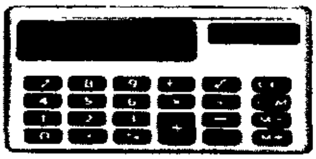
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT2615

May/June 2013

CALCULUS IN HIGHER DIMENSIONS

Duration 2 Hours

100 Marks

EXAMINERS :
FIRST DR JE SINGLETON
SECOND PROF I NAIDOO

Use of a non-programmable pocket calculator is permissible

Closed book examination.

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This paper consists of 4 pages

Show **ALL** calculations

Try to answer **ALL** the questions

1 Let L be the line in $\mathbb{R}^3$ that passes through the points $(1, -1, -2)$ and $(2, -1, 1)$ Let V_1 be the plane defined by

$$x + y - 3z + 6 = 0$$

(a) Find an equation for L (3)

(b) Find an equation for the plane V_2 that contains the line L and is perpendicular to V_1 (7)

[10]

2 Consider the $\mathbb{R}^2 - \mathbb{R}$ function f defined by

$$f(x, y) = \frac{y^2}{x^2 - y^2}$$

(a) Write down the largest possible domain D_f of f (2)

(b) Show that

$$\lim_{(x,y)\to(0,0)} f(x, y)$$

does not exist (7)

[TURN OVER]

(c) Is f continuous at $(x, y) = (0, 0)$? Give a reason for your answer (2)

[11]

3 Consider the $\mathbb{R}^2$–$\mathbb{R}$ function f given by

$$f(x, y) = x^2 - 2xy + y^2$$

Let V be the tangent plane to the graph of f at the point $(1, -1, 4)$

(a) Calculate the rate of change in f at $(1, -1)$ in the direction of the vector $(3, 4)$ (5)

(b) Calculate the maximum rate of change in f at $(1, -1)$ (2)

(c) Find a vector perpendicular to the plane V (4)

(d) Find an equation for the plane V (2)

[13]

4 (a) State the Chain Rule for determining the derivative of the function $f \circ \underline{r}$, if $\underline{r}$ is an $\mathbb{R} - \mathbb{R}^2$ function and f is an $\mathbb{R}^2 - \mathbb{R}$ function (2)

(b) Consider the $\mathbb{R}^2 - \mathbb{R}$ function f defined by

$$f(x, y) = \ln xy$$

and the $\mathbb{R} - \mathbb{R}^2$ function $\underline{r}$ defined by

$$\underline{r}(t) = \left(2\sqrt{t},\ t^2\right)$$

Use the Chain Rule that you stated in (a) to calculate $\left(f \circ \underline{r}\right)'(1)$ (5)

[7]

5 Use the method of Lagrange to set up a system of equations whose solution will give the answer to the following problem

Determine the radius and the height of an open cylindrical tank that will have maximum volume subject to the constraint that the area of its outer surface (bottom included but top excluded) must be 100 m^3

Let f denote the function to be maximized, and define all symbols used

DO NOT SOLVE THE EQUATIONS **[8]**

[TURN OVER]

6 Consider the integral

$$I = \int_0^2 \int_{-x}^{\sqrt{4-(x-2)^2}} 1 \, dydx$$

(a) Sketch the region of integration (3)

(b) Write the integral I as a sum of integrals with reversed order of integration (6)

DO NOT EVALUATE THE INTEGRALS. **[9]**

7 Let D be the region in $\mathbb{R}^3$ that is bounded below by the surface $z = \sqrt{x^2 + y^2}$ and bounded above by the surface $z = 4$

(a) Sketch the region D and show its XY–projection on your sketch (3)

(b) Express the volume of D by means of

(i) a **double** integral and **polar** coordinates (5)

(ii) a **triple** integral and **spherical** coordinates (5)

DO NOT EVALUATE THE INTEGRALS. **[13]**

8 (a) State Green's Theorem for converting a line integral over a closed curve in the plane to a double integral over the region enclosed by the curve (4)

(b) Let C be the curve in $\mathbb{R}^2$ with parametric equation

$$(x, y) = (3\cos t,\ 3\sin t),\ t \in [0, 2\pi]$$

Evaluate the integral

$$\oint_C y dx - x dy$$

by

(i) converting the given line integral to an ordinary single integral (5)

(ii) using Green's Theorem (5)

[14]

[TURN OVER]

9 Consider the surface

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid z = 6 - x^2 - y^2 \text{ and } z \geq 1\}$$

Assume that S is oriented upward

Consider the flux integral

$$I = \iint_S \underline{F} \cdot \underline{n} dS,$$

where

$$\underline{F}(x, y, z) = (y, -x, 1)$$

(a) Sketch the surface S (3)

(b) Explain why Gauss' Theorem cannot be used to evaluate the integral I (2)

(c) Evaluate the integral I (10)

[15]

TOTAL: 100 marks