

Appendix 15

A15.1 Chi-squared goodness-of-fit test

$$H_0 : p_1 = .50, p_2 = .20, p_3 = .15, p_4 = .10, p_5 = .05$$

H_1 : At least one p_i is not equal to its specified value.

$$\chi^2 = \sum \frac{(f_i - e_i)^2}{e_i}$$

| | C | D | E |
|---|--------|----------|--------|
| 1 | Actual | Expected | |
| 2 | 183 | 175 | |
| 3 | 63 | 70 | |
| 4 | 55 | 52.5 | |
| 5 | 29 | 35 | |
| 6 | 20 | 17.5 | |
| 7 | | p-value= | 0.6321 |

p-value = .6321. There is not enough evidence to conclude that applicants to WLU's MBA program are different in terms of their undergraduate degrees from the population of MBA applicants?

A15.2 t-test of μ_D

$$H_0 : \mu_D = 0$$

$$H_1 : \mu_D < 0$$

$$t = \frac{\bar{x}_D - \mu_D}{s_D / \sqrt{n_D}}$$

| | A | B | C |
|----|-------------------------------------|------------------|-------------------|
| 1 | t-Test: Paired Two Sample for Means | | |
| 2 | | | |
| 3 | | <i>First Sat</i> | <i>Second SAT</i> |
| 4 | Mean | 1175 | 1190 |
| 5 | Variance | 28422 | 35392 |
| 6 | Observations | 40 | 40 |
| 7 | Pearson Correlation | 0.91 | |
| 8 | Hypothesized Mean Difference | 0 | |
| 9 | df | 39 | |
| 10 | t Stat | -1.20 | |
| 11 | P(T<=t) one-tail | 0.1182 | |
| 12 | t Critical one-tail | 1.6849 | |
| 13 | P(T<=t) two-tail | 0.2365 | |
| 14 | t Critical two-tail | 2.0227 | |

$t = -1.20$, $p\text{-value} = .1182$. There is not enough evidence to indicate that repeating the SAT produces higher exam scores.

A15.3 Time to solve the 48 problems: Equal-variances t-test of $\mu_1 - \mu_2$

$$H_0 : (\mu_1 - \mu_2) = 0$$

$$H_1 : (\mu_1 - \mu_2) > 0$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

| | A | B | C |
|----|---|-------------|------------|
| 1 | t-Test: Two-Sample Assuming Equal Variances | | |
| 2 | | | |
| 3 | | <i>Diet</i> | <i>Not</i> |
| 4 | Mean | 581.95 | 551.5 |
| 5 | Variance | 2716.6 | 2221.5 |
| 6 | Observations | 20 | 20 |
| 7 | Pooled Variance | 2469.1 | |
| 8 | Hypothesized Mean Difference | 0 | |
| 9 | df | 38 | |
| 10 | t Stat | 1.94 | |
| 11 | P(T<=t) one-tail | 0.0300 | |
| 12 | t Critical one-tail | 1.6860 | |
| 13 | P(T<=t) two-tail | 0.0601 | |
| 14 | t Critical two-tail | 2.0244 | |

$t = 1.94$, $p\text{-value} = .0300$. There is enough evidence to conclude that dieters take longer to solve the 48 problems than do nondieters.

Successfully repeat string of five letters: z-test of $p_1 - p_2$ (case 1)

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 < 0$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

| | A | B | C | D |
|----|--------------------------------|---|-------------|------------|
| 1 | z-Test: Two Proportions | | | |
| 2 | | | | |
| 3 | | | <i>Diet</i> | <i>Not</i> |
| 4 | Sample Proportions | | 0.50 | 0.80 |
| 5 | Observations | | 20 | 20 |
| 6 | Hypothesized Difference | | 0 | |
| 7 | z Stat | | -1.99 | |
| 8 | P(Z<=z) one tail | | 0.0234 | |
| 9 | z Critical one-tail | | 1.6449 | |
| 10 | P(Z<=z) two-tail | | 0.0468 | |
| 11 | z Critical two-tail | | 1.96 | |

$z = -1.99$, $p\text{-value} = .0234$. There is enough evidence to conclude that dieters are less successful at repeating string of five letters.

Successfully repeat string of five words: z-test of $p_1 - p_2$ (case 1)

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 > 0$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

| | A | B | C | D |
|----|--------------------------------|---|-------------|------------|
| 1 | z-Test: Two Proportions | | | |
| 2 | | | | |
| 3 | | | <i>Diet</i> | <i>Not</i> |
| 4 | Sample Proportions | | 0.35 | 0.60 |
| 5 | Observations | | 20 | 20 |
| 6 | Hypothesized Difference | | 0 | |
| 7 | z Stat | | -1.58 | |
| 8 | P(Z<=z) one tail | | 0.0567 | |
| 9 | z Critical one-tail | | 1.6449 | |
| 10 | P(Z<=z) two-tail | | 0.1134 | |
| 11 | z Critical two-tail | | 1.96 | |

$z = -1.58$, $p\text{-value} = .0567$. There is not enough evidence to conclude that dieters are less successful at repeating string of five words.

A15.4 a t-estimator of μ

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

| | A | B | C | D |
|---|-------------------------|---|---|----------------|
| 1 | t-Estimate: Mean | | | |
| 2 | | | | |
| 3 | | | | <i>Overdue</i> |
| 4 | Mean | | | 7.09 |
| 5 | Standard Deviation | | | 6.97 |
| 6 | LCL | | | 6.40 |
| 7 | UCL | | | 7.77 |

LCL = 6.40, UCL = 7.77

b LCL = 50,000(\$0.25)(6.40) = \$80,000

UCL = 50,000(\$0.25)(7.77) = \$97,125

It does appear that not all fines are collected

A15.5 One-way analysis of variance

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

H_1 : At least two means differ

| | A | B | C | D | E | F | G |
|----|----------------------------|-----------|-----------|-----------|----------|----------------|---------------|
| 10 | ANOVA | | | | | | |
| 11 | <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> | <i>P-value</i> | <i>F crit</i> |
| 12 | Between Groups | 57512 | 2 | 28756 | 3.23 | 0.0468 | 3.16 |
| 13 | Within Groups | 506984 | 57 | 8894 | | | |
| 14 | | | | | | | |
| 15 | Total | 564496 | 59 | | | | |

F = 3.23; p-value = .0468. There is enough evidence to conclude that differences in sales exist between the three advertising strategies.

A15.6 Chi-squared test of a contingency table

H_0 : The two variables (income category and mutual fund ownership) are independent

H_1 : The two variables are dependent

$$\chi^2 = \sum \frac{(f_i - e_i)^2}{e_i}$$

| | A | B | C | D | E |
|----|--------------------------|------------------------|-----|-----|---------|
| 1 | Contingency Table | | | | |
| 2 | | | | | |
| 3 | | <i>Income category</i> | | | |
| 4 | <i>Mutual fund</i> | | 1 | 2 | TOTAL |
| 5 | | 1 | 71 | 13 | 84 |
| 6 | | 2 | 59 | 28 | 87 |
| 7 | | 3 | 86 | 55 | 141 |
| 8 | | 4 | 87 | 157 | 244 |
| 9 | | 5 | 32 | 145 | 177 |
| 10 | | 6 | 58 | 205 | 263 |
| 11 | | TOTAL | 393 | 603 | 996 |
| 12 | | | | | |
| 13 | | | | | |
| 14 | | chi-squared Stat | | | 196.77 |
| 15 | | df | | | 5 |
| 16 | | p-value | | | 0 |
| 17 | | chi-squared Critical | | | 11.0705 |

$\chi^2 = 196.77$; p-value = 0. There is overwhelming evidence to infer that household income and ownership of mutual funds are related

A15.7 Two-factor analysis of variance

| | A | B | C | D | E | F | G |
|----|----------------------------|-----------|-----------|-----------|----------|----------------|---------------|
| 23 | ANOVA | | | | | | |
| 24 | <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> | <i>P-value</i> | <i>F crit</i> |
| 25 | Sample | 13172 | 1 | 13172 | 1.42 | 0.2387 | 4.02 |
| 26 | Columns | 98839 | 2 | 49419 | 5.33 | 0.0077 | 3.17 |
| 27 | Interaction | 1610 | 2 | 805 | 0.09 | 0.9171 | 3.17 |
| 28 | Within | 501137 | 54 | 9280 | | | |
| 29 | | | | | | | |
| 30 | Total | 614757 | 59 | | | | |

Interaction: $F = .09$; p-value = .9171. There is no evidence of interaction.

Advertising strategy (Columns): $F = 5.33$; p-value = .0077. There is sufficient evidence to conclude that advertising strategies differ with respect to sales.

Media (Sample): $F = 1.42$; p-value = .2387. There is not enough evidence to conclude that differences in the medium for advertising differ in terms of sales.

A15.8 z-test of $p_1 - p_2$ (case 1) Code 3 results were omitted.

$$H_0 : (p_1 - p_2) = 0$$

$$H_1 : (p_1 - p_2) < 0$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

| | A | B | C | D |
|----|--------------------------------|---|-------------------|----------------|
| 1 | z-Test: Two Proportions | | | |
| 2 | | | | |
| 3 | | | <i>Folic acid</i> | <i>Placebo</i> |
| 4 | Sample Proportions | | 0.0101 | 0.0343 |
| 5 | Observations | | 597 | 612 |
| 6 | Hypothesized Difference | | 0 | |
| 7 | z Stat | | -2.85 | |
| 8 | P(Z<=z) one tail | | 0.0022 | |
| 9 | z Critical one-tail | | 1.6449 | |
| 10 | P(Z<=z) two-tail | | 0.0044 | |
| 11 | z Critical two-tail | | 1.9600 | |

$z = -2.85$, $p\text{-value} = .0022$. There is overwhelming evidence to conclude that folic acid reduces the incidence of spina bifida.

A15.9 Unequal-variances t-test of $\mu_1 - \mu_2$

$$H_0 : (\mu_1 - \mu_2) = 0$$

$$H_1 : (\mu_1 - \mu_2) < 0$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} =$$

| | A | B | C |
|----|--|----------------|-----------------|
| 1 | t-Test: Two-Sample Assuming Unequal Variances | | |
| 2 | | | |
| 3 | | <i>British</i> | <i>American</i> |
| 4 | Mean | 238.0 | 252.0 |
| 5 | Variance | 149.9 | 220.2 |
| 6 | Observations | 263 | 279 |
| 7 | Hypothesized Mean Difference | 0 | |
| 8 | df | 531 | |
| 9 | t Stat | -12.00 | |
| 10 | P(T<=t) one-tail | 7.64E-30 | |
| 11 | t Critical one-tail | 1.6477 | |
| 12 | P(T<=t) two-tail | 1.53E-29 | |
| 13 | t Critical two-tail | 1.9644 | |

$z = -12.00$, $p\text{-value} = 0$. There is enough evidence to infer that British golfers play golf in less time than do American golfers.

A15.10 one-way analysis of variance

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

H_1 : At least two means differ

| | A | B | C | D | E | F | G |
|----|----------------------------|-----------|-----------|-----------|----------|----------------|---------------|
| 10 | ANOVA | | | | | | |
| 11 | <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> | <i>P-value</i> | <i>F crit</i> |
| 12 | Between Groups | 626046 | 2 | 313023 | 58.37 | 0.0000 | 3.03 |
| 13 | Within Groups | 1523047 | 284 | 5363 | | | |
| 14 | | | | | | | |
| 15 | Total | 2149093 | 286 | | | | |

$F = 58.37$, $p\text{-value} = 0$. There is enough evidence to conclude that there are differences between the three groups.

Multiple Comparisons

| | A | B | C | D | E |
|---|-----------------------------|-------------------|------------|----------------|--------------|
| 1 | Multiple Comparisons | | | | |
| 2 | | | | | |
| 3 | | | | LSD | Omega |
| 4 | Treatment | Treatment | Difference | Alpha = 0.0167 | Alpha = 0.05 |
| 5 | <i>Before 1976</i> | <i>After 1986</i> | 122.62 | 28.03 | 25.46 |
| 6 | | <i>Canadian</i> | 78.04 | 24.67 | 25.46 |
| 7 | <i>After 1986</i> | <i>Canadian</i> | -44.58 | 25.75 | 25.46 |

All three groups differ from each other.

A15.11 Chi-squared test of a contingency table

H_0 : The two variables (year and sport) are independent

H_1 : The two variables are dependent

$$\chi^2 = \sum \frac{(f_i - e_i)^2}{e_i}$$

| | A | B | C | D | E |
|----|--------------------------|----------------------|-----|-----|---------|
| 1 | Contingency Table | | | | |
| 2 | | | | | |
| 3 | | <i>Sport</i> | | | |
| 4 | <i>Year</i> | | 1 | 2 | TOTAL |
| 5 | | 1 | 116 | 122 | 238 |
| 6 | | 2 | 119 | 92 | 211 |
| 7 | | 3 | 29 | 58 | 87 |
| 8 | | 4 | 52 | 39 | 91 |
| 9 | | 5 | 48 | 34 | 82 |
| 10 | | 6 | 16 | 33 | 49 |
| 11 | | 7 | 26 | 29 | 55 |
| 12 | | 8 | 24 | 21 | 45 |
| 13 | | 9 | 70 | 72 | 142 |
| 14 | | TOTAL | 500 | 500 | 1000 |
| 15 | | | | | |
| 16 | | | | | |
| 17 | | chi-squared Stat | | | 23.8101 |
| 18 | | df | | | 8 |
| 19 | | p-value | | | 0.0025 |
| 20 | | chi-squared Critical | | | 15.5073 |

$\chi^2 = 23.8101$, p-value = .0025. There is enough evidence to infer that North Americans changed their favorite sport between 1985 and 1992.

A15.12 t-estimator of μ

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

| | A | B | C | D |
|---|-------------------------|---|---|-------------|
| 1 | t-Estimate: Mean | | | |
| 2 | | | | |
| 3 | | | | <i>Cars</i> |
| 4 | Mean | | | 165.79 |
| 5 | Standard Deviation | | | 51.59 |
| 6 | LCL | | | 157.17 |
| 7 | UCL | | | 174.41 |

Five minute interval: LCL = 157.17, UCL = 174.41

Twenty-four hour day (12 5-minute intervals, 24 hours per day):

$$LCL = 12 \times 24 \times 157.17 = 45,265$$

$$UCL = 12 \times 24 \times 174.41 = 50,230$$

A15.13 z-estimator of p

$$\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}$$

| | A | B |
|---|-------------------------------|------------------|
| 1 | z-Estimate: Proportion | |
| 2 | | <i>Exercise?</i> |
| 3 | Sample Proportion | 0.551 |
| 4 | Observations | 671 |
| 5 | LCL | 0.514 |
| 6 | UCL | 0.589 |

Total number of adults who exercise:

$$\text{LCL} = 205.9 \text{ million } (.514) = 105.8 \text{ million}$$

$$\text{UCL} = 205.9 \text{ million } (.589) = 121.3 \text{ million}$$

A15.14 Equal-variances t-test of $\mu_1 - \mu_2$

$$H_0 : (\mu_1 - \mu_2) = 0$$

$$H_1 : (\mu_1 - \mu_2) < 0$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

| | A | B | C |
|----|--|-----------------|--------------|
| 1 | t-Test: Two-Sample Assuming Equal Variances | | |
| 2 | | | |
| 3 | | <i>Activity</i> | <i>Usual</i> |
| 4 | Mean | 57.06 | 87.28 |
| 5 | Variance | 296.18 | 215.42 |
| 6 | Observations | 67 | 67 |
| 7 | Pooled Variance | 255.80 | |
| 8 | Hypothesized Mean Difference | 0.00 | |
| 9 | df | 132 | |
| 10 | t Stat | -10.94 | |
| 11 | P(T<=t) one-tail | 0.0000 | |
| 12 | t Critical one-tail | 1.6565 | |
| 13 | P(T<=t) two-tail | 0.0000 | |
| 14 | t Critical two-tail | 1.9781 | |

$t = -10.94$, $p\text{-value} = 0$. There is enough evidence to indicate to infer that graded activity is effective.

A15.15 Chi-squared test of a contingency table

H_0 : The two variables (group and improvement) are independent

H_1 : The two variables are dependent

$$\chi^2 = \sum \frac{(f_i - e_i)^2}{e_i}$$

| | A | B | C | D | E |
|----|--------------------------|----------------------|----|----|--------|
| 1 | Contingency Table | | | | |
| 2 | | | | | |
| 3 | | Group | | | |
| 4 | Improvement | | 1 | 2 | TOTAL |
| 5 | | 1 | 42 | 8 | 50 |
| 6 | | 2 | 32 | 18 | 50 |
| 7 | | 3 | 13 | 37 | 50 |
| 8 | | TOTAL | 87 | 63 | 150 |
| 9 | | | | | |
| 10 | | | | | |
| 11 | | chi-squared Stat | | | 35.63 |
| 12 | | df | | | 2 |
| 13 | | p-value | | | 0 |
| 14 | | chi-squared Critical | | | 5.9915 |

$\chi^2 = 35.63$, p-value = 0. There is sufficient evidence to infer there are differences between the three groups.

A15.16 z-estimator of p

$$\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}$$

| | A | B | C | D | E |
|---|-----------------------------------|-------|-------------------------------------|----------|---------------|
| 1 | z-Estimate of a Proportion | | | | |
| 2 | | | | | |
| 3 | Sample proportion | 0.774 | Confidence Interval Estimate | | |
| 4 | Sample size | 780 | 0.774 | ± | 0.0294 |
| 5 | Confidence level | 0.95 | Lower confidence limit | | 0.7446 |
| 6 | | | Upper confidence limit | | 0.8034 |

Total number of on-time departures:

$$LCL = 7,140,596(.7446) = 5,316,888$$

$$UCL = 7,140,596(.8034) = 5,736,755$$

Case A15.1 One-way analysis of variance

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

H_1 : At least two means differ

Weight loss:

| | A | B | C | D | E | F | G |
|----|----------------------------|-----------|-----------|-----------|----------|----------------|---------------|
| 11 | ANOVA | | | | | | |
| 12 | <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> | <i>P-value</i> | <i>F crit</i> |
| 13 | Between Groups | 189.4 | 3 | 63.14 | 1.78 | 0.1532 | 2.66 |
| 14 | Within Groups | 5533.6 | 156 | 35.47 | | | |
| 15 | | | | | | | |
| 16 | Total | 5723.0 | 159 | | | | |

$F = 1.78$, $p\text{-value} = .1532$. There is not enough evidence to conclude that there are differences in weight loss between the four diets.

Percent LDL decrease:

| | A | B | C | D | E | F | G |
|----|----------------------------|-----------|-----------|-----------|----------|----------------|---------------|
| 11 | ANOVA | | | | | | |
| 12 | <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> | <i>P-value</i> | <i>F crit</i> |
| 13 | Between Groups | 2569.4 | 3 | 856.48 | 32.56 | 0.0000 | 2.66 |
| 14 | Within Groups | 4104.0 | 156 | 26.31 | | | |
| 15 | | | | | | | |
| 16 | Total | 6673.4 | 159 | | | | |

$F = 32.56$, $p\text{-value} = 0$. There is enough evidence to conclude that there are differences in bad cholesterol reduction between the four diets.

Percent HDL Increase:

| | A | B | C | D | E | F | G |
|----|----------------------------|-----------|-----------|-----------|----------|----------------|---------------|
| 11 | ANOVA | | | | | | |
| 12 | <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> | <i>P-value</i> | <i>F crit</i> |
| 13 | Between Groups | 5595.0 | 3 | 1864.99 | 92.62 | 0.0000 | 2.66 |
| 14 | Within Groups | 3141.3 | 156 | 20.14 | | | |
| 15 | | | | | | | |
| 16 | Total | 8736.2 | 159 | | | | |

$F = 96.62$, $p\text{-value} = 0$. There is enough evidence to conclude that there are differences in good cholesterol increase between the four diets.

