Tutorial letter 202/2/2018

Forecasting

STA2604

Semester 2

Department of Statistics

Solutions to Assignment 2
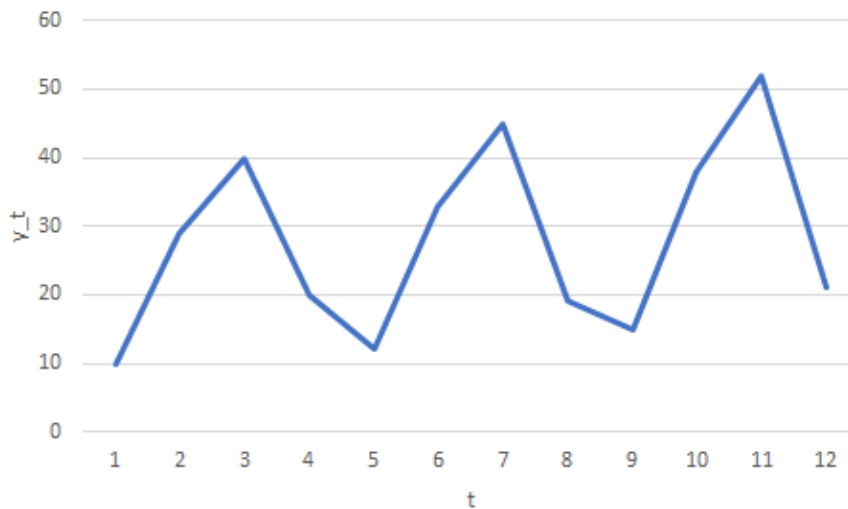
Define tomorrow.

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QUESTION 1

The multiplicative decomposition model is:

$$y_t = TR_t \times SN_t \times CL_t \times IR_t$$

where TR_t, SN_t, CL_t and IR_T are the trend, seasonal, cyclical and irregular factors, respectively.

(1.1) The plot of sales versus time is given by the following graph:

This graph indicates that the trend for the data appears to be increasing.

(1.2) The four-point moving averages are given in the fifth column of Table 1.

Table 1: Analysis of sales data using multiplicative decomposition of a time series

Yr	Quart	t	y_t	4MA	CMA_t	$sn_t \times ir_t$	sn_t	d_t	tr_t	$\widehat{y}_t$	$cl_t \times irt_t$	cl_t	ir_t
1	1	1	10				0.47	21.28	23.13	10.87	0.92		
1	2	2	29				1.21	23.97	23.91	28.93	1.00	0.98	1.03
1	3	3	40	24.75	25	1.6000	1.61	24.84	24.69	39.75	1.01	1.03	0.97
1	4	4	20	25.25	25.75	0.7767	0.72	27.78	25.47	18.34	1.09	1.02	1.07
2	1	5	12	26.25	26.875	0.4465	0.47	25.53	26.25	12.34	0.97	1.02	0.95
2	2	6	33	27.5	27.375	1.2055	1.21	27.27	27.03	32.70	1.01	1.00	1.01
2	3	7	45	27.25	27.625	1.6290	1.61	27.95	27.81	44.77	1.01	0.98	1.03
2	4	8	19	28	28.625	0.6638	0.72	26.39	28.59	20.58	0.92	1.01	0.92
3	1	9	15	29.25	30.125	0.4979	0.47	31.91	29.37	13.80	1.09	1.02	1.07
3	2	10	38	31	31.25	1.2160	1.21	31.40	30.15	36.48	1.04	1.06	0.98
3	3	11	52	31.5			1.61	32.30	30.93	49.79	1.04	1.00	1.04
3	4	12	21				0.72	29.17	31.71	22.83	0.92		

(1.3) The centered moving averages, $CMA_t = tr_t \times cl_t$, are given in the sixth column of Table 1.

(1.4) The $sn_t \times ir_t = y_t/CMA_t$ values are given in the seventh column of Table 1.

(1.5) Steps for the calculation of seasonal indices are given in the following tables:

Year	Quarter			
	1	2	3	4
1	-	-	1.600	0.7767
2	0.4465	1.2055	1.629	0.6638
3	0.4979	1.216	-	-
Total	0.9444	2.4215	3.229	1.4405
Mean (unadjusted seasonal indices)	0.4722	1.2108	1.6145	0.7203

The total index is: $0.4722 + 1.2108 + 1.6145 + 0.7203 = 4.0178$.

Adjusted seasonal indices (must sum to 4, the number of seasons; quarters here):

Quarter	Value
1	$\frac{0.4722 \times 4}{4.0178} = 0.4701$
2	$\frac{1.2108 \times 4}{4.0178} = 1.2054$
3	$\frac{1.6145 \times 4}{4.0178} = 1.6073$
4	$\frac{0.7203 \times 4}{4.0178} = 0.7171$

These adjusted seasonal indices are copied in the eighth column of Table 1

(1.6) The deseasonalised observations, $d_t = y_t/sn_t$, are given in the ninth column of Table 1.

(1.7) The plots of these deseasonalised observations versus time is given in the following figure. The figure indicates an overall increasing trend.
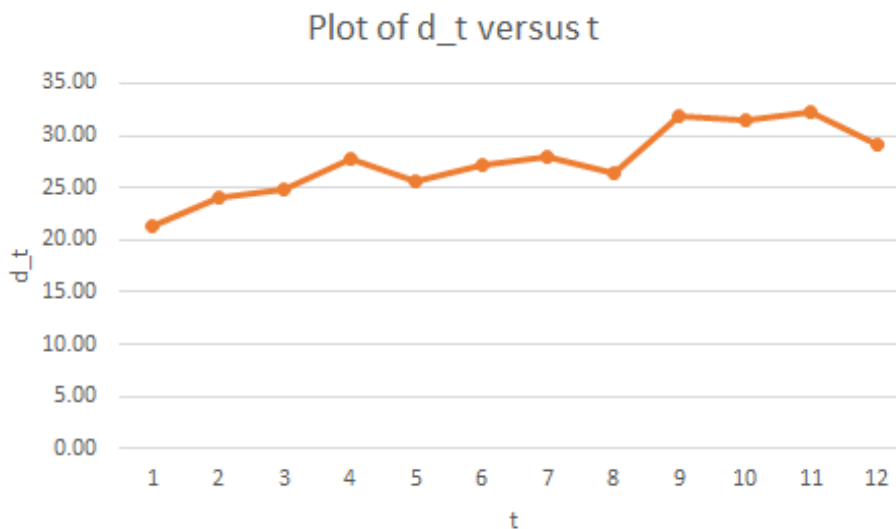
Plot of d_t versus t

(1.8) This can easily be done using Excel, but must be done by hand in an exam.

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Year	Quarter	t	d_t	td_t	t^2	$(t - \bar{t})^2$
1	1	1	21	21	1	30.25
1	2	2	24	48	4	20.25
1	3	3	25	75	9	12.25
1	4	4	28	112	16	6.25
2	1	5	26	130	25	2.25
2	2	6	27	162	36	0.25
2	3	7	28	196	49	0.25
2	4	8	26	208	64	2.25
3	1	9	32	288	81	6.25
3	2	10	31	310	100	12.25
3	3	11	32	352	121	20.25
3	4	12	29	348	144	30.25
Total		78	329	2250	650	143
Mean		6.5	27.4167			

$$\widehat{\beta}_1 = \frac{\sum_{t=1}^{12} td_t - \frac{1}{12}\left(\sum_{t=1}^{12} t\right)\left(\sum_{t=1}^{12} d_t\right)}{\sum_{t=1}^{12} t^2 - \frac{\left(\sum_{t=1}^{12} t\right)^2}{12}} = \frac{2250 - \frac{78 \times 329}{12}}{650 - \frac{78^2}{12}} = 0.7797$$

and

$$\widehat{\beta}_0 = \bar{d} - \widehat{\beta}_1 \bar{t} = \frac{1}{12}\sum_{t=1}^{12} d_t - \frac{1}{12}\widehat{\beta}_1\sum_{t=1}^{12} t = \frac{1}{12} \times 329 - \frac{1}{12} \times 0.7797 \times 78 = 22.3486.$$

Hence the estimate of the trend is: $tr_t = 22.3486 + 0.7797t$.

(1.9) The trend values tr_t are given in the tenth column of Table 1, then the values $\widehat{y}_t = tr_t \times sn_t$ are given in the eleventh column of Table 1. Finally, the values $cl_t \times ir_t$ are given in the twelfth column of Table 1.

(1.10) The cyclical factors calculated as the three-point moving averages

$$cl_t = \frac{cl_{t-1}ir_{t-1} + cl_t ir_t + cl_{t+1}ir_{t+1}}{3}$$

(see textbook, page 335) are given the thirteenth column of Table 1.

(1.11) The irregular values, $ir_t = \dfrac{cl_t \times ir_t}{cl_t}$, are given in the fourteenth column of Table 1. All the values are close to 1, and thus this time series seems not having the irregular variation component.

(1.12) All values of cl_t are close to 1, thus we cannot determine any well-defined cycle for these data. Following the result in this part and part (1.11), this time series can only be characterised by trend and seasonal components.

(1.13) At year 4, the values of t are 13, 14, 15 and 16 for the four quarters, respectively. The trend was found to be $tr_t = 22.3486 + 0.7797t$., and the four seasonal indices were found to be 0.4701, 1.2054, 1.6073 and 0.7171. Now using the prediction equation $\widehat{y}_t = tr_t \times sn_t$, we obtain:

$\widehat{y}_{13} = (22.3486 + 0.7797 \times 13) \times 0.4701 = 15.2711$
$\widehat{y}_{14} = (22.3486 + 0.7797 \times 14) \times 1.2054 = 40.0969$
$\widehat{y}_{15} = (22.3486 + 0.7797 \times 15) \times 1.6073 = 54.7191$
$\widehat{y}_{16} = (22.3486 + 0.7797 \times 16) \times 0.7171 = 24.9721$

(1.14) Here we first calculate prediction intervals of the trend at the four time points then use the formula $[\widehat{y} \pm B_t(1-\alpha)]$ where $B_t(1-\alpha)$ is the error band in the prediction interval $[tr_t \pm B_t(1-\alpha)]$ (see textbook on pages 335-336). This time, we fit the trend using Excel to obtain the standard error, that $s = 1.8433$, or equivalently $s = \sqrt{MSE} = \sqrt{3.3978} = 1.8433$.

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.8479
R Square	0.7190
Adjusted R Square	0.6909
Standard Error	1.8433
Observations	12

ANOVA					
	df	SS	MS	F	Significance F
Regression	1	86.9388	86.9388	15.8302	0.0026
Residual	10	33.9779	3.3978		
Total	11	120.9167			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	22.3485	1.1345	19.6994	< 0.0001	19.8207	24.8763
X Variable 1	0.7797	0.1541	5.0583	0.0005	0.4363	1.1232

The mean value of t is: $\bar{t} = 6.5$.
For time 13, the error band of the trend is:

$$B_{13}(0.95) = s \times t_{[0.025]}^{(12-2)} \sqrt{1 + \frac{1}{12} + \frac{(13 - \bar{t})^2}{\displaystyle\sum_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 1.84 \times 2.228 \sqrt{1 + \frac{1}{12} + \frac{(13 - 6.5)^2}{143}}$$

$$= 4.8137.$$

Therefore, the prediction interval is: $[15.3 \pm 4.8] = [10.5; 20.1]$.
For time 14, the error band of the trend is:

$$B_{14}(0.95) = s \times t_{[0.25]}^{(10)} \sqrt{1 + \frac{1}{12} + \frac{(14 - \bar{t})^2}{\displaystyle\sum_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 1.84 \times 2.228 \sqrt{1 + \frac{1}{12} + \frac{(14 - 6.5)^2}{143}}$$

$$= 4.9817.$$

Therefore, the prediction interval is: $[40.1 \pm 5.0] = [35.1; 45.1]$.
For time 15, the error band is:

$$B_{15}(0.95) = s \times t_{[0.25]}^{(10)} \sqrt{1 + \frac{1}{12} + \frac{(15 - \bar{t})^2}{\sum\limits_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 1.84 \times 2.228 \sqrt{1 + \frac{1}{12} + \frac{(15 - 6.5)^2}{143}}$$

$$= 5.1670.$$

Therefore, the prediction interval is: $[54.7 \pm 5.2] = [49.5; 59.9]$.
For time 16, the error band is:

$$B_{16}(0.95) = s \times t_{[0.25]}^{(10)} \sqrt{1 + \frac{1}{12} + \frac{(16 - \bar{t})^2}{\sum\limits_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 1.84 \times 2.228 \sqrt{1 + \frac{1}{12} + \frac{(16 - 6.5)^2}{143}}$$

$$= 5.3678.$$

Therefore, the prediction interval is: $[25.0 \pm 5.4] = [19.6; 29.4]$.

QUESTION 2

(2.1) The additive decomposition model is

$$y_t = TR_t + SN_t + CL_t + IR_t$$

where TR_t, SN_t, CL_t and IR_T are the trend, seasonal, cyclical and irregular terms, respectively. All calculations will be done as in Question 1 by replacing the sign $\times$ by the sign $+$, and the sign $/$ by the sign $-$.

Table 2: Analysis of sales data using additive decomposition of a time series

Yr	Quart	t	y_t	4MA	CMA_t	$sn_t + ir_t$	sn_t	d_t	tr_t	$\widehat{y}_t$	$cl_t + irt_t$	cl_t	ir_t
1	1	1	10				-14.92	24.92	23.68	8.76	1.24		
1	2	2	29				6.27	22.73	24.48	30.75	-1.75	-0.69	-1.06
1	3	3	40	24.75	25	15	16.27	23.73	25.28	41.55	-1.55	-0.59	-0.96
1	4	4	20	25.25	25.75	-5.75	-7.61	27.61	26.08	18.47	1.53	0.01	1.52
2	1	5	12	26.25	26.875	-14.875	-14.92	26.92	26.88	11.96	0.04	0.20	-0.17
2	2	6	33	27.5	27.375	5.625	6.27	26.73	27.68	33.95	-0.95	-0.22	-0.73
2	3	7	45	27.25	27.625	17.375	16.27	28.73	28.48	44.75	0.25	-1.13	1.37
2	4	8	19	28	28.625	-9.625	-7.61	26.61	29.28	21.67	-2.67	-0.86	-1.81
3	1	9	15	29.25	30.125	-15.125	-14.92	29.92	30.09	15.17	-0.17	-0.67	0.50
3	2	10	38	31	31.25	6.75	6.27	31.73	30.89	37.16	0.84	1.57	-0.73
3	3	11	52	31.5			16.27	35.73	31.69	47.96	4.04	0.34	3.71
3	4	12	21				-7.61	28.61	32.49	24.88	-3.88		

(2.1.1) The plot of sales versus time is the same as in Question 1, part (1.1).

(2.1.2) The four-point moving averages are given in the fifth column of Table 2, but same as those in Table 1.

(2.1.2) The centered moving averages are given in the sixth column of Table 2, but same as those in Table 1.

(2.1.4) The $sn_t + ir_t = y_t - CMA_t$ are given in the seventh column of Table 2.

(2.1.5) Steps for the calculation of seasonal indices are given in the following tables:

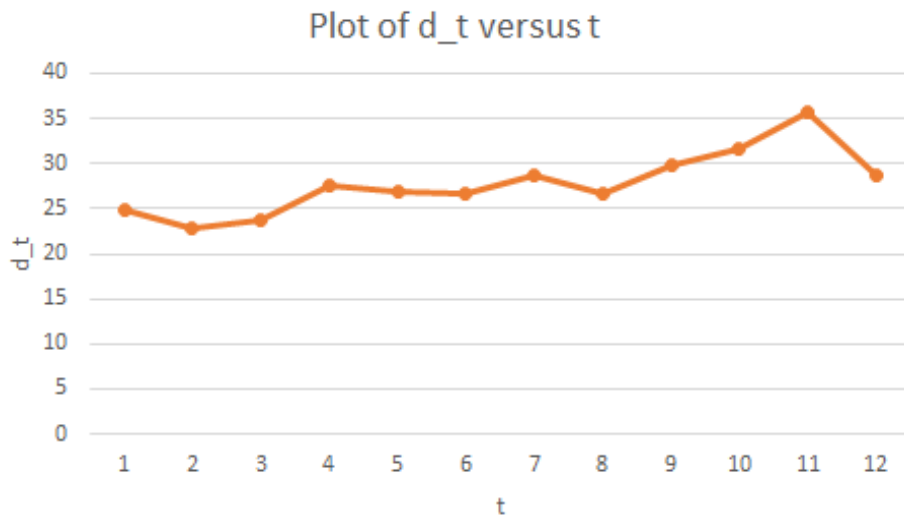| Year	Quarter			
	1	2	3	4
1	-	-	15	-5.75
2	-14.875	5.625	17.375	-9.625
3	-15.125	6.75	-	-
Total	-30	12.375	32.375	-15.375
Mean (unadjusted seasonal indices)	-15	6.1875	16.1875	-7.6875

The total index is: $-15 + 6.1875 + 16.1875 - 7.6875 = -0.3125$.

Adjusted seasonal or normalised indices (sum must be equal to zero):

Quarter	Value
1	$-15 - \frac{-0.3125}{4} = -14.9219$
2	$6.1875 - \frac{-0.3125}{4} = 6.2656$
3	$16.1875 - \frac{-0.3125}{4} = 16.2656$
4	$-7.6875 - \frac{-0.3125}{4} = -7.6094$

These adjusted seasonal indices are copied in the eighth column of Table 2

(2.1.6) The deseasonalised observations $d_t = y_t - sn_t$ are given in the ninth column of Table 2.

(2.1.7) The plots of these observations versus time is given in the following figure. The figure indicates an overall slightly increasing trend.

Plot of d_t versus t

(2.1.8) This can easily be done using Excel, but must be done by hand in an exam. We need to use a different table here.

Year	Quarter	t	d_t	td_t	t^2	$(t-\bar{t})^2$
1	1	1	25	25	1	30.25
1	2	2	23	46	4	20.25
1	3	3	24	72	9	12.25
1	4	4	28	112	16	6.25
2	1	5	27	135	25	2.25
2	2	6	27	162	36	0.25
2	3	7	29	203	49	0.25
2	4	8	27	216	64	2.25
3	1	9	30	270	81	6.25
3	2	10	32	320	100	12.25
3	3	11	36	396	121	20.25
3	4	12	29	348	144	30.25
Total		78	337	2305	650	143
Mean		6.5	28.0833			

$$\widehat{\beta}_1 = \frac{\sum_{t=1}^{12} td_t - \frac{1}{12}\left(\sum_{t=1}^{12} t\right)\left(\sum_{t=1}^{12} d_t\right)}{\sum_{t=1}^{12} t^2 - \frac{\left(\sum_{t=1}^{12} t\right)^2}{12}} = \frac{2305 - \frac{78\times337}{12}}{650 - \frac{78^2}{12}} = 0.8007$$

and

$$\widehat{\beta}_0 = \bar{d} - \widehat{\beta}_1\bar{t} = \frac{1}{12}\sum_{t=1}^{12} d_t - \frac{1}{12}\widehat{\beta}_1\sum_{t=1}^{12} t = \frac{1}{12}\times 337 - \frac{1}{12}\times 0.8007 \times 78 = 22.8788.$$

Hence the estimate of the trend is: $tr_t = 22.8788 + 0.8007t$.

This equation is slightly different from the one found in Question 1, part (1.8) since the deseasonalised data are based on a different model. The trend values tr_t are given in the tenth column of Table 2.

(2.1.9) The point forecast values $\widehat{y}_t = tr_t + sn_t$ are given in the eleventh column of Table 2. Finally, the values $cl_t + ir_t$ are given in the twelfth column of Table 2.

(2.1.10) The cyclical terms calculated as the three-point moving averages

$$cl_t = \frac{(cl_{t-1} + ir_{t-1}) + (cl_t + ir_t) + (cl_{t+1} + ir_{t+1})}{3}$$

(see textbook, page 339) are given the thirteenth column of Table 2.

(2.1.11) The irregular values $ir_t = (cl_t + ir_t) - cl_t$ are given in the fourteenth column of Table 2. Notice that some, but a few, irregular values are not close to zero.

(2.1.12) There is no much evidence to state that the data contain a well defined cycle since the cl_t values do not largely vary.

(2.1.13) At year 4, the values of t are 13, 14, 15 and 16 for the four quarters, respectively. The trend was found to be $tr_t = 22.8788 + 0.8007t$, and the four seasonal indices were found to be -14.9219, 6.2656, 16.2656 and -7.6094. Now using the prediction equation $\widehat{y}_t = tr_t + sn_t$, we obtain:
$\widehat{y}_{13} = (22.8788 + 0.8007 \times 13) - 14.9219 = 18.366$
$\widehat{y}_{14} = (22.8788 + 0.8007 \times 14) + 6.2656 = 40.3542$
$\widehat{y}_{15} = (22.8788 + 0.8007 \times 15) + 16.2656 = 51.1549$
$\widehat{y}_{16} = (22.8788 + 0.8007 \times 16) - 7.6094 = 28.0806$

(2.1.14) The prediction intervals are given by the formula $[\widehat{y} \pm B_t(1 - \alpha)]$ where $B_t(1 - \alpha)$ is the error band in the prediction interval $[tr_t \pm B_t(1 - \alpha)]$ (see textbook on pages 335-336). This time, we fit the trend using Excel to obtain the standard error. We only report the relevant output here.

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.8124
R Square	0.6599
Adjusted R Square	0.6260
Standard Error	2.1734
Observations	12

The mean value of t is: $\bar{t} = 6.5$.
For time 13, the error band is:

$$B_{13}(0.95) = s \times t_{[0.025]}^{(12-2)} \sqrt{1 + \frac{1}{12} + \frac{(13 - \bar{t})^2}{\sum\limits_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 2.2 \times 2.228 \sqrt{1 + \frac{1}{12} + \frac{(13 - 6.5)^2}{143}}$$

$$= 5.7555.$$

Therefore, the prediction interval is: $[18.4 \pm 5.8] = [12.6; 24.2]$.

For time 14, the error band of the trend is:

$$B_{14}(0.95) = s \times t^{(10)}_{[0.025]} \sqrt{1 + \frac{1}{12} + \frac{(14 - \bar{t})^2}{\sum\limits_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 2.2 \times 2.228\sqrt{1 + \frac{1}{12} + \frac{(14 - 6.5)^2}{143}}$$

$$= 5.9564.$$

Therefore, the prediction interval is: $[40.4 \pm 6.0] = [34.4; 46.4]$.
For time 15, the error band is:

$$B_{15}(0.95) = s \times t^{(10)}_{[0.025]} \sqrt{1 + \frac{1}{12} + \frac{(15 - \bar{t})^2}{\sum\limits_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 2.2 \times 2.228\sqrt{1 + \frac{1}{12} + \frac{(15 - 6.5)^2}{143}}$$

$$= 6.1779.$$

Therefore, the prediction interval is: $[51.2 \pm 6.2] = [45.0; 57.4]$.
For time 16, the error band is:

$$B_{16}(0.95) = s \times t^{(10)}_{[0.025]} \sqrt{1 + \frac{1}{12} + \frac{(16 - \bar{t})^2}{\sum\limits_{t=1}^{12}(t - \bar{t})^2}}$$

$$= 2.2 \times 2.228\sqrt{1 + \frac{1}{12} + \frac{(16 - 6.5)^2}{143}}$$

$$= 6.4180.$$

Therefore, the prediction interval is: $[28.1 \pm 6.4] = [21.7; 28.1]$.

(2.2) Overall both models perform similarly in term of the detection of the trend and seasonality, and also perform similarly in terms of predicting future values. Therefore, the additive decomposition model does not seem to be more appropriate than the multiplicative decomposition model. Rather, we have noticed that with the multiplicative model there is no evidence of irregular variations while some irregular variations are evident when the additive is used. This may probably be due to lack-of-fit. Moreover, the following table indicates that the ranges of prediction intervals in year 4 are larger (i.e. wider prediction intervals), except the last one for the additive model than for the multiplicative model.

Time	Multiplicative Model		Additive Model	
	95% Prediction interval	Range	95% Prediction interval	Range
13	[10.5;20.1]	9.5	[12.6;24.2]	11.6
14	[35.1;45.1]	10.0	[34.4;46.4]	12.0
15	[49.5;59.9]	10.4	[45.0;57.4]	12.4
16	[19.6;29.4]	9.8	[21.7;28.1]	7.4

QUESTION 3

(3.1) Here, we don't use deseasonalised data, but the observed data, because we are only asked to fit a simple linear regression model to the data.

Year	Quarter	t	y_t	ty_t	t^2
1	1	1	10	10	1
1	2	2	29	58	4
1	3	3	40	120	9
1	4	4	20	80	16
2	1	5	12	60	25
2	2	6	33	198	36
2	3	7	45	315	49
2	4	8	19	152	64
3	1	9	15	135	81
3	2	10	38	380	100
3	3	11	52	572	121
3	4	12	21	252	144
Total		78	334	2332	650

$$\widehat{\beta}_1 = \frac{\sum_{t=1}^{12} ty_t - \frac{1}{12}\left(\sum_{t=1}^{12} t\right)\left(\sum_{t=1}^{12} y_t\right)}{\sum_{t=1}^{12} t^2 - \frac{\left(\sum_{t=1}^{12} t\right)^2}{12}} = \frac{2332 - \frac{78 \times 334}{12}}{650 - \frac{78^2}{12}} = 1.1259.$$

$$\widehat{\beta}_0 = \bar{y} - \widehat{\beta}_1 \bar{t} = \frac{1}{12}\sum_{t=1}^{12} y_t - \frac{1}{12}\widehat{\beta}_1 \sum_{t=1}^{12} t = \frac{1}{12} \times 334 - \frac{1}{12} \times 1.1259 \times 78 = 20.5150.$$

Hence the estimate of the trend is: $tr_t = 20.5150 + 1.1259t$.

Excel would give the following output.

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	20.5152	8.4755	2.4205	0.0360	1.6307	39.3996
X Variable 1	1.1259	1.1516	0.9777	0.3513	-1.4400	3.6918

(3.2) There is no well defined way of choosing the initial level value l_0 and b_0. For a large dataset, a linear model is fit on part of the data, say half, so that $l_0 = \widehat{\beta}_0$ and $b_0 = \widehat{\beta}_1$. But, in this case we have a small dataset, and thus we use the entire dataset [No penalty will be applied if you use half of the data, or data for one year]. In this situation, $l_0 = \widehat{\beta}_0 = 20.5150$ and $b_0 = \widehat{\beta}_1 = 1.1259$. To determine the smoothed levels we start with $\widehat{y}_1(0) = l_0 + b_0 = 20.5150 + 1.1259 = 21.6276$. Hence, using $l_T = \alpha y_T + (1 - \alpha)[l_{T-1} + b_{T-1}]$ and $b_T = \gamma [l_T - l_{T-1}] + (1 - \gamma)b_{T-1}$ we obtain:

$l_1 = \alpha y_1 + (1 - \alpha)(l_0 + b_0) = 0.15(10) + 0.85(21.6276) = 19.8835;$
$b_1 = \gamma (l_1 - l_0) + (1 - \gamma)b_0 = 0.1(19.8835 - 20.5150) + 0.9(1.1259) = 0.9502.$

$l_2 = \alpha y_2 + (1 - \alpha)(l_1 + b_1) = 0.15(29) + 0.85(19.8835 + 0.9502) = 22.0586;$
$b_2 = \gamma (l_2 - l_1) + (1 - \gamma)b_1 = 0.1(22.0586 - 19.8835) + 0.9(0.9502) = 1.0727.$

This process continues until calculating l_{12} and b_{12}. Table 3, columns 3 and 4 contains all the smoothed levels and growth rates manually calculated using Excel spreadsheet.

Table 3: Holt's trend corrected exponential smoothing for the sales data

Time (t)	Sales (y_t)	Level (l_t)	Growth (b_t)	FMLP	e_t	e_t^2
0		20.515	1.1259			
1	10	19.8835	0.9502	21.6409	-11.6409	135.5106
2	29	22.0586	1.0727	20.8337	8.1663	66.6891
3	40	25.6616	1.3257	23.1313	16.8687	284.5542
4	20	25.9392	1.2209	26.9873	-6.9873	48.8218
5	12	24.8860	0.9935	27.1601	-15.1601	229.8271
6	33	26.9476	1.1003	25.8795	7.1205	50.7012
7	45	30.5907	1.3546	28.0479	16.9521	287.3745
8	19	30.0035	1.1604	31.9453	-12.9453	167.5797
9	15	28.7393	0.9179	31.1639	-16.1639	261.2703
10	38	30.9086	1.0431	29.6572	8.3428	69.6022
11	52	34.9589	1.3438	31.9517	20.0483	401.9345
12	21	34.0073	1.1143	36.3027	-15.3027	234.1738
Total						2238.0390

(3.3) The growth rates $b_T = \gamma [l_T - l_{T-1}] + (1 - \gamma)b_{T-1}$ where $l_T = \alpha y_T + (1 - \alpha)[l_{T-1} + b_{T-1}]$ are given in the fourth column of Table 3.

(3.4) The forecast made last time ($FMLP = F_t = l_{t-1} + b_{t-1}$) are given in column 5 of Table 3.

(3.5) The forecast errors $e_t = y_t - F_t$ are given in the sixth column of Table 3.

(3.6) The squared forecast errors are given in the seventh column of Table 3.

(3.7) The mean squared forecast error is $MSFE = \frac{\sum_{i=1}^{12} e_t^2}{12} = \frac{2238.0390}{12} = 186.5033.$