Mechanics – PHY1505
Assignment 01 Semester 1 2018
Solutions

2.42. Model: Represent the spaceship as a particle.

Solve: **(a)** The known information is: $x_0 = 0$ m, $v_0 = 0$ m/s, $t_0 = 0$ s, $a = g = 9.8$ m/s^2, and $v_1 = 3.0 \times 10^8$ m/s. Constant acceleration kinematics gives

$$v_1 = v_0 + a\Delta t \Rightarrow \Delta t = t_1 = \frac{v_1 - v_0}{a} = 3.06 \times 10^7 \text{ s}$$

The problem asks for the answer in days, so we need a conversion:

$$t_1 = (3.06 \times 10^7 \text{ s}) \times \frac{1 \text{ hour}}{3600 \text{ s}} \times \frac{1 \text{ day}}{24 \text{ hour}} = 3.54 \times 10^2 \text{ days} \approx 3.6 \times 10^2 \text{ days}$$

(b) The distance traveled is

$$x_1 - x_0 = v_0 \Delta t + \frac{1}{2} a (\Delta t)^2 = \frac{1}{2} a t_1^2 = 4.6 \times 10^{15} \text{ m}$$

(c) The number of seconds in a year is

$$1 \text{ year} = 365 \text{ days} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{3600 \text{ s}}{1 \text{ hour}} = 3.15 \times 10^7 \text{ s}$$

In one year light travels a distance

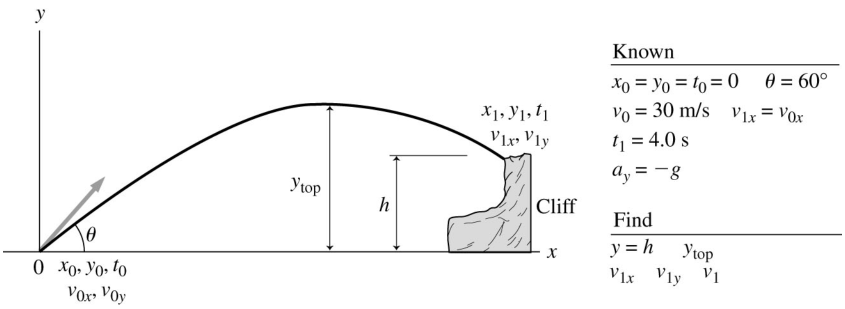
$$1 \text{ light year} = (3.0 \times 10^8 \text{ m/s})(3.15 \times 10^7 \text{ s}) = 9.47 \times 10^{15} \text{ m}$$

The distance traveled by the spaceship is $4.6 \times 10^{15} \text{ m} / 9.46 \times 10^{15} \text{ m} = 0.49$ of a light year.

Assess: Note that x_1 gives "Where is it?" rather than "How far has it traveled?" "How far" is represented by $x_1 - x_0$. They happen to be the same number in this problem, but that isn't always the case. **[15]**

4.44. Model: The particle model for the ball and the constant-acceleration equations of motion are assumed.
Visualize:

Pictorial representation



Solve: **(a)** Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2} a_y (t_1 - t_0)^2$,

$$h = 0 \text{ m} + (30 \text{ m/s}) \sin 60° (4 \text{ s} - 0 \text{ s}) + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(4 \text{ s} - 0 \text{ s})^2 = 25.5 \text{ m}$$

The height of the cliff is 26 m.

(b) Using $(v_y^2)_{top} = v_y^2 + 2a_y(y_{top} - y_0)$,

$$0 \text{ m}^2/\text{s}^2 = (v_0 \sin\theta)^2 + 2(-g)(y_{top}) \Rightarrow y_{top} = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(30 \text{ m/s}) \sin 60°]^2}{2(9.8 \text{ m/s}^2)} = 34.4 \text{ m}$$

The maximum height of the ball is 34 m.

(c) The x and y components are

$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = v_0 \sin\theta - g t_1 = (30 \text{ m/s}) \sin 60° - (9.8 \text{ m/s}^2) \times (4.0 \text{ s}) = -13.22 \text{ m/s}$$

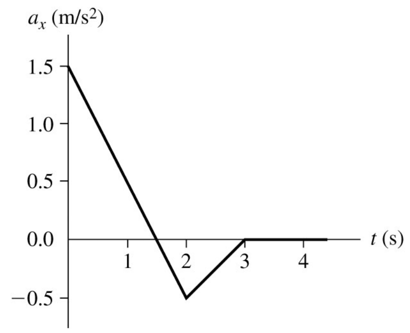
$$v_{1x} = v_{0y} = v_0 \cos 60° = (30 \text{ m/s}) \cos 60° = 15.0 \text{ m/s}$$

$$\Rightarrow v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = 20.0 \text{ m/s}$$

The impact speed is 20 m/s.

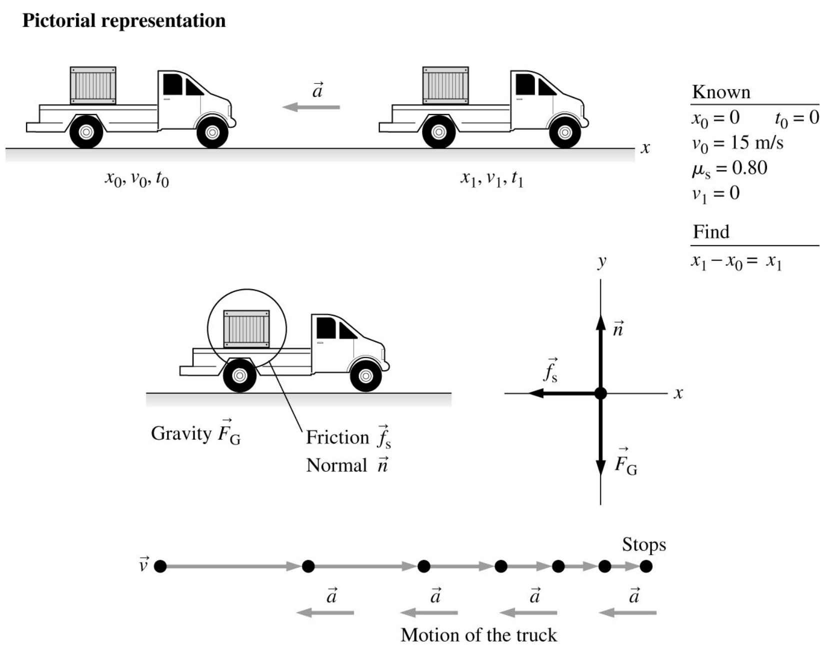
Assess: Compared to a maximum height of 34.4 m, a height of 25.5 for the cliff is reasonable. **[20]**

5.32. Visualize:



Solve: According to Newton's second law $F = ma$, the acceleration at any time is found simply by dividing the value of the force by the mass of the object. Thus, for example, the point at (0 s, 3 N) become [0 s, (3 N)/(2.0 kg)] = (0 s, 1.5 m/s^2). **[15]**

6.51. Model: Model the steel cabinet as a particle. It touches the truck's bed, so only the steel bed can exert contact forces on the cabinet. As long as the cabinet does not slide, the acceleration *a* of the cabinet is equal to the acceleration of the truck.
Visualize: First use Newton's second law in both directions on the cabinet. The force that is responsible for the small box's acceleration is the static friction force. We use this to determine *a*.



Solve:
(a)

$$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$$
$$\Sigma F_x = 2\, f_s = ma_x$$
$$-(f_s)_{max} = -\mu_s n = -\mu_s mg = ma_x$$
$$a_x = -\mu_s g$$

Now use the kinematic equation $v_1^2 = v_0^2 + 2a\Delta x$ where $\Delta x = d_{min}$ and $v_1 = 0$.

$$d_{min} = \frac{v_1^2 - v_0^2}{2a_x} = \frac{-v_0^2}{2(-\mu_s g)} = \frac{v_0^2}{2(\mu_s g)}$$

(b) Insert the known value for v_0 and look up μ_s for steel on steel in the table.

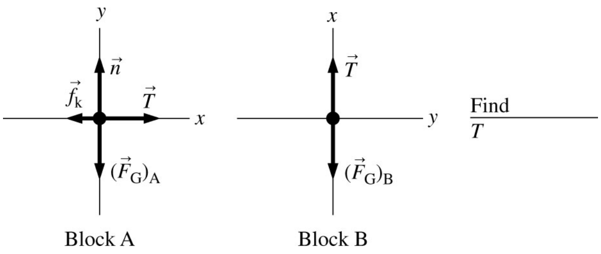
$$d_{min} = \frac{(15 \text{ m/s})^2}{2(0.80)(9.8 \text{ m/s}^2)} = 14.35 \text{ m} \approx 14 \text{ m}$$

Assess: Check the reasonableness of our answer by examining the dependence of $T_{\max}$ on μ_s: if the cabinet were glued to the truck $(\mu_s \to \infty)$ then one could stop in an arbitrarily small distance without the cabinet slipping; if the friction between the cabinet and truck were zero then $d_{\min} = \infty$ and there is no minimum stopping distance without causing the cabinet to slip.

[20]

7.36. Model: Blocks 1 and 2 make up the system of interest and will be treated as particles. We shall use the kinetic friction model.

Visualize:



Notice that the coordinate system of for block B is rotated so that the motion in the positive x-direction is consistent between the two free-body diagrams.

Solve: The blocks are constrained to have the same magnitude acceleration. Applying Newton's second law to block B gives

$$\Sigma(F)_y = -T + (F_G)_B = ma \quad \Rightarrow \quad T - mg = -ma$$

Applying Newton's second law in both the x-and y-directions to the block A gives

$$\Sigma(F)_y = n - (F_G)_A = 0 \quad \Rightarrow \quad n = Mg$$

$$\Sigma(F)_x = T = Ma \quad \Rightarrow \quad T = Ma$$

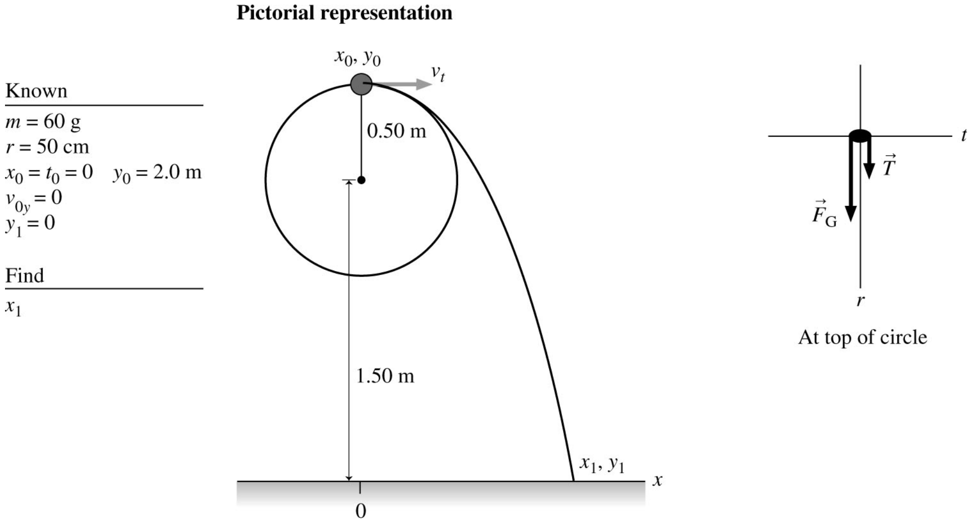
Using the first equation to eliminate the acceleration a gives the tension:

$$T = Ma = M(g - T/m) \quad \Rightarrow \quad T = \frac{mMg}{m+M}$$

Assess: The result is positive, as it should be for our choice of coordinate system. Consider $m = 0$. In this case, $T = 0$, as expected. For $m \gg M$, the tension is independent of the mass of the hanging block because its acceleration will be g, as we can see by solving for the acceleration: **[15]**

$$a = -\frac{T}{m} + g = g - \frac{Mg}{m+M} \to g \text{ for } m \gg M$$

8.51. Model: Use the particle model for a ball in motion in a vertical circle and then as a projectile.
Visualize:



Solve: For the circular motion, Newton's second law along the r-direction is

$$\Sigma F_r = T + F_G = \frac{mv_t^2}{r}$$

Since the string goes slack as the particle makes it over the top, $T = 0$ N. That is,

$$F_G = mg = \frac{mv_t^2}{r} \Rightarrow v_t = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(0.5 \text{ m})} = 2.21 \text{ m/s}$$

The ball begins projectile motion as the string is released. The time it takes for the ball to hit the floor can be found as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0 \text{ m} = 2.0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0 \text{ s})^2 \Rightarrow t_1 = 0.639 \text{ s}$$

The place where the ball hits the ground is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = 0 \text{ m} + (+2.21 \text{ m/s})(0.639 \text{ s} - 0 \text{ s}) = +1.41 \text{ m}$$

The ball hits the ground 1.4 m to the right of the point beneath the center of the circle. **[15]**