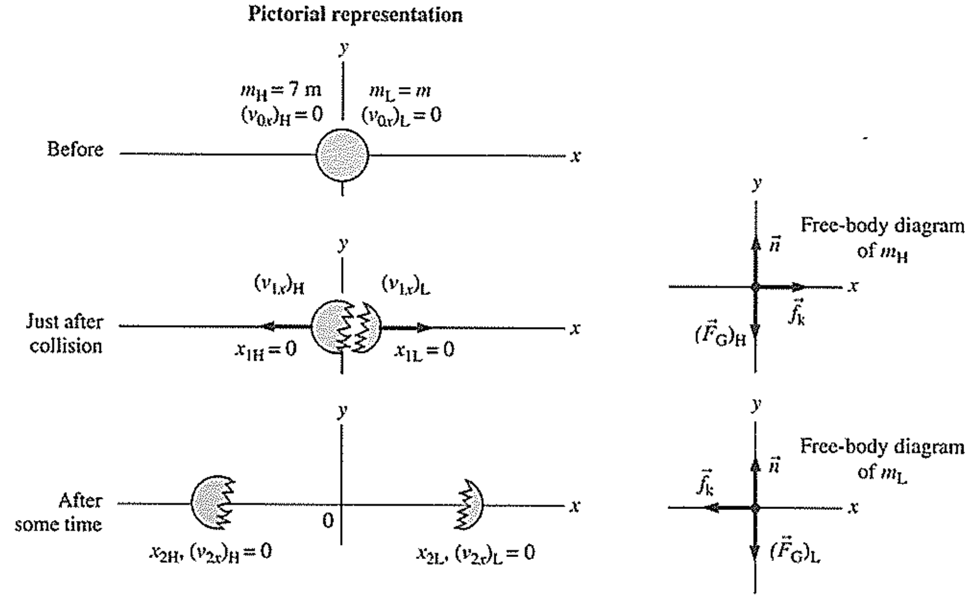
Mechanics PHY1505
Keys to Assignment 02
Semester 2

9.46. Model: This is a two-part problem. First, we have an explosion that creates two particles. The momentum of the system, comprised of two fragments, is conserved in the explosion. Second, we will use kinematic equations and the model of kinetic friction to find the displacement of the lighter fragment.

Visualize:



Solve: The initial momentum is zero. Using momentum conservation $p_{fx} = p_{ix}$ during the explosion,

$$m_H(v_{1x})_H + m_L(v_{1x})_L = m_H(v_{0x})_H + m_L(v_{0x})_L \Rightarrow 7m(v_{1x})_H + m(v_{1x})_L = 0 \text{ kg m/s} \Rightarrow (v_{1x})_H = -\left(\tfrac{1}{7}\right)(v_{1x})_L$$

Because m_H slides to $x_{2H} = -8.2$ m before stopping, we have

$$f_k = \mu_k n_H = \mu_k w_H = \mu_k m_H g = m_H a_H \Rightarrow a_H = \mu_k g$$

Using kinematics,

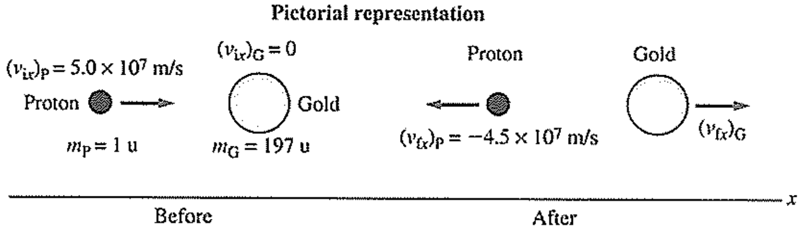
$$(v_{2x})_H^2 = (v_{1x})_H^2 + 2a_H(x_{2H} - x_{1H}) \Rightarrow 0 \text{ m}^2/\text{s}^2 = \left(\tfrac{1}{7}\right)^2 (v_{1x})_L^2 + 2\mu_k g(-8.2 \text{ m} - 0 \text{ m}) \Rightarrow (v_{1x})_L = -88.74\sqrt{\mu_k} \text{ m/s}$$

How far does m_L slide? Using the information obtained above in the following kinematic equation,

$$(v_{2x})_L^2 = (v_{1x})_L^2 + 2a_L(x_{2L} - x_{1L}) \Rightarrow 0 \text{ m}^2/\text{s}^2 = \mu_k(88.74)^2 - 2\mu_k g x_{2L} \Rightarrow x_{2L} = 4.0\times10^2 \text{ m}$$

Assess: Note that a_H is positive, but a_L is negative, and both are equal in magnitude to $\mu_k g$. Also, x_{2H} is negative but x_{2L} is positive.

9.58. Model: Model the proton (P) and the gold atom (G) as particles. The two constitute our system, and momentum is conserved in the collision between the proton and the gold atom.
Visualize:



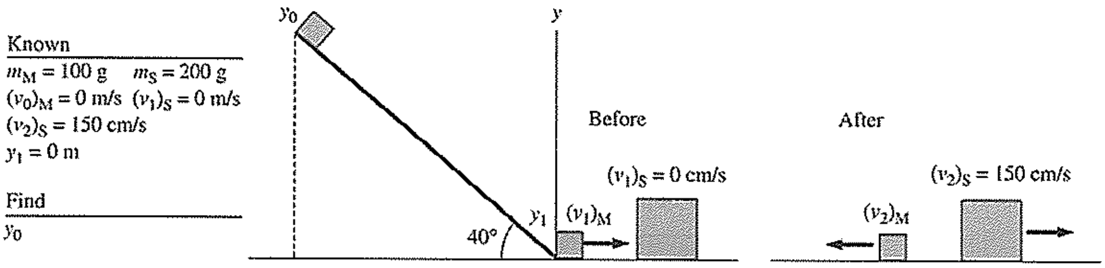
Solve: The conservation of momentum equation $p_{fx} = p_{ix}$ is

$$m_G(v_{fx})_G + m_P(v_{fx})_P = m_P(v_{ix})_P + m_G(v_{ix})_G$$

$$\Rightarrow (197\text{ u})(v_{fx})_G + (1\text{ u})(-0.90 \times 5.0 \times 10^7\text{ m/s}) = (1\text{ u})(5.0 \times 10^7\text{ m/s}) + 0\text{ u m/s}$$

$$\Rightarrow (v_{fx})_G = 4.8 \times 10^5\text{ m/s}$$

10.44. Model: Model the marble and the steel ball as particles. We will assume an elastic collision between the marble and the ball, and apply the conservation of momentum and the conservation of energy equations. We will also assume zero rolling friction between the marble and the incline.
Visualize:



This is a two-part problem. In the first part, we will apply the conservation of energy equation to find the marble's speed as it exits onto a horizontal surface. We have put the origin of our coordinate system on the horizontal surface just where the marble exits the incline. In the second part, we will consider the elastic collision between the marble and the steel ball.
Solve: The conservation of energy equation $K_1 + U_{g1} = K_0 + U_{g0}$ gives us:

$$\frac{1}{2}m_M(v_1)_M^2 + m_M g y_1 = \frac{1}{2}m_M(v_0)_M^2 + m_M g y_0$$

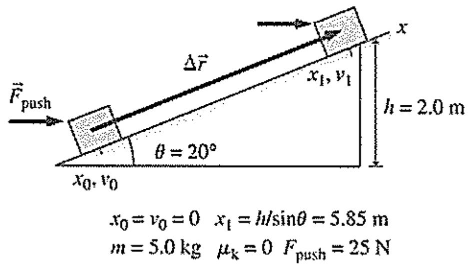
Using $(v_0)_M = 0$ m/s and $y_1 = 0$ m, we get $\frac{1}{2}(v_1)_M^2 = g y_0 \Rightarrow (v_1)_M = \sqrt{2gy_0}$. When the marble collides with the steel ball, the elastic collision gives the ball velocity

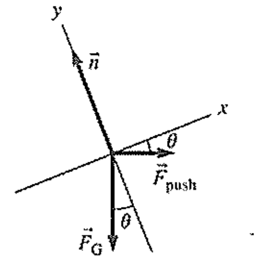
$$(v_2)_S = \frac{2m_M}{m_M + m_S}(v_1)_M = \frac{2m_M}{m_M + m_S}\sqrt{2gy_0}$$

Solving for y_0 gives

$$y_0 = \frac{1}{2g}\left[\frac{m_M + m_S}{2m_M}(v_2)_S\right]^2 = 0.258\text{ m} = 25.8\text{ cm}$$

11.44. Model: Model the crate as a particle, and use the work-kinetic energy theorem.
Visualize:





Solve: (a) The work-kinetic energy theorem is $\Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 = W_{total}$. Three forces act on the box, so $W_{total} = W_{grav} + W_n + W_{push}$. The normal force is perpendicular to the motion, so $W_n = 0$ J. The other two forces do the following amount of work:

$$W_{push} = \vec{F}_{push} \cdot \Delta\vec{r} = F_{push}\Delta x\cos 20° = 137.4 \text{ J} \qquad W_{grav} = \vec{F}_G \cdot \Delta\vec{r} = \left(F_G\right)_x \Delta x = (-mg\sin 20°)\Delta x = -98.0 \text{ J}$$

Thus, $W_{total} = 39.4$ J, leading to a speed at the top of the ramp equal to

$$v_1 = \sqrt{\frac{2W_{total}}{m}} = \sqrt{\frac{2(39.4 \text{ J})}{5.0 \text{ kg}}} = 4.0 \text{ m/s}$$

(b) The x-component of Newton's second law is

$$a_x = a = \frac{(F_{net})_x}{m} = \frac{F_{push}\cos 20° - F_G\sin 20°}{m} = \frac{F_{push}\cos 20° - mg\sin 20°}{m} = 1.35 \text{ m/s}^2$$

Constant-acceleration kinematics with $x_1 = h/\sin 20° = 5.85$ m gives the final speed

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) = 2ax_1 \Rightarrow v_1 = \sqrt{2ax_1} = \sqrt{2(1.35 \text{ m/s}^2)(5.85 \text{ m})} = 4.0 \text{ m/s}$$

11.62. Solve: Power output during the push-off period is equal to the work done by the cat divided by the time the cat applied the force. Since the force on the floor by the cat is equal in magnitude to the force on the cat by the floor, work done by the cat can be found using the work-kinetic energy theorem during the push-off period, $W_{net} = W_{floor} = \Delta K$. We do not need to explicitly calculate W_{cat}, since we know that the cat's kinetic energy is transformed into its potential energy during the leap. That is,

$$\Delta U_g = mg(y_2 - y_1) = (5.0 \text{ kg})(9.8 \text{ m/s}^2)(0.95 \text{ m}) = 46.55 \text{ J}$$

Thus, the average power output during the push-off period is

$$P = \frac{W_{net}}{t} = \frac{46.55 \text{ J}}{0.20 \text{ s}} = 0.23 \text{ kW}$$

12.58. Solve: From Equation 12.16,

$$I = \int\left(x^2 + y^2\right)dm$$

A small region of area dA has mass dm, and

$$dm = \frac{M}{A}dA = \frac{M}{A}dx\,dy$$

The area of the plate is $\frac{1}{2}(0.20 \text{ m})(0.30 \text{ m}) = 0.030 \text{ m}^2$. So

$$\frac{M}{A} = \left(\frac{0.800 \text{ kg}}{0.030 \text{ m}^2}\right) = 26.67 \text{ kg m}^2$$

The limits for x are $0 \le x \le 30$ cm. For a particular value of x, $-\frac{1}{3}x \le y \le \frac{1}{3}x$. Note that $\pm\frac{1}{3}$ is the slope of the top and bottom edges of the triangle. Therefore,

$$I = \int_0^{30\text{ cm}} \int_{-\frac{1}{3}x}^{\frac{1}{3}x} (x^2 + y^2)(26.67\text{ kg/m}^2)\,dx\,dy$$

$$= (26.67\text{ kg/m}^2) \int_0^{30\text{ cm}} \left(x^2 y + \frac{y^3}{3} \right)\Bigg|_{-\frac{1}{3}x}^{\frac{1}{3}x} dx$$

$$= (26.67\text{ kg/m}^2) \int_0^{30\text{ cm}} \left(\frac{2}{3}x^3 + \frac{2x^3}{81} \right) dx$$

$$= (26.67\text{ kg/m}^2) \left(\frac{56}{81} \right) \left(\frac{1}{4}x^4 \right) \Bigg|_0^{30\text{ cm}} = 0.037\text{ kg m}^2$$

13.32. Model: Model the earth (e) as a spherical mass and the satellite (s) as a point particle.
Visualize: Let h be the height from the surface of the earth where the acceleration due to gravity (g_{altitude}) is 10% of the surface value (g_{surface}).
Solve: (a) Since $g_{\text{altitude}} = (0.10)\,g_{\text{surface}}$, we have

$$\frac{GM_e}{(R_e + h)^2} = (0.10)\frac{GM_e}{R_e^2} \Rightarrow (R_e + h)^2 = 10R_e^2$$

$$\Rightarrow h = 2.162R_e \Rightarrow h = (2.162)(6.37 \times 10^6\text{ m}) = 1.377 \times 10^7\text{ m} = 1.38 \times 10^7\text{ m}$$

(b) For a satellite orbiting the earth at a height h above the surface of the earth, the gravitational force between the earth and the satellite provides the centripetal acceleration necessary for circular motion. For a satellite orbiting with velocity v_s,

$$\frac{GM_e m_s}{(R_e + h)^2} = \frac{m_s v_s^2}{(R_e + h)} \Rightarrow v_s = \sqrt{\frac{GM_e}{R_e + h}} = \sqrt{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{(6.37 \times 10^6\text{ m} + 1.377 \times 10^7\text{ m})}} = 4.45\text{ km/s}$$

14.56 The equation of motion for the small angle oscillations of the rod is

$$\frac{d^2\theta}{dt^2} = -\frac{MgL}{I}\theta$$

The angular frequency of the rod is

$$\omega = 2\pi f = \sqrt{\frac{MgL}{I}}$$

An axis $\frac{1}{4}L$ from one end is also $d = \frac{1}{4}L$ from the center of mass of the rod. Therefore the moment of inertia of the rod is

$$I = I_{cm} + Md^2 = \frac{1}{12}ML^2 + M\left(\frac{1}{4}L\right)^2$$

$$= \frac{7}{48}ML^2$$

Therefore the frequency of the oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{Mg\frac{3L}{4}}{I}} = \frac{1}{2\pi}\sqrt{\frac{36}{7}\cdot\frac{g}{L}}$$

15.26. Model: Turning the tuning screws on a guitar string creates tensile stress in the string.
Solve: The tensile stress in the string is given by T/A, where T is the tension in the string and A is the cross-sectional area of the string. From the definition of Young's modulus,

$$Y = \frac{T/A}{\Delta L/L} \Rightarrow \Delta L = \frac{T}{A}\left(\frac{L}{Y}\right)$$

Using $T = 2000$ N, $L = 0.80$ m, $A = \pi(0.00050 \text{ m})^2$, and $Y = 20\times10^{10}$ N/m^2 (from Table 15.3), we obtain $\Delta L = 0.010$ m = 1.02 cm.
Assess: 1.02 cm is a large stretch for a length of 80 cm, but 2000 N is a large tension.

15.62. Model: Treat the air as an ideal fluid obeying Bernoulli's equation.
Solve: (a) The pressure above the roof is lower due to the higher velocity of the air.
(b) Bernoulli's equation, with $y_{\text{inside}} \approx y_{\text{outside}}$, is

$$p_{\text{inside}} = p_{\text{outside}} + \tfrac{1}{2}\rho_{\text{air}}v^2 \Rightarrow \Delta p = \frac{1}{2}\rho_{\text{air}}v^2 = \frac{1}{2}\left(1.28 \text{ kg/m}^3\right)\left(\frac{130\times1000 \text{ m}}{3600 \text{ s}}\right)^2 = 835 \text{ Pa}$$

The pressure difference is 0.83 kPa
(c) The force on the roof is $(\Delta p)A = (835 \text{ Pa})(6.0 \text{ m}\times15.0 \text{ m}) = 7.5\times10^4$ N. The roof will blow up, because pressure inside the house is greater than pressure on the top of the roof.