UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

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MAT3706

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Ordinary Differential Equations

Duration 2 Hours

100 Marks

EXAMINERS
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Closed book examination

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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS
ALL CALCULATIONS MUST BE SHOWN
The notation is the same as in the Study Guide
Answer **ALL** the questions

[TURN OVER]

QUESTION 1

(a) Determine whether the system

$$x + x - y - y = e^{\frac{t}{2}},$$
$$2x - x - 2y + y = 5e^{-t}$$

is degenerate In the degenerate case, decide whether it has no solution or infinitely many solutions If it has no solution, explain why, else find the general form of the solutions **(8)**

(b) Reduce the system

$$(D-2)[x] + 2D[y] = 2 - 4e^{2t}$$
$$(2D-3)[x] + (3D-1)[y] = 0$$

to an equivalent *triangular system* of the form **(10)**

$$P_1(D)[x] = f_1(t),$$
$$P_2(D)[x] + P_3(D)[y] = f_2(t),$$

and DETERMINE THE SOLUTION

(c) Solve the system

$$(D^2+1)[x] - 2D[y] = 2t$$
$$(2D-1)[x] - (2-D)[y] = 5$$

by using the elimination method (operator method) **(10)**
Hint Eliminate y first

[28]

QUESTION 2

(a) Use the eigenvalue-eigenvectors to solve the initial value problem **(10)**

$$\dot{\mathbf{X}} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \mathbf{X}, \qquad \mathbf{X}(0) = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

(b) Define the concept *root vector of order* k, $(k \geq 1)$ of a matrix $\mathbf{A}$ **(2)**

(c) Use root vector to solve the initial value problem

$$\mathbf{X} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \mathbf{X}, \qquad \mathbf{X}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

(14)

[26]

QUESTION 3

(a) If Ψ is a normalized fundamental matrix of $\mathbf{X} = \mathbf{A}\mathbf{X}$ at $t_0 = 0$ and $\mathbf{F}(t)$ is continuous in t then show that

$$\mathbf{X}_p(t) = \int_{t_0}^{t} \Psi\,(t-s)\,\mathbf{F}\,(s)\,ds$$

is a particular solution of $\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{F}(t)$ **(6)**

(b) Use the result in 3(a) with $t_0 = 0$ to find a *general* real solution of **(14)**

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \mathbf{X} + e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

[20]

QUESTION 4

(a) Write the companion system for the equation given below **(4)**

$$y^{(3)} - y \sin t = f(t)$$

(b) Write the equation below as a system of two first order differential equations and then find the series solution using the ***power series method***

$$x - 2x + x = 0, \qquad x(0) = 1, \quad x(0) = 1$$

Write your final answer in terms of the appropriate exponential functions **(10)**

[14]

QUESTION 5

Classify the critical points of the plane autonomous system corresponding to the second order nonlinear differential equation

$$x + \alpha\left(\frac{x^3}{3} - x\right) + x = 0$$

in terms of α where α is a real-valued constant

[12]

First examiner Prof H Jafari
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TOTAL MARKS: [100]