UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

MAT3706

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ORDINARY DIFFERENTIAL EQUATIONS

Duration 2 Hours

100 Marks

EXAMINERS

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Closed book examination.

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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS

ALL CALCULATIONS MUST BE SHOWN

The notation is the same as in the Study Guide

QUESTION 1

(a) Determine whether the system

$$\begin{aligned}(D-1)[x]+(D-1)[y]&=e^{-2t},\\ (D+2)[x]+(D+2)[y]&=e^{t},\end{aligned}$$

is degenerate In the degenerate case, decide whether it has no solution or infinitely many solutions If it has no solution, explain why, else find the general form of the solutions **(8)**

(b) Reduce the system

$$\begin{aligned}(D^2+1)[x]+(4D-4)[y]&=4e^{t},\\ (D-1)[x]+(D+9)[y]&=0,\end{aligned}$$

to an equivalent *triangular system* of the form

$$\begin{aligned}P_1(D)[y]&=f_1(t),\\ P_2(D)[x]+\quad P_3(D)[y]&=f_2(t),\end{aligned}$$

and DETERMINE THE SOLUTION **(20)**

[28]

QUESTION 2

(a) Define the concept *root vector of order* $k, k \geq 1$ of a matrix $\mathbf{A}$ **(2)**

(b) Let k be a positive integer Use induction to prove that if $(\mathbf{A}-\lambda\mathbf{I})^{\mathbf{k}}\mathbf{u}=\mathbf{0}$ then $\mathbf{u}$ is an eigenvector which corresponds to the (non-repeated) eigenvalue λ of $\mathbf{A}$ **(10)**

(c) Use the eigenvalue-eigenvectors to solve the initial value problem

$$\dot{\mathbf{X}}=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}\mathbf{X},\quad \mathbf{X}(0)=\begin{bmatrix}0\\ -4\end{bmatrix}$$

(16)

[28]

QUESTION 3

(a) If $\mathbf{F}(t)$ is continuous and $\mathbf{\Phi}$ is a fundamental matrix of $\dot{\mathbf{X}}=\mathbf{AX}$, then show that a particular solution of the non-homogeneous system $\mathbf{X}=\mathbf{AX}+\mathbf{F}(t)$ is given by **(8)**

$$\mathbf{X}_p(t)=\mathbf{\Phi}(t)\int_{t_0}^{t}\mathbf{\Phi}^{-1}(s)\,\mathbf{F}(s)\,ds$$

(b) Use the result in 3(a) with $t_0=0$ to find a *general* real solution of **(12)**

$$\dot{\mathbf{X}}=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\mathbf{X}+2\begin{bmatrix}\cos t\\ \sin t\end{bmatrix}e^{2t}$$

[20]

QUESTION 4

Write the equation below as a system of two first order differential equations and then find the series solution using the *power series method*

$$x + 2x = 0, \qquad x(0) = 1, \quad x(0) = -2$$

Write your final answer in terms of the appropriate exponential functions

[14]

QUESTION 5

Classify the critical points of the plane autonomous system corresponding to the second order nonlinear differential equation

$$x + x = \alpha x^3,$$

where α is a positive real-valued constant

[10]

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TOTAL MARKS: [100]