Chapter 9

9.1a. 1/6

b. 1/6

9.2 a $P(\bar{X} = 1)$ =P(1,1)= 1/36

b $P(\bar{X} = 6)$ = P(6,6) = 1/36

9.3a $P(\bar{X} = 1) = (1/6)^5 = .0001286$

b $P(\bar{X} = 6) = (1/6)^5 = .0001286$

9.4 The variance of $\bar{X}$ is smaller than the variance of X.

9.5 The sampling distribution of the mean is normal with a mean of 40 and a standard deviation of $12/\sqrt{100}$ = 1.2.

9.6 No, because the sample mean is approximately normally distributed.

9.7 a $P(\bar{X} > 1050) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1050-1000}{200/\sqrt{16}}\right)$ = P(Z > 1.00) = 1 – P(Z < 1.00) = 1 – .8413 = .1587

b $P(\bar{X} < 960) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{960-1000}{200/\sqrt{16}}\right)$ = P(Z < –.80) = .2119

c $P(\bar{X} > 1100) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1100-1000}{200/\sqrt{16}}\right)$ = P(Z > 2.00) = 1 – P(Z < 2.00) = 1 – .9772 = .0228

9.8 a $P(\bar{X} > 1050) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1050-1000}{200/\sqrt{25}}\right)$ = P(Z > 1.25) = 1 – P(Z < 1.25) = 1 – .8944 = .1056

b $P(\bar{X} < 960) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{960-1000}{200/\sqrt{25}}\right)$ = P(Z < –1.00) = .1587

c $P(\bar{X} > 1100) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1100-1000}{200/\sqrt{25}}\right)$ = P(Z > 2.50) = 1 – P(Z < 2.50) = 1 – .9938 = .0062

9.9 a $P(\bar{X} > 1050) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1050-1000}{200/\sqrt{100}}\right)$ = P(Z > 2.50) = 1 – P(Z < 2.50) = 1 – .9938 = .0062 b $P(\overline{X} < 960) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{960-1000}{200/\sqrt{100}}\right) = P(Z < -2.00) = .0228$

c $P(\overline{X} > 1100) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{1100-1000}{200/\sqrt{100}}\right) = P(Z > 5.00) = 0$

9.10 a $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{5/\sqrt{4}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{5/\sqrt{4}}\right) = P(-.40 < Z < .80)$

$= P(Z < .80) - P(Z < -.40) = .7881 - .3446 = .4435$

b $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{5/\sqrt{16}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{5/\sqrt{16}}\right) = P(-.80 < Z < 1.60)$

$= P(Z < 1.60) - P(Z < -.80) = .9452 - .2119 = .7333$

c $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{5/\sqrt{25}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{5/\sqrt{25}}\right) = P(-1.00 < Z < 2.00)$

$= P(Z < 2.00) - P(Z < -1.00) = .9772 - .1587 = .8185$

9.11 a $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{10/\sqrt{4}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{10/\sqrt{4}}\right) = P(-.20 < Z < .40)$

$= P(Z < .40) - P(Z < -.20) = .6554 - .4207 = .2347$

b $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{10/\sqrt{16}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{10/\sqrt{16}}\right) = P(-.40 < Z < .80)$

$= P(Z < .80) - P(Z < -.40) = .7881 - .3446 = .4435$

c $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{10/\sqrt{25}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{10/\sqrt{25}}\right) = P(-.50 < Z < 1.00)$

$= P(Z < 1.00) - P(Z < -.50) = .8413 - .3085 = .5328$

9.12 a $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{20/\sqrt{4}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{20/\sqrt{4}}\right) = P(-.10 < Z < .20)$

$= P(Z < .20) - P(Z < -.10) = .5793 - .4602 = .1191$

b $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{20/\sqrt{16}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{20/\sqrt{16}}\right) = P(-.20 < Z < .40)$

$= P(Z < .40) - P(Z < -.20) = .6554 - .4207 = .2347$

c $P(49 < \overline{X} < 52) = P\left(\frac{49-50}{20/\sqrt{25}} < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{52-50}{20/\sqrt{25}}\right) = P(-.25 < Z < .50)$

$= P(Z < .50) - P(Z < -.25) = .6915 - .4013 = .2902$

9.13 a $\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{1{,}000-100}{1{,}000-1}} = .9492$

b $\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{3{,}000-100}{3{,}000-1}} = .9834$

c $\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{5{,}000-100}{5{,}000-1}} = .9900$

d. The finite population correction factor is approximately 1.

9.14 a $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{500}{\sqrt{1{,}000}}\sqrt{\frac{10{,}000-1{,}000}{10{,}000-1}} = 15.00$

b $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{500}{\sqrt{500}}\sqrt{\frac{10{,}000-500}{10{,}000-1}} = 21.80$

c $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{500}{\sqrt{100}}\sqrt{\frac{10{,}000-100}{10{,}000-1}} = 49.75$

9.15 a $P(X > 66) = P\left(\frac{X-\mu}{\sigma} > \frac{66-64}{2}\right) = P(Z > 1.00) = 1 – P(Z < 1.00) = 1 – .8413 = .1587$

b $P(\bar{X} > 66) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{66-64}{2/\sqrt{4}}\right) = P(Z > 2.00) = 1 – P(Z < 2.00) = 1 – .9772 = .0228$

c $P(\bar{X} > 66) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{66-64}{2/\sqrt{100}}\right) = P(Z > 10.00) = 0$

9.16 We can answer part (c) and possibly part (b) depending on how nonnormal the population is.

9.17 a $P(X > 120) = P\left(\frac{X-\mu}{\sigma} > \frac{120-117}{5.2}\right) = P(Z > 0.58) = 1 – P(Z < .58) = 1 – .7190 = .2810$

b $P(\bar{X} > 120) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{120-117}{5.2/\sqrt{4}}\right) = P(Z > 1.15) = 1 – P(Z < 1.15) = 1 – .8749 = .1251$

c $[P(X > 120)]^4 = [.2810]^4 = .00623$

9.18 a $P(X > 60) = P\left(\frac{X-\mu}{\sigma} > \frac{60-52}{6}\right) = P(Z > 1.33) = 1 – P(Z < 1.33) = 1 – .9082 = .0918$

b $P(\bar{X} > 60) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{60-52}{6/\sqrt{3}}\right) = P(Z > 2.31) = 1 – P(Z < 2.31) = 1 – .9896 = .0104$

c $[P(X > 60)]^3 = [.0918]^3 = .00077$

9.19 a $P(X > 12) = P\left(\frac{X-\mu}{\sigma} > \frac{12-10}{3}\right) = P(Z > .67) = 1 - P(Z < .67) = 1 - .7486 = .2514$

b $P(\overline{X} > 275/25) = P(\overline{X} > 11) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{11-10}{3/\sqrt{25}}\right) = P(Z > 1.67) = 1 - P(Z < 1.67) = 1 - .9525 = 0475$

9.20 a $P(X < 75) = P\left(\frac{X-\mu}{\sigma} < \frac{75-78}{6}\right) = P(Z < -.50) = .3085$

b $P(\overline{X} < 75) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{75-78}{6/\sqrt{50}}\right) = P(Z < -3.54) = 1 - P(Z < 3.54) = 1 - 1 = 0$

9.21 a $P(X > 7) = P\left(\frac{X-\mu}{\sigma} > \frac{7-6}{1.5}\right) = P(Z > .67) = 1 - P(Z < .67) = 1 - .7486 = .2514$

b $P(\overline{X} > 7) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{7-6}{1.5/\sqrt{5}}\right) = P(Z > 1.49) = 1 - P(Z < 1.49) = 1 - .9319 = .0681$

c $[P(X > 7)]^5 = [.2514]^5 = .00100$

9.22 a $P(\overline{X} < 5.97) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < \frac{5.97-6.05}{.18/\sqrt{36}}\right) = P(Z < -2.67) = .0038$

b It appears to be false.

9.23 $P(\overline{X} > 10{,}000/16) = P(\overline{X} > 625) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{625-600}{200/\sqrt{16}}\right) = P(Z > .50) = 1 - P(Z < .50)$

$= 1 - .6915 = .3085$

9.24 The professor needs to know the mean and standard deviation of the population of the weights of elevator users and that the distribution is not extremely nonnormal.

9.25 $P(\overline{X} > 1{,}140/16) = P(\overline{X} > 71.25) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{71.25-75}{10/\sqrt{16}}\right) = P(Z > -1.50)$

$= 1 - P(Z < -1.50) = 1 - 0668 = .9332$

9.26 P(Total time > 300) = $P(\overline{X} > 300/60) = P(\overline{X} > 5) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{5-4.8}{1.3/\sqrt{60}}\right) = P(Z > 1.19)$

$= 1 – P(Z < 1.19) = 1 – .8830 = .1170$

9.27 No because the central limit theorem says that the sample mean is approximately normally distributed.

9.28 $P(\text{Total number of cups} > 240) = P(\overline{X} > 240/125) = P(\overline{X} > 1.92) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{1.92-2.0}{.6/\sqrt{125}}\right)$

$= P(Z > –1.49) = 1 - P(Z < -1.49) = 1 - .0681 = .9319$

9.29 $P(\text{Total number of faxes} > 1500) = P(\overline{X} > 1500/5) = P(\overline{X} > 300) = P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} > \frac{300-275}{75/\sqrt{5}}\right)$

$= P(Z > .75) = 1 – P(Z < .75) = 1 – .7734 = .2266$

9.30a $P(\hat{P} > .60) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.60-.5}{\sqrt{(.5)(1-.5)/300}}\right) = P(Z > 3.46) = 0$

b. $P(\hat{P} > .60) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.60-.55}{\sqrt{(.55)(1-.55)/300}}\right) = P(Z > 1.74) = 1 – P(Z < 1.74)$

$= 1 – .9591 = .0409$

c. $P(\hat{P} > .60) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.60-.6}{\sqrt{(.6)(1-.6)/300}}\right) = P(Z > 0) = 1 – P(Z < 0) = 1 - .5 = .5$

9.31a $P(\hat{P} < .22) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.22-.25}{\sqrt{(.25)(1-.25)/500}}\right) = P(Z < –1.55) = .0606$

b. $P(\hat{P} < .22) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.22-.25}{\sqrt{(.25)(1-.25)/800}}\right) = P(Z < –1.96) = .0250$

c. $P(\hat{P} < .22) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.22-.25}{\sqrt{(.25)(1-.25)/1000}}\right) = P(Z < –2.19) = .0143$

9.32 $P(\hat{P} < .75) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.75-.80}{\sqrt{(.80)(1-.80)/100}}\right) = P(Z < –1.25) = .1056$

9.33 $P(\hat{P} > .35) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.35-.40}{\sqrt{(.40)(1-.40)/60}}\right) = P(Z > –.79) = .1 - P(Z < -.79)$

$1 - .2148 = .7852$

9.34 $P(\hat{P} < .49) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} < \frac{.49-.55}{\sqrt{(.55)(1-.55)/500}}\right) = P(Z < -2.70) = .0035$

9.35 $P(\hat{P} > .04) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.04-.02}{\sqrt{(.02)(1-.02)/800}}\right) = P(Z > 4.04) = 1 - P(Z < 4.04) = 1 - 1 = 0;$

The defective rate appears to be larger than 2%.

9.36 a $P(\hat{P} < .50) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} < \frac{.50-.53}{\sqrt{(.53)(1-.53)/400}}\right) = P(Z < -1.20) = .1151$; the claim may be true

b $P(\hat{P} < .50) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} < \frac{.50-.53}{\sqrt{(.53)(1-.53)/1{,}000}}\right) = P(Z < -1.90) = .0287$; the claim appears to be false

9.37 $P(\hat{P} > .10) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.10-.14}{\sqrt{(.14)(1-.14)/100}}\right) = P(Z > -1.15) = 1 - P(Z < -1.15)$

$= 1 - .1251 = .8749$

9.38 $P(\hat{P} > .05) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.05-.03}{\sqrt{(.03)(1-.03)/400}}\right) = P(Z > 2.34) = 1 - P(Z < 2.34) = 1 - .9904$

$= .0096$; the commercial appears to be dishonest

9.39 $P(\hat{P} > .32) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.32-.30}{\sqrt{(.30)(1-.30)/1{,}000}}\right) = P(Z > 1.38) = 1 - P(Z < 1.38)$

$= 1 - .9162 = .0838$

9.40 a $P(\hat{P} < .45) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} < \frac{.45-.50}{\sqrt{(.50)(1-.50)/600}}\right) = P(Z < -2.45) = .0071$

b The claim appears to be false.

9.41 $P(\hat{P} < .75) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} < \frac{.75-.80}{\sqrt{(.80)(1-.80)/350}}\right) = P(Z < -2.34) = .0096$

9.42 $P(\hat{P} < .70) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} < \frac{.70-.75}{\sqrt{(.75)(1-.75)/460}}\right) = P(Z < -2.48) = .0066$

9.43 $P(\hat{P} > .28) = P\left(\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} > \frac{.28-.25}{\sqrt{(.25)(1-.25)/1200}}\right) = P(Z > 2.40) = 1 - P(Z < 2.40)$

$= 1 - .9918 = .0082$

9.44 The claim appears to be false.

9.45 $P(\overline{X}_1 - \overline{X}_2 > 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0-(18-15)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right) = P(Z > -2.24) = 1 - P(Z < -2.24)$

$= 1 - .0125 = .9875$

9.46 $P(\overline{X}_1 - \overline{X}_2 > 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0-(75-65)}{\sqrt{\frac{20^2}{5} + \frac{21^2}{5}}}\right) = P(Z > -.77) = 1 - P(Z < -.77)$

$= 1 - .2206 = .7794$

9.47 $P(\overline{X}_1 - \overline{X}_2 > 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0-(73-77)}{\sqrt{\frac{12^2}{4} + \frac{10^2}{4}}}\right) = P(Z > .51) = 1 - P(Z < .51)$

$= 1 - .6950 = .3050$

9.48 $P(\overline{X}_1 - \overline{X}_2 < 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < \frac{0-(10-15)}{\sqrt{\frac{3^2}{25} + \frac{3^2}{25}}}\right) = P(Z < 5.89) = 1$

9.49 $P(\overline{X}_1 - \overline{X}_2 > 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0-(40-38)}{\sqrt{\frac{6^2}{25} + \frac{8^2}{25}}}\right) = P(Z > -1.00) = 1 - P(Z < -1.00)$

$1 - .1587 = .8413$

9.50 $P(\overline{X}_1 - \overline{X}_2 > 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0 - (40 - 38)}{\sqrt{\frac{12^2}{25} + \frac{16^2}{25}}}\right)$ = P(Z > –.50) = 1 – P(Z < –.50)

= 1 – .3085 = .6915

9.51 $P(\overline{X}_1 - \overline{X}_2 > 0) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{0 - (140 - 138)}{\sqrt{\frac{6^2}{25} + \frac{8^2}{25}}}\right)$ = P(Z > –1.00) = 1 – P(Z < –1.00)

= 1 – .1587 = .8413

9.52 $P(\overline{X}_1 - \overline{X}_2 > 25) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{25 - (280 - 270)}{\sqrt{\frac{25^2}{10} + \frac{30^2}{10}}}\right)$ = P(Z > 1.21) = 1 – P(Z < 1.21)

= 1 – .8869 = .1131

9.53 $P(\overline{X}_1 - \overline{X}_2 > 25) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{25 - (280 - 270)}{\sqrt{\frac{25^2}{50} + \frac{30^2}{50}}}\right)$ = P(Z > 2.72) = 1 – P(Z < 2.72)

= 1 – .9967 = .0033

9.54 $P(\overline{X}_1 - \overline{X}_2 > 25) = P\left(\frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{25 - (280 - 270)}{\sqrt{\frac{25^2}{100} + \frac{30^2}{100}}}\right)$ = P(Z > 3.84) = 1 – P(Z < 3.84)

= 1 – 1= 0