Tutorial letter 202/1/2019

Basic Statistics

STA1510

Semester 1

Department of Statistics

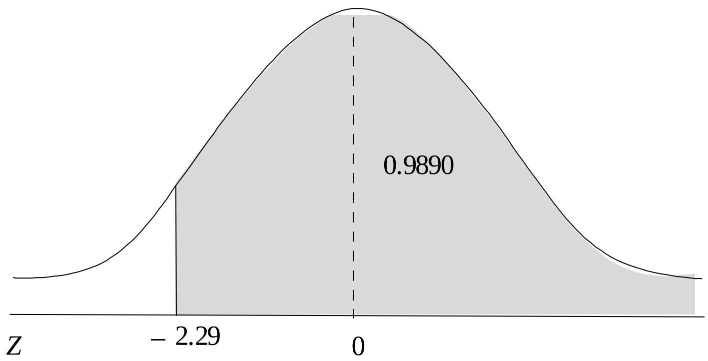
SOLUTIONS TO ASSIGNMENT 02

QUESTION 1

Sampling distribution of the proportion

$$\begin{aligned} \pi &= 0.95 & \sigma_p &= \sqrt{\frac{0.95\,(0.05)}{100}} \\ p &= \frac{91}{100} = 0.91 & &= 0.0218 \end{aligned}$$

$$\begin{aligned} \therefore \quad & P\,(p > 0.90) \\ & P\left(Z > \frac{0.90 - 0.95}{0.0218}\right) \\ & P\,(Z > -2.29) \\ = \;& 0.9890 \text{ from the } Z \text{ tables} \end{aligned}$$



Option 2

QUESTION 2

Confidence interval for μ (σ known)

$$n = 50 \qquad \bar{X} = \text{R}2500 \qquad \sigma = \text{R}600$$

90% confidence interval for μ

$$\begin{aligned} & \bar{X} \pm Z_{\frac{\sigma}{\sqrt{n}}} \\ & 2500 \pm 1.645 \;\; (84.8528) \\ & [2360.42 \le \mu \le 2639.58] \end{aligned}$$

We are 90% confident that the mean monthly rent will be between R2360.42 and R2639.58.

Option 1

QUESTION 3

Confidence interval for μ (σ unknown).

$$\begin{aligned} n &= 20 \\ \bar{X} &= 600 \text{ km} \\ S &= 50 \text{ km} \end{aligned}$$

95% confidence interval for μ.

$$\begin{gathered}\bar{X} \pm t_{\frac{s}{\sqrt{n}}} \\ 600 \pm (2.093)\,(11.1803) \\ [576.60 \leq \mu \leq 623.40]\end{gathered}$$

We are 95% confident that the average distance covered on one tank will be between 576.60 km and 623.40 km.

Option 1

QUESTION 4

Confidence interval for proportion

$$p = \frac{30}{100} = 0.3$$

90% confidence interval for π

$$\begin{gathered}p \pm Z\sqrt{\frac{p\,(1-p)}{n}} \\ 0.3 \pm (1.645)\,(0.0458) \\ 0.3 \pm 0.0753 \\ [0.2247 \leq \pi \leq 0.3753]\end{gathered}$$

We are 90% confident that the proportion of students who pay their own fees will between 22.47% and 37.53%.

Option 3

QUESTION 5

Hypothesis test for proportion, Z_{STAT}

$$\begin{aligned}p &= \frac{36}{200} = 0.18 \qquad \alpha = 0.01 \\ H_0 &: \pi \geq 0.24 \\ H_1 &: \pi < 0.24 \rightarrow \text{ less than} \\ \text{Reject } H_0 \text{ if } Z_{STAT} &< -2.33 \\ \sigma_p &= 0.0302 \\ \therefore \quad Z_{STAT} &= \frac{0.18 - 0.24}{0.0302} = -1.99\end{aligned}$$

Option 4

QUESTION 6

Hypothesis test for proportion, Z_{STAT}

$$\begin{aligned}
\pi &= 0.60 \qquad p = \frac{300}{450} = 0.67 \\
\alpha &= 0.05 \\
H_0 &: \pi = 0.6 \rightarrow \text{ exactly } 60\% \\
H_1 &: \pi \neq 0.6 \\
\therefore \quad Z_{STAT} &= \frac{0.67 - 0.6}{0.0231} \\
&= 3.03
\end{aligned}$$

Option 4

QUESTION 7

Hypothesis testing: State H_0 and H_1 using information in Question 6.

$$\begin{aligned}
H_0 &: \pi = 0.6 \rightarrow \text{ exactly } 60\% \\
H_1 &: \pi \neq 0.6, \text{ a two–tailed test.}
\end{aligned}$$

Option 3

QUESTION 8

χ^2 critical value

$$\begin{aligned}
\alpha &= 0.01 \qquad df = (3-1)(2-1) = 2 \\
\therefore \quad \chi^2_{2;0.01} &= 9.210 \text{ from } \chi^2 \text{ table.}
\end{aligned}$$

Option 5

QUESTION 9

Simple linear regression analysis

$$\begin{aligned}
b_0 &= 2112.80 \\
b_1 &= 0.67 \\
\hat{y} &= 2112.80 + 0.67x
\end{aligned}$$

When $x = 600$ then $\hat{y} = 2112.80 + 0.67\ (600) =$ R2514.80

When $x = 940$ then $\hat{y} = 2112.80 + 0.67\,(940) =$ R2742.60

Option 3

QUESTION 10

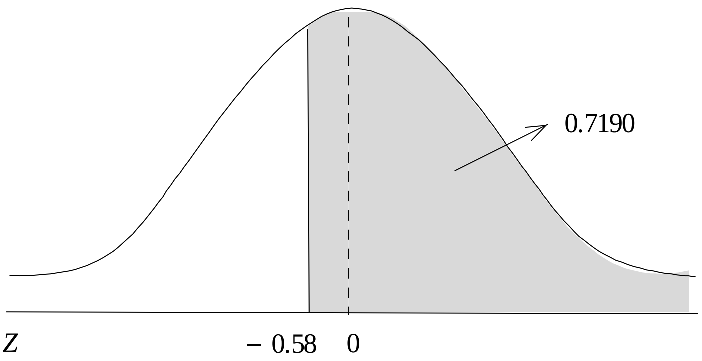
Simple linear regression analysis.
Referring to Question 9, since $r = 0.7996$ or 0.80 then $r^2 = 63.94\%$
$r > 0$ means there is a positive relationship between quantity of units sold and the annual sales.

Option 3

QUESTION 11

Sample distribution of the proportion

$$\begin{aligned} & n = 200 \\ & \pi = 0.6 \\ & P\left(p > 0.58\right) \\ & P\left(z > \frac{0.58 - 0.6}{\sqrt{\frac{0.6(0.4)}{200}}}\right) \\ = \; & P\left(Z > -0.578\right) \\ = \; & 0.7190 \end{aligned}$$

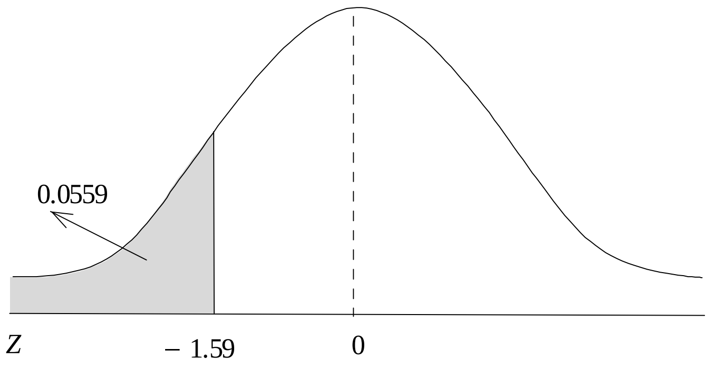


Option 5

QUESTION 12

Hypothesis testing for μ the p–value

$Z_{STAT} = -1.59$ $\quad\quad$ $H_0 : \mu = 50$
$\therefore \quad P\left(Z < -1.59\right)$ $\quad\quad$ $H_1 : \mu < 50$ lower tail test



$= 0.0559$

Option 2

QUESTION 13

χ^2 hypothesis testing: Conclusions.
$\alpha = 0.10$
$df = (2-1)(2-1) = 1$
$\therefore \quad \chi^2_{\text{critical}} = \chi^2_{1;0.10} = 2.706$

	Yes	No	
Cold	10 (6.3)	110 (113.7)	120
Warm	11 (14.7)	269 (265.3)	280
	21	379	400

$$\begin{aligned}\chi^2_{STAT} &= 2.173 + 0.1204 + 0.9313 + 0.0516 \\ &= 3.2763\end{aligned}$$

Since $\chi^2_{STAT} > \chi^2_{\text{critical}}$ $\quad \therefore$ Reject H_0 at 10% level.

Option 3

QUESTION 14

Simple linear regression and correlation analysis. Interpretation of r^2.

$$r^2 = 0.82 \quad \text{or} \quad 82\%$$

The interpretation is that 82% of the variation in the dependent variable can be explained by the variation in the independent variable.

Option 2

QUESTION 15

Correlation analysis. Determine r

$$\begin{aligned}
SSR &= b_0 \sum Y_i + b_1 \sum X_i Y_i - \frac{\left(\sum Y_i\right)^2}{n} \\
&= -0.3517\,(59.97) + 0.1156\,(1496.69) - \frac{(59.97)^2}{30} \\
&= -21.0914 + 173.0174 - 119.88 \\
&= 32.046 \\
SST &= \sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n} = 155.3025 - \frac{(59.97)^2}{30} \\
&= 35.4225 \\
&\therefore \quad r^2 = \frac{SSR}{SST} = \frac{32.046}{35.4225} = 0.9047
\end{aligned}$$

Since b_1 is positive, r will be the positive square root of 0.9047.

$$r = 0.9512$$

Option 2