 1 & -1 & : & 0 \\ 2 & -2 & : & 0 \end{bmatrix} \underset{R_2 - 2R_1}{\sim} \begin{bmatrix} 1 & -1 & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}$$

This gives

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

so that

$$E_{\lambda_2} = \left\{ x_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} : x_1 \in \mathbb{R} \right\} = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

A basis for E_{λ_2} is thus $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

Then $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ diagonalizes A and the diagonal matrix is $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. (11)

[22]

QUESTION 4: 12 Marks

(4.1) Let $p(x) = a + bx, q(x) = c + dx \in P_1$ and $k \in \mathbb{R}$. Then

$$\begin{aligned} T[p(x)+q(x)] &= T[(a+c)+(b+d)x] \\ &= A\begin{bmatrix} a+c \\ b+d \end{bmatrix} \\ &= A\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) && \text{...matrix addition} \\ &= A\begin{bmatrix} a \\ b \end{bmatrix} + A\begin{bmatrix} c \\ d \end{bmatrix} && \text{...left matrix multiplication} \\ &= T(p(x)) + T(q(x)) \end{aligned}$$

and

$$\begin{aligned} T[kp(x)] &= T[(ka)+(kb)x] \\ &= A\begin{bmatrix} ka \\ kb \end{bmatrix} \\ &= A\left(k\begin{bmatrix} a \\ b \end{bmatrix}\right) \\ &= k\left(A\begin{bmatrix} a \\ b \end{bmatrix}\right) \\ &= k\,T(p(x)). \end{aligned}$$

Hence, T is a linear transformation. (5)

(4.2) (a) We apply Theorem 8.2.2.

$$\begin{aligned} & p(x) = a + b\,x \in \ker(T) \\ \Leftrightarrow\ & T(p(x)) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \Leftrightarrow\ & \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \Leftrightarrow\ & \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} && \text{...by Theorem 2.3.8 since } \det\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = 1 \neq 0 \\ \Leftrightarrow\ & a = b = 0 \\ \Leftrightarrow\ & p(x) = 0_{P_1}. \end{aligned}$$

Thus $\ker(T) = \{0_{P_1}\}$ so that by Theorem 8.2.2, T is one-to-one. (3)

(b)

$$
\begin{aligned}
p(x) = a + bx \in T^{-1}\left(\begin{bmatrix}2\\3\end{bmatrix}\right) &\Leftrightarrow T(p(x)) = \begin{bmatrix}2\\3\end{bmatrix}\\
&\Leftrightarrow \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}\\
&\Leftrightarrow \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}^{-1}\begin{bmatrix}2\\3\end{bmatrix} \qquad \because \det(A) = 1 \neq 0\\
&\Leftrightarrow \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix} \qquad \ldots \text{Theorem 1.4.5}\\
&\Leftrightarrow \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}\\
&\Leftrightarrow p(x) = 1 + x.
\end{aligned}
$$

Hence $T^{-1}\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = \{1 + x\}$. (4)

[12]

QUESTION 5: 16 Marks

(5.1) Let $P = P_{B' \leftarrow B}$. Then $P_{B \leftarrow B'} = P^{-1}$. Now,

$$
\left[\begin{array}{ccc:ccc}1 & -1 & 0 & 1 & 0 & 0\\0 & 1 & -2 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right] \underset{\sim}{R_1 + R_2} \left[\begin{array}{ccc:ccc}1 & 0 & -2 & 1 & 1 & 0\\0 & 1 & -2 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right] \begin{array}{c}R_1 + 2R_3\\R_2 + 2R_3\\\sim\end{array} \left[\begin{array}{ccc:ccc}1 & 0 & 0 & 1 & 1 & 2\\0 & 1 & 0 & 0 & 1 & 2\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]
$$

so that $P_{B \leftarrow B'} = P^{-1} = \begin{bmatrix}1 & 1 & 2\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}$. (3)

(5.2) Since $P_{B' \leftarrow B}[p(x)]_B = [p(x)]_{B'}$ and $[p(x)]_B = \begin{bmatrix}a+b\\0\\c\end{bmatrix}$ we have that

$$
[p(x)]_{B'} = \begin{bmatrix}1 & -1 & 0\\0 & 1 & -2\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}a+b\\0\\c\end{bmatrix} = \begin{bmatrix}a+b\\-2c\\c\end{bmatrix}
$$

so that $p(x) = (a + b) + (-2c)(1 + x) + c(1 + x)^2$. (3)

(5.3) Since $[T]_{B'} = P^{-1}[T]_B P$ with $P = P_{B' \leftarrow B}$ and $P^{-1} = P_{B \leftarrow B'}$ we have that

$$\begin{aligned}
[T]_B &= P[T]_{B'}P^{-1} \\
&= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4 & 3 & 3 \\ -4 & -3 & -5 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 4 & 7 & 17 \\ -4 & -7 & -19 \\ 1 & 2 & 6 \end{bmatrix}
\end{aligned}$$

(6)

(5.4) Since $[T]_B[p(x)]_B = [T(p(x))]_B$, for $p(x) = a + bx + cx^2$,

$$\begin{aligned}
[T(a + bx + cx^2)]_B &= [T]_B[p(x)]_B \\
&= \begin{bmatrix} 4 & 7 & 17 \\ -4 & -7 & -19 \\ 1 & 2 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\
&= \begin{bmatrix} 4a + 7b + 17c \\ -4a - 7b - 19c \\ a + 2b + 6c \end{bmatrix}
\end{aligned}$$

(3)

(5.5) $T(a + bx + cx^2) = (4a + 7b + 17) + (-4a - 7b - 19c)x + (a + 2b + 6c)x^2$. (1)

[16]

[Total: 100 Marks]