UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT3706

October/November 2016

ORDINARY DIFFERENTIAL EQUATIONS

Duration 2 Hours

110 Marks

EXAMINERS

FIRST DR PP GHOSH

SECOND PROF Y HARDY

Use of a non-programmable pocket calculator is permissible.

Closed book examination.

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This paper consists of 3 pages

Instructions

- The notation is the same as in the Study Guide

[TURN OVER]

Question 1.

Mark Distribution for this question $8 + 22 = 30$ *marks*

(a) Determine whether the system

$$(D+3)[x] + (D+3)[y] = e^{2t}$$
$$(D-2)[x] + (D-2)[y] = e^{-3t}$$

is degenerate In case it is degenerate decide whether it has no solution or infinitely many solutions If it has no solutions, explain why, else find the general form of the solutions

(b) Reduce the system

$$(D^2+1)[x] - 2D[y] = 2t$$
$$(2D-1)[x] + (D-2)[y] = 7$$

to an equivalent triangular system of the form

$$P_1(D)[y] = f_1(t)$$
$$P_2(D)[x] + P_3(D)[y] = f_2(t)$$

and hence **determine the solution.**

Question 2

Mark Distribution for this question $2 + 12 + 16 = 30$ *marks*

(a) Define the concept of a *root vector of order* k, $k \geq 1$, of a matrix $\mathbf{A}$

(b) Let k be a positive integer Use induction to prove that if $(\mathbf{A} - \lambda\mathbf{I})^k\mathbf{u} = \mathbf{0}$ then $\mathbf{u}$ is an eigenvector which corresponds to the *non-repeated* eigenvalue λ of $\mathbf{A}$

(c) Use the eigenvalue-eigenvector method to solve the initial value problem

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix} \mathbf{X}, \quad \mathbf{X}(0) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Question 3.

Mark Distribution for this question $6 + 19 = 25$ *marks*

(a) Prove if $\boldsymbol{\Phi}$ is a normalized fundamental matrix of $\boldsymbol{X} = \boldsymbol{AX}$ at $t_0 = 0$ and $\boldsymbol{F}$ is continuous in t then

$$\boldsymbol{X}_p(t) = \int_0^t \boldsymbol{\Phi}(t-s)\boldsymbol{F}(s)\,\mathrm{d}\,s$$

is a particular solution of $\boldsymbol{X} = \boldsymbol{AX} + \boldsymbol{F}$

(b) Use the result in 3((a)) to find a general solution of

$$\boldsymbol{X} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} \cos t \\ 1 \end{bmatrix}$$

You may use the trigonometric identities:

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

and

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

Question 4.

Mark Distribution for this question 15 *marks*

Use the *power series method* to find a series solution for

$$\boldsymbol{X} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{X}, \quad \boldsymbol{X}(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Write your final answer in terms of the appropriate exponential function

Question 5.

Mark Distribution for this question 10 *marks*

Determine and classify the critical point(s) in the second and third quadrants of the system

$$x = x^2 + y^2 - 5$$
$$y = x^2 + 2y^2 - 9$$

TOTAL: [110]