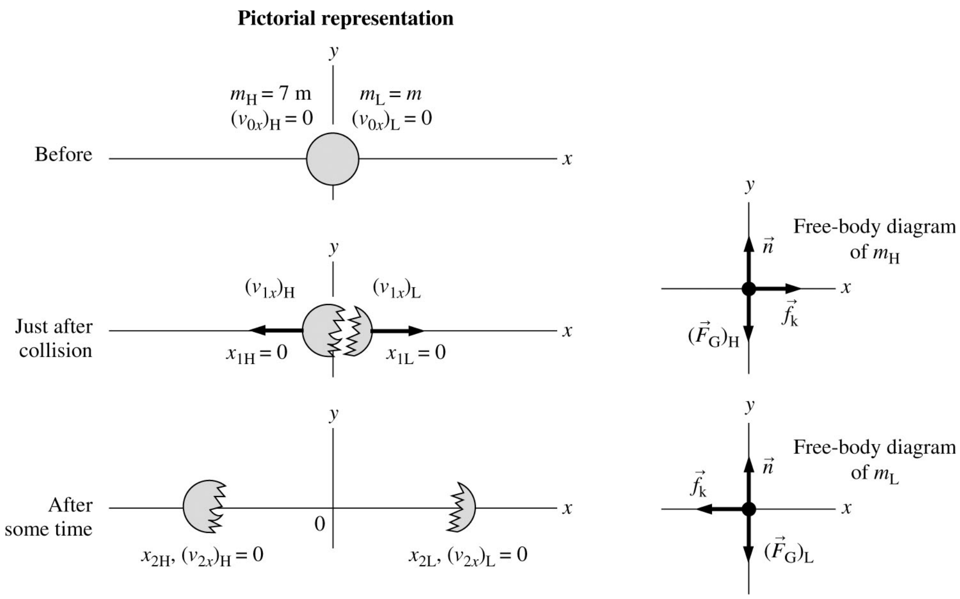
Mechanics – PHY1505
Assignment 02 Semester 1 2018
Solutions

9.46. Model: This is a two-part problem. First, we have an explosion that creates two particles. The momentum of the system, comprised of two fragments, is conserved in the explosion. Second, we will use kinematic equations and the model of kinetic friction to find the displacement of the lighter fragment.

Visualize:



Solve: The initial momentum is zero. Using momentum conservation $p_{fx} = p_{ix}$ during the explosion,

$$m_H(v_{1x})_H + m_L(v_{1x})_L = m_H(v_{0x})_H + m_L(v_{0x})_L \quad \Rightarrow \quad 7m(v_{1x})_H + m(v_{1x})_L = 0 \text{ kg m/s} \quad \Rightarrow \quad (v_{1x})_H = -\left(\tfrac{1}{7}\right)(v_{1x})_L$$

Because m_H slides to $x_{2H} = -8.2$ m before stopping, we have

$$f_k = \mu_k n_H = \mu_k w_H = \mu_k m_H g = m_H a_H \quad \Rightarrow \quad a_H = \mu_k g$$

Using kinematics,

$$(v_{2x})_H^2 = (v_{1x})_H^2 + 2a_H(x_{2H} - x_{1H}) \quad \Rightarrow \quad 0 \text{ m}^2/\text{s}^2 = \left(\tfrac{1}{7}\right)^2 (v_{1x})_L^2 + 2\mu_k g(-8.2 \text{ m} - 0 \text{ m})$$

$$(v_{1x})_L = -88.74\sqrt{\mu_k} \text{ m/s}$$

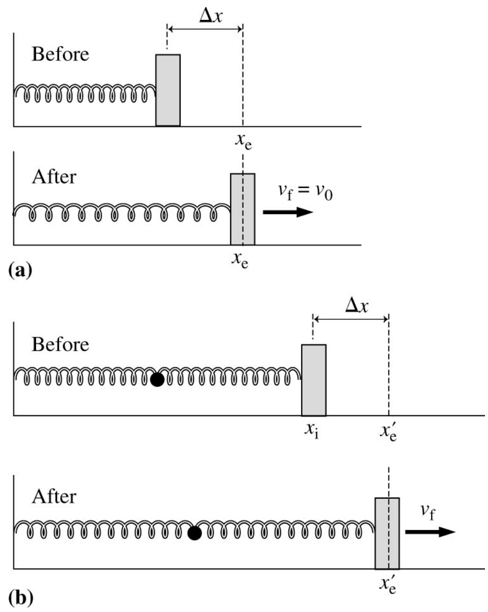
How far does m_L slide? Using the information obtained above in the following kinematic equation,

$$(v_{2x})_L^2 = (v_{1x})_L^2 + 2a_L(x_{2L} - x_{1L}) \quad \Rightarrow \quad 0 \text{ m}^2/\text{s}^2 = \mu_k(88.74)^2 - 2\mu_k g x_{2L} \quad \Rightarrow \quad x_{2L} = 4.0 \times 10^2 \text{ m}$$

Assess: Note that a_H is positive, but a_L is negative, and both are equal in magnitude to $\mu_k g$. Also, x_{2H} is negative but x_{2L} is positive. **[15]**

10.41. Model: Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K + U_s$ is conserved.

Visualize:



The springs in both cases have the same compression Δx. We put the origin of the coordinate system at the equilibrium position of the free end of the spring for the single-spring case (a), and at the free end of the two connected springs for the two-spring case (b).

Solve: The conservation of energy for the single-spring case:

$$K_f + U_{sf} = K_i + U_{si} \Rightarrow \frac{1}{2}mv_f^2 + \frac{1}{2}k(x_f - x_e)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_i - x_e)^2$$

Using $x_f = x_e = 0$ m, $v_i = 0$ m/s, and $v_f = v_0$, this equation simplifies to

$$\frac{1}{2}mv_0^2 = \frac{1}{2}k(\Delta x)^2$$

Conservation of energy in the case of the two springs in series, where each spring compresses by $\Delta x/2$, is

$$K_f + U_{sf} = K_i + U_{si} \Rightarrow \frac{1}{2}mV_f^2 + 0 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(\Delta x/2)^2 + \frac{1}{2}k(\Delta x/2)^2$$

Using $x_f = x_e' = 0$ m and $v_i = 0$ m/s, we get

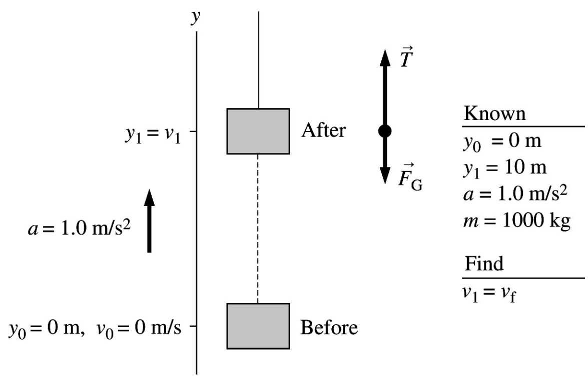
$$\frac{1}{2}mV_f^2 = \frac{1}{2}\left[\frac{1}{2}k(\Delta x)^2\right]$$

Comparing the two results we see that $V_f = v_0/\sqrt{2}$.

Assess: The block pushes on the spring until the spring has returned to its equilibrium length. **[15]**

11.42. Model: Model the elevator as a particle.

Visualize:



Solve: (a) The work done by gravity on the elevator is

$$W_g = -\Delta U_g = mgy_0 - mgy_1 = -mg(y_1 - y_0) = -(1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = -9.8\times10^4\text{ J}$$

(b) The work done by the tension in the cable on the elevator is

$$W_T = T(\Delta y)\cos(0^\circ) = T(y_1 - y_0) = T(10\text{ m})$$

To find T we write Newton's second law for the elevator:

$$\sum F_y = T - F_G = ma_y \quad\Rightarrow\quad T = F_G + ma_y = m(g + a_y) = (1000\text{ kg})(9.8\text{ m/s}^2 + 1.0\text{ m/s}^2)$$

$$= 1.08\times10^4\text{ N} \quad\Rightarrow\quad W_T = (1.08\times10^4\text{ N})(10\text{ m}) = 1.1\times10^5\text{ J}$$

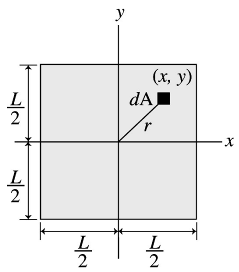
(c) The work–kinetic-energy theorem is

$$W_{\text{net}} = W_g + W_T = \Delta K = K_f - K_i = K_f - \tfrac{1}{2}mv_0^2 \Rightarrow K_f = W_g + W_T + \tfrac{1}{2}mv_0^2$$

$$K_f = (-9.8\times10^4\text{ J}) + (1.08\times10^5\text{ J}) + \tfrac{1}{2}(1000\text{ kg})(0\text{ m/s})^2 = 1.0\times10^4\text{ J}$$

(d) $K_f = \tfrac{1}{2}mv_f^2 \Rightarrow 1.0\times10^4\text{ J} = \tfrac{1}{2}(1000\text{ kg})v_f^2 \Rightarrow v_f = 4.5\text{ m/s}$ **[15]**

12.57. Model: The plate has uniform density.
Visualize:



Solve: The moment of inertia is

$$I = \int r^2 dm.$$

Let the mass of the plate be M. Its area is L^2. A region of area dA located at (x, y) has mass $dm = \frac{M}{A}dA = \frac{M}{L^2}dx\,dy$. The distance from the axis of rotation to the point (x, y) is $r = \sqrt{x^2 + y^2}$. With $-\frac{L}{2} \le x \le \frac{L}{2}$ and $-\frac{L}{2} \le y \le \frac{L}{2}$,

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{-\frac{L}{2}}^{\frac{L}{2}}(x^2 + y^2)\left(\frac{M}{L^2}\right)dx\,dy = \frac{M}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{x^3}{3} + y^2x\right)\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}}dy$$

$$= \frac{M}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{L^3}{24} + \frac{y^2L}{2} - \left(\frac{-L^3}{24} - \frac{y^2L}{2}\right)\right)dy = \frac{M}{L^2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{L^3}{12} + Ly^2\right)dy = \frac{M}{L^2}\left(\frac{L^4}{12} + L\frac{y^3}{3}\Bigg|_{-\frac{L}{2}}^{\frac{L}{2}}\right)$$ **[20]**

$$= \frac{M}{L^2}\left(\frac{L^4}{12} + L\left(\frac{L^3}{24} + \frac{L^3}{24}\right)\right) = \frac{1}{6}ML^2$$

13.39. Model: The projectile is a particle. The earth and moon are spherical masses.
Solve: The projectile is attracted to both the moon and earth. Its final velocity and potential energy are zero. Since the projectile is fired from the far side of the moon, its initial distance from the center of the earth is the earth-moon distance $R_{\text{e-m}}$ plus the radius of the moon R_m. Let the projectile have mass m. The conservation of mechanical energy equation is

$$\frac{1}{2}mv_{\text{escape}}^2 - G\frac{M_e m}{(R_{\text{e-m}} + R_m)} - G\frac{M_m m}{R_m} = 0\text{ J} + 0\text{ J}$$

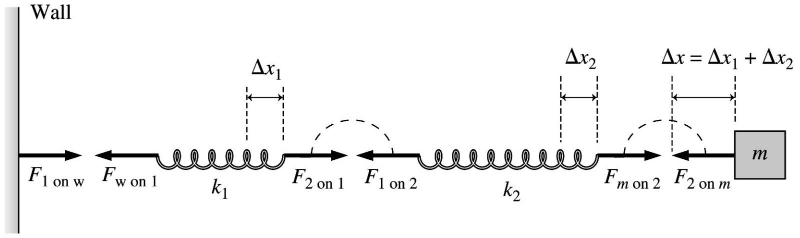
$$v_{\text{escape}}^2 = 2G\left(\frac{M_e}{R_{\text{e-m}} + R_m} + \frac{M_m}{R_m}\right)$$

From Table 13.2, $R_{\text{e-m}} = 3.84 \times 10^8$ m, $R_m = 1.74 \times 10^6$ m, $M_e = 5.98 \times 10^{24}$ kg, and $M_m = 7.36 \times 10^{22}$ kg, which gives $v_{\text{escape}} = 2.78$ km/s.

Assess: The escape velocity does not depend on the mass of the object which is trying to escape. **[10]**

14.74. Model: The two springs obey Hooke's law. Assume massless springs.

Visualize: Each spring is shown separately. Note that $\Delta x = \Delta x_1 + \Delta x_2$.



Solve: Only spring 2 touches the mass, so the net force on the mass is $F_m = F_{2\text{ on }m}$. Newton's third law tells us that $F_{2\text{ on }m} = F_{m\text{ on 2}}$ and that $F_{2\text{ on 1}} = F_{1\text{ on 2}}$. From $F_{\text{net}} = ma$, the net force on a massless spring is zero. Thus $F_{\text{w on 1}} = F_{2\text{ on 1}} = k_1\Delta x_1$ and $F_{m\text{ on 2}} = F_{1\text{ on 2}} = k_2\Delta x_2$. Combining these pieces of information,

$$F_m = k_1\Delta x_1 = k_2\Delta x_2$$

The net displacement of the mass is $\Delta x = \Delta x_1 + \Delta x_2$, so

$$\Delta x = \Delta x_1 + \Delta x_2 = \frac{F_m}{k_1} + \frac{F_m}{k_2} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)F_m = \frac{k_1 + k_2}{k_1 k_2}F_m$$

Turning this around, the net force on the mass is

$$F_m = \frac{k_1 k_2}{k_1 + k_2}\Delta x = k_{\text{eff}}\Delta x \text{ where } k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}$$

k_{eff}, the proportionality constant between the force on the mass and the mass's displacement, is the effective spring constant. Thus the mass's angular frequency of oscillation is

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{1}{m}\frac{k_1 k_2}{k_1 + k_2}}$$

Using $\omega_1^2 = k_1/m$ and $\omega_2^2 = k_2/m$ for the angular frequencies of either spring acting alone on m, we have

$$\omega = \sqrt{\frac{(k_1/m)(k_2/m)}{(k_1/m) + (k_2/m)}} = \sqrt{\frac{\omega_1^2\omega_2^2}{\omega_1^2 + \omega_2^2}}$$

Since the actual frequency f is simply a multiple of ω, this same relationship holds for f:

$$f = \sqrt{\frac{f_1^2 f_2^2}{f_1^2 + f_2^2}}$$

[15]

15.64. Model: The ideal fluid (i.e., air) obeys Bernoulli's equation.

Visualize: Please refer to Figure P15.64. There is a streamline connecting points 1 and 2. The air speeds at points 1 and 2 are v_1 and v_2, and the cross-sectional areas of the pipes at these points are A_1 and A_2. Points 1 and 2 are at the same height, so $y_1 = y_2$.

Solve: (a) The height of the mercury is 10 cm. So, the pressure at point 2 is larger than at point 1 by

$$\rho_{\text{Hg}}g(0.10\text{ m}) = (13{,}600\text{ kg/m}^3)(9.8\text{ m/s}^2)(0.10\text{ m}) = 13{,}328\text{ Pa} \quad \Rightarrow \quad p_2 = p_1 + 13{,}328\text{ Pa}$$

Using Bernoulli's equation,

$$p_1 + \tfrac{1}{2}\rho_{\text{air}}v_1^2 + \rho_{\text{air}}gy_1 = p_2 + \tfrac{1}{2}\rho_{\text{air}}v_2^2 + \rho_{\text{air}}gy_2 \quad \Rightarrow \quad p_2 - p_1 = \tfrac{1}{2}\rho_{\text{air}}(v_1^2 - v_2^2)$$

$$v_1^2 - v_2^2 = \frac{2(p_2 - p_1)}{\rho_{\text{air}}} = \frac{2(13{,}328\text{ Pa})}{(1.28\text{ kg/m}^3)} = 20{,}825\text{ m}^2/\text{s}^2$$

From the continuity equation, we can obtain another equation connecting v_1 and v_2:

$$A_1 v_1 = A_2 v_2 \quad \Rightarrow \quad v_1 = \frac{A_2}{A_1} v_2 = \frac{\pi (0.0050 \text{ m})^2}{\pi (0.0010 \text{ m})^2} v_2 = 25 v_2$$

Substituting $v_1 = 25 v_2$ in the Bernoulli equation, we get

$$(25\, v_2)^2 - v_2^2 = 20{,}825 \text{ m}^2/\text{s}^2 \quad \Rightarrow \quad v_2 = 5.78 \text{ m/s}$$

Thus $v_2 = 5.8$ m/s and $v_1 = 25 v_2 = 1.4 \times 10^2$ m/s.

(b) The volume flow rate $A_2 v_2 = \pi (0.0050 \text{ m})^2 (5.78 \text{ m/s}) = 4.5 \times 10^{-3} \text{ m}^3/\text{s}$. **[10]**