

Tutorial letter 202/2/2017

Basic Statistics

STA1510

Semester 2

Department of Statistics

SOLUTIONS TO ASSIGNMENT 02

CHAPTER 7

QUESTION 1

Sampling distribution (mean)

The standard deviation of the sampling distribution of the sample mean is also called the standard error of the mean

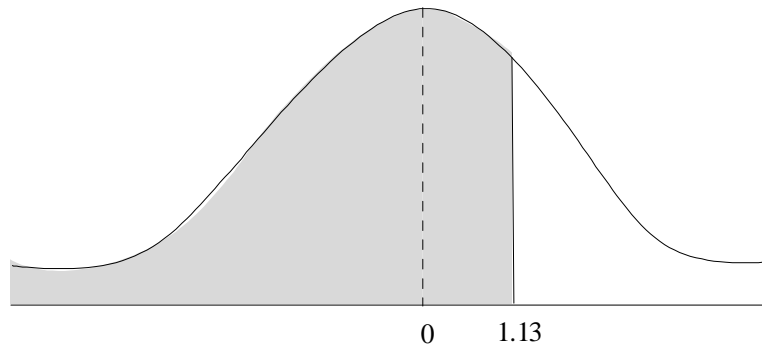
Option 4

QUESTION 2

Sampling distribution (proportion)

$$\pi = 0.12 \quad \text{and} \quad 150 = n$$
$$P(p < 0.15) = P\left(Z < \frac{p - \pi}{\sigma_p}\right)$$

$$\text{then } \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.12(0.88)}{150}} = 0.0265$$



$$\therefore P\left(z < \frac{0.15 - 0.12}{0.0265}\right)$$
$$= P(Z < 1.13)$$
$$= 0.8708$$

Option 1

QUESTION 3

$$\mu = 60, \quad n = 50, \quad \bar{X} = 57.1 \quad \text{and} \quad \sigma = 10$$
$$P(\bar{X} > 57.1) \Rightarrow P\left(Z > \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}\right) \quad \text{then} \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{50}} = 1.4142$$

$$\therefore P\left(Z > \frac{57.1 - 60}{1.4142}\right) \Rightarrow P(Z > -2.05) = 0.9798$$

Option 1

CHAPTER 8**QUESTION 4**

Confidence interval (proportion)

$$\text{Given : } n = 400, X = 80 \quad \text{then } p = \frac{X}{n} = \frac{80}{400} = 0.2$$

$$95\% \text{ CI, } p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{P(1-p)}{n}} \quad \text{then } p = 0.20$$

$$Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.2)(0.8)}{400}} = 0.02$$

$$\therefore p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \Rightarrow 0.2 \pm (1.96)(0.02)$$

$$0.2 \pm 0.0392$$

$$[0.1608; 0.2392]$$

Option 1

QUESTION 5Confidence interval (mean, σ known)

$$\text{Given : } \bar{X} = 160, \quad \sigma = 45, \quad n = 36$$

$$95\% \text{ CI for } \mu, \bar{X} \pm t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \quad \text{then}$$

$$\therefore \bar{X} = 160, \quad Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

$$\frac{\sigma}{\sqrt{n}} = \frac{45}{\sqrt{36}} = 7.5$$

$$\therefore \bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \Rightarrow 160 \pm (1.96)(7.5)$$

$$160 \pm 14.70$$

$$[145.30; 174.70]$$

We are 95% confident that the average lifespan of this light bulbs is between 145.30 and 174.70 hours

Option 1

QUESTION 6

Confidence interval (proportion)

$$\text{Given : } n = 1000, \quad \bar{X} = 410 \quad \text{then } p = \frac{X}{n} = 0.41$$

$$95\% \text{ CI for } \pi, p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \quad \text{with}$$

$$p = 0.41$$

$$Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.41(0.59)}{1.000}} = 0.0156$$

$$\therefore p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \Rightarrow 0.41 \pm (1.96)(0.0156)$$

0.41 ± 0.0306
[0.3794; 0.4406]

We are 95% confident that the proportion of first year loan defaults that are approved on the basis of falsified applications is between 37.94% and 44.06%.

Option 1

CHAPTER 9

QUESTION 7

Hypothesis test (proportion)

Given:

$$H_0 : \pi = 0.40 \quad \alpha = 0.05 \quad \text{and}$$

$$H_1 : \pi > 0.40 \quad p = 0.45, \quad n = 49$$

One tailed (right) test.

The standard error of the proportion is given by:

$$\sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{(0.4)(0.6)}{49}} = 0.07$$

Option 2

QUESTION 8

Hypothesis test (proportion)

$$Z_{\text{critical}} = Z_{\alpha} = Z_{0.05} = 1.645$$

Option 2

QUESTION 9

Hypothesis test (mean, σ known)

$$\text{Given : } H_0 : \mu = 50 \quad H_1 : \mu \neq 50$$

$$n = 64 \quad \bar{X} = 53.5 \quad \text{and} \quad \sigma = 10$$

The value of the standardized test statistic is given by

$$Z_{\text{stat}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \quad \text{with}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{64}} = 1.25$$

$$Z_{\text{stat}} = \frac{53.5 - 50}{1.25} = 2.80$$

Option 4

QUESTION 10

Hypothesis test (mean, p -value)

Given,

$$H_0 : \mu = 50$$

$$H_1 : \mu \neq 50 \quad \text{two-tailed test}$$

$$Z_{stat} = 2.80$$

$$\begin{aligned} p\text{-value} &= P(Z > 2.80) \times 2 \\ &= 0.0026 \times 2 \\ &= 0.0052 \text{ or } 0.52\% \end{aligned}$$

Option 2

CHAPTER 11

QUESTION 11

χ^2 distribution

Option 5 is the incorrect statement as the χ^2 curve is positively skewed.

Option 5

QUESTION 12

χ^2 test of independence

χ^2 test statistic or χ_{stat}^2 is given by

$$= \sum \frac{(f_0 - f_e)^2}{f_e}$$

please find the expected frequencies = $f_e = \frac{\text{row total} \times \text{column total}}{n}$

	Beat	Youth	Grow	Live	Total
G	21.84	14.82	21.06	20.28	78
B	34.16	23.18	32.94	31.72	122
Total	56	38	54	52	200

$$\begin{aligned} \chi_{stat}^2 &= 0.6752 + 0.5366 + 0.0534 + 2.9388 + \\ &\quad 0.4317 + 0.3431 + 0.0341 + 1.8789 \\ &= 6.8918 \end{aligned}$$

Option 3

CHAPTER 12

QUESTION 13

Simple linear regression

The coefficient of determination is given by: $r^2 = \frac{SSR}{SST}$ or just using the statistics mode of your calculator to get $r = 0.9798$, then $r^2 = 0.96$.

Option 2

QUESTION 14

Simple regression

Using the calculator or fomrula, $b_0 = 12.0354$ and $b_1 = 1.416$

$$\therefore \hat{Y}_i = 12.0354 + 1.416X_i$$

Option 4

QUESTION 15

Simple regression

Given that $X = 23$

$$\begin{aligned}\hat{Y} &= 12.0354 + 1.416 (23) \\ &= 12.0354 + 32.568 \\ &= 44.60 \text{ approximate profit of R44.60}\end{aligned}$$

Option 1