Mechanics PHY1505
Oct/Nov 2015 Exam
Memorandum

Question 1

(a) Rearrange the equation $a=\frac{dv}{dt}$ to the form $a\cdot dt=dv$ and integrate to obtain $\int^{t} a\cdot dt=\int_{v_0}^{v} dv$. If a is constant in time, then $at=v-v_0$ or $v=v_0+at$. (7)

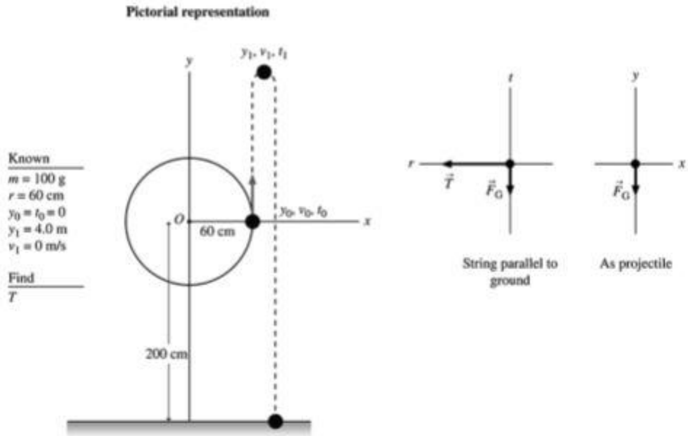
(b) Rearrange the equation $a=\frac{dv}{dt}$ to the form $a=v\frac{dv}{dx}$ or $a\cdot dx=v\cdot dv$ and integrate to obtain $\int^{x} a\cdot dx=\int_{v_0}^{v} v\cdot dv$. If a does not depend on x, then $a\cdot x=\frac{1}{2}(v^2-v_0^2)$ or $v^2=v_0^2+2ax$.

(8)
[15]

Question 2

(a) For every action there is an equal opposite reaction. OR
If object A exerts a force $\vec{F}$ on object B then object B will exert a force $-\vec{F}$ on object A. (5)

(b) Model the ball as a particle undergoing circular motion in a vertical circle.
Visualize:



Solve: Initially, the ball is moving in circular motion. Once the string breaks, it becomes a projectile. The final circular-motion velocity is the initial velocity for the projectile, which we can find by using the kinematic equation

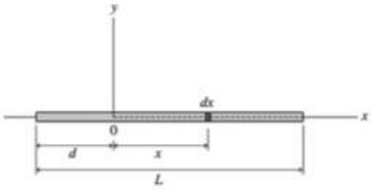
$$v_1^2=v_0^2+2a_y(y_1-y_0)\Rightarrow 0\ \text{m}^2/\text{s}^2=(v_0)^2+2(-9.8\ \text{m/s}^2)(4.0\ \text{m}-0\ \text{m})\Rightarrow v_0=8.85\ \text{m/s}$$

This is the speed of the ball as the string broke. The tension in the string at that instant can be found by using the r-component of the net force on the ball:

$$\sum F_r=T=m\left(\frac{v_{0y}^2}{r}\right)\Rightarrow T=(0.100\ \text{kg})\frac{(8.85\ \text{m/s})^2}{0.60\ \text{m}}=13.1\ \text{N}$$ (15)

[20]

Question 3
Visualize:



We chose the origin of the coordinate system to be on the axis of rotation, that is, at a distance d from one end of the rod.

(a) The moment of inertia can be calculated as follows:

$$I = \int x^2\, dm \quad \text{and} \quad \frac{dm}{M} = \frac{dx}{L} \Rightarrow dm = \frac{M}{L} dx$$

$$\Rightarrow I = \frac{M}{L}\int_{-d}^{L-d} x^2 dx = \left(\frac{M}{L}\right)\frac{x^3}{3}\bigg|_{-d}^{L-d} = \left(\frac{1}{3}\right)\left(\frac{M}{L}\right)[(L-d)^3 - (-d)^3] = \frac{M}{3L}[(L-d)^3 + d^3] \qquad (10)$$

(b) For $d = \frac{1}{2}L$,

$$I = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 + \left(\frac{L}{2}\right)^3\right] = \frac{1}{12}ML^2 \qquad (5)$$

[15]

Question 4

(a) Linear momentum and kinetic energy (6)

(b) In the case of a perfectly elastic collision, the two velocities $(v_f)_A$ and $(v_f)_B$ can be determined by combining the conservation equations of momentum and mechanical energy.

Momentum conservation $\quad m_A(v_i)_A + m_B(v_i)_B = m_A(v_f)_A + m_B(v_f)_B$

Energy conservation $\quad \frac{1}{2}m_A(v_i)_A^2 + \frac{1}{2}m_B(v_i)_B^2 = \frac{1}{2}m_A(v_f)_A^2 + \frac{1}{2}m_B(v_f)_B^2$

(i) Because $(v_i)_B = 0\, m/s$ we have $m_A(v_i)_A = m_A(v_f)_A + m_B(v_f)_B$ from conservation of momentum. Therefore, $(v_f)_A = (v_i)_A - \frac{m_B}{m_A}(v_f)_B$. Substitute this in the energy equation to obtain

$m_A(v_i)_A^2 = m_A[(v_i)_A - \frac{m_B}{m_A}(v_f)_B]^2 + m_B(v_f)_B^2$, which leads to $(v_f)_B[(1+\frac{m_B}{m_A})(v_f)_B - 2(v_i)_A] = 0$.

Therefore, $(v_f)_B = \frac{2m_A}{m_A + m_B}(v_i)_A$. (8)

(ii) From the equation $(v_f)_A = (v_i)_A - \frac{m_B}{m_A}(v_f)_B$ we obtain $(v_f)_A = (v_i)_A - \frac{m_B}{m_A}[\frac{2m_A}{m_A+m_B}(v_i)_A]$ we is solved to give $(v_f)_B = \frac{m_A - m_B}{m_A + m_B}(v_i)_A$. (8)

[20]

Question 5

(a) $F_G = G\frac{m_1 m_2}{r^2}$

where F_G = gravitational force

G = Universal gravitational constant

m_1 = mass of object 1

m_2 = mass of object 2

r = distance between the two objects (5)

(b) We are given $M_1 + M_2 = 150\ \text{kg}$ which means $M_1 = 150\ \text{kg} - M_2$. We also have

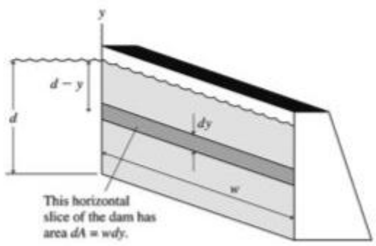
$$\frac{GM_1M_2}{(0.20\ \text{m})^2} = 8.00\times10^{-6}\ \text{N} \Rightarrow M_1M_2 = \frac{(8.00\times10^{-6}\ \text{N})(0.20\ \text{m})^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 4798\ \text{kg}^2$$

Thus, $(150\ \text{kg} - M_2)M_2 = 4798\ \text{kg}^2$ or $M_2^2 - (150\ \text{kg})M_2 + (4798\ \text{kg}^2) = 0$. Solving this equation gives $M_2 = 103.75\ \text{kg}$ and $46.25\ \text{kg}$. The two masses are 104 kg and 46 kg. (10)

[15]

Question 6

Visualize:



The figure shows a dam with water height *d*. We chose a coordinate system with the origin at the bottom of the dam. The horizontal slice has height *dy*, width *w*, and area $dA = wdy$. The slice is at the depth of $d - y$.

(a) The water exerts a small force $dF = pdA = pwdy$ on this small piece of the dam, where $p = \rho g(d-y)$ is the pressure at depth $d - y$. Altogether, the force on this small horizontal slice at position y is $dF = \rho gw(d-y)dy$. Note that a force from atmospheric pressure is not included. This is because atmospheric pressure exerts a force on *both* sides of the dam. The total force *of the water* on the dam is found by adding up all the small forces *dF* for the small slices *dy* between $y = 0\ \text{m}$ and $y = d$. This summation is expressed as the integral

$$F_{\text{total}} = \int_{\text{all slices}} dF = \int_0^d dF = \rho gw\int_0^d (d-y)\,dy = \rho gw\left(yd - \frac{1}{2}y^2\right)\Big|_0^d = \frac{1}{2}\rho gwd^2 \quad (6)$$

(b) The total force is

$$F_{\text{total}} = \tfrac{1}{2}(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(100\ \text{m})(60\ \text{m})^2 = 1.76\times10^9\ \text{N} \quad (5)$$

(c) $p + \frac{1}{2}\rho v^2 + \rho gy = constant$ OR $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

where p = pressure
ρ = density of the fluid
v = velocity of the fluid
g = acceleration due to gravity
y = height (4)

[15]