Appendix 14

A14.1 One-way Analysis of variance

$H_0: \mu_1 = \mu_2 = \mu_3$

$H_1:$ At least two means differ

	A	B	C	D	E	F	G
10	ANOVA						
11	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
12	Between Groups	91.43	2	45.72	1.09	0.3441	3.16
13	Within Groups	2397.5	57	42.06			
14							
15	Total	2488.9	59				

F = 1.09, p-value = .3441. There is no evidence to infer that sales of candy differ according to placement.

A14.2 t-test of μ_D

$H_0: \mu_D = 0$

$H_1: \mu_D < 0$

$$t = \frac{\bar{x}_D - \mu_D}{s_D / \sqrt{n_D}}$$

	A	B	C
1	t-Test: Paired Two Sample for Means		
2			
3		*Price shown*	*Price not shown*
4	Mean	56.15	60.31
5	Variance	243.68	467.71
6	Observations	100	100
7	Pearson Correlation	0.79	
8	Hypothesized Mean Difference	0	
9	df	99	
10	t Stat	-3.12	
11	P(T<=t) one-tail	0.0012	
12	t Critical one-tail	1.6604	
13	P(T<=t) two-tail	0.0024	
14	t Critical two-tail	1.9842	

t = –3.12, p-value = .0012. There is overwhelming evidence to conclude that ads with no price shown are more effective in generating interest than ads that show the price.

A14.3 Unequal-variances t-test of $\mu_1 - \mu_2$

$H_0: (\mu_1 - \mu_2) = 0$

$H_1: (\mu_1 - \mu_2) < 0$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

	A	B	C
1	t-Test: Two-Sample Assuming Unequal Variances		
2			
3		*Leftover*	*Returned*
4	Mean	61.71	70.57
5	Variance	48.99	203.98
6	Observations	14	53
7	Hypothesized Mean Difference	0	
8	df	44	
9	t Stat	-3.27	
10	P(T<=t) one-tail	0.0011	
11	t Critical one-tail	1.6802	
12	P(T<=t) two-tail	0.0021	
13	t Critical two-tail	2.0154	

t = –3.27, p-value = .0011. There is enough evidence to support the professor's theory.

A14.4 a z-test of $p_1 - p_2$ (case 1)

$$H_0: p_1 - p_2 = 0$$

$$H_1: p_1 - p_2 > 0$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

	A	B	C	D
1	**z-Test: Two Proportions**			
2				
3			*Topiramate*	*Placebo*
4	Sample Proportions		0.2364	0.1042
5	Observations		55	48
6	Hypothesized Difference		0	
7	z Stat		1.76	
8	P(Z<=z) one tail		0.0390	
9	z Critical one-tail		1.6449	
10	P(Z<=z) two-tail		0.0780	
11	z Critical two-tail		1.96	

z = 1.76, p-value = .0390. There is enough evidence to conclude that topiramate is effective in causing abstinence for the first month.

b z-test of $p_1 - p_2$ (case 1)

$$H_0: p_1 - p_2 = 0$$

$H_1: p_1 - p_2 > 0$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

	A	B	C	D
1	**z-Test: Two Proportions**			
2				
3			*Topiramate*	*Placebo*
4	Sample Proportions		0.5091	0.1667
5	Observations		55	48
6	Hypothesized Difference		0	
7	z Stat		3.64	
8	P(Z<=z) one tail		0.0001	
9	z Critical one-tail		1.6449	
10	P(Z<=z) two-tail		0.0002	
11	z Critical two-tail		1.96	

z = 3.64, p-value = .0001. There is enough evidence to conclude that topiramate is effective in causing alcoholics to refrain from binge drinking in the final month.

A14.5a Equal-variances t-test of $\mu_1 - \mu_2$

$H_0: (\mu_1 - \mu_2) = 0$

$H_1: (\mu_1 - \mu_2) < 0$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

	A	B	C
1	t-Test: Two-Sample Assuming Equal Variances		
2			
3		*Expenses MSA*	*Expenses Regular*
4	Mean	347.24	479.25
5	Variance	21043	21128
6	Observations	63	141
7	Pooled Variance	21102	
8	Hypothesized Mean Difference	0	
9	df	202	
10	t Stat	-6.00	
11	P(T<=t) one-tail	0.0000	
12	t Critical one-tail	1.6524	
13	P(T<=t) two-tail	0.0000	
14	t Critical two-tail	1.9718	

t = –6.00, p-value = 0. There is enough evidence to infer that medical expenses for those under the MSA plan are lower than those who are not.

b z-test of $p_1 - p_2$ (case 1)

$$H_0: p_1 - p_2 = 0$$

$$H_1: p_1 - p_2 < 0$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

	A	B	C	D
1	**z-Test: Two Proportions**			
2				
3			*Health MSA*	*Health Regular*
4	Sample Proportions		0.7619	0.7801
5	Observations		63	141
6	Hypothesized Difference		0	
7	z Stat		-0.29	
8	P(Z<=z) one tail		0.3867	
9	z Critical one-tail		1.6449	
10	P(Z<=z) two-tail		0.7734	
11	z Critical two-tail		1.9600	

z = –.29, p-value = .3867. There is not enough evidence to support the critics of MSA.

A14.6 a One-way analysis of variance

	A	B	C	D	E	F	G
11	ANOVA						
12	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
13	Between Groups	25113	3	8371.2	85.98	0.0000	2.63
14	Within Groups	30766	316	97.36			
15							
16	Total	55880	319				

F = 85.98, p-value = 0.

Two-factor analysis of variance

	A	B	C	D	E	F	G
23	ANOVA						
24	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
25	Sample	17024	1	17024	174.85	0.0000	3.87
26	Columns	7411	1	7411	76.12	0.0000	3.87
27	Interaction	679	1	678.6	6.97	0.0087	3.87
28	Within	30766	316	97.36			
29							
30	Total	55880	319				

Interaction: F = 6.97; p-value = .0087. There is enough evidence to infer that differences are caused by interaction.

There is no need to conduct the other two tests.

A14.7 t-tests of μ

45 minutes: $H_0: \mu = 45$

$H_1: \mu < 45$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

	A	B	C	D
1	**t-Test: Mean**			
2				
3				*45 minutes*
4	Mean			41.75
5	Standard Deviation			3.63
6	Hypothesized Mean			45
7	df			19
8	t Stat			-4.01
9	P(T<=t) one-tail			0.0004
10	t Critical one-tail			1.7291
11	P(T<=t) two-tail			0.0008
12	t Critical two-tail			2.0930

60 minutes: $H_0: \mu = 60$

$H_1: \mu < 60$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

	A	B	C	D
1	**t-Test: Mean**			
2				
3				*60 minutes*
4	Mean			58.75
5	Standard Deviation			5.02
6	Hypothesized Mean			60
7	df			19
8	t Stat			-1.11
9	P(T<=t) one-tail			0.1399
10	t Critical one-tail			1.7291
11	P(T<=t) two-tail			0.2798
12	t Critical two-tail			2.0930

80 minutes: $H_0: \mu = 80$

$H_1: \mu < 80$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

	A	B	C	D
1	**t-Test: Mean**			
2				
3				*80 minutes*
4	Mean			69.05
5	Standard Deviation			6.31
6	Hypothesized Mean			80
7	df			19
8	t Stat			-7.76
9	P(T<=t) one-tail			0.0000
10	t Critical one-tail			1.7291
11	P(T<=t) two-tail			0.0000
12	t Critical two-tail			2.0930

100 minutes: $H_0 : \mu = 100$

$H_1 : \mu < 100$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

	A	B	C	D
1	**t-Test: Mean**			
2				
3				*100 minutes*
4	Mean			90.40
5	Standard Deviation			12.35
6	Hypothesized Mean			100
7	df			19
8	t Stat			-3.48
9	P(T<=t) one-tail			0.0013
10	t Critical one-tail			1.7291
11	P(T<=t) two-tail			0.0026
12	t Critical two-tail			2.0930

125 minutes: $H_0 : \mu = 125$

$H_1 : \mu < 125$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

	A	B	C	D
1	**t-Test: Mean**			
2				
3				*125 minutes*
4	Mean			110.05
5	Standard Deviation			17.11
6	Hypothesized Mean			125
7	df			19
8	t Stat			-3.91
9	P(T<=t) one-tail			0.0005
10	t Critical one-tail			1.7291
11	P(T<=t) two-tail			0.0010
12	t Critical two-tail			2.0930

Overall Conclusion: p-values are .0004, .1399, 0, .0013, and .0005, respectively. In four of the jobs there is overwhelming evidence to conclude that the times specified by the schedule are greater than the actual times.

A14.8 Two-way analysis of variance

$H_0: \mu_1 = \mu_2 = \mu_3$

H_1: At least two means differ.

	A	B	C	D	E	F	G
35	ANOVA						
36	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
37	Rows	31,154,590	24	1,298,108	15.05	5.20E-15	1.75
38	Columns	913,217	2	456,608	5.29	0.0084	3.19
39	Error	4,141,276	48	86,277			
40							
41	Total	36,209,083	74				

F = 5.29; p-value = .0084. There is sufficient evidence to infer that differences exist between the estimated repair costs from different appraisers.

A14.9 One-way analysis of variance

$H_0: \mu_1 = \mu_2 = \mu_3$

H_1: At least two means differ

	A	B	C	D	E	F	G
10	ANOVA						
11	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
12	Between Groups	126.0	2	63.02	3.30	0.0439	3.16
13	Within Groups	1087.3	57	19.08			
14							
15	Total	1213.3	59				

F = 3.30, p-value = .0439. There is evidence to infer that at least one rust-proofing method is different from the others.

A14.10 z-test of $p_1 - p_2$ (case 2)

$$H_0: p_1 - p_2 = -.15$$

$$H_1: p_1 - p_2 < -.15$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

	A	B	C
1	**z-Test: Two Proportions**		
2			
3		*Comm 1*	*Comm 2*
4	Sample Proportions	0.268	0.486
5	Observations	500	500
6	Hypothesized Difference	-0.15	
7	z Stat	-2.28	
8	P(Z<=z) one tail	0.0114	
9	z Critical one-tail	1.6449	
10	P(Z<=z) two-tail	0.0228	
11	z Critical two-tail	1.9600	

z = –2.28, p-value = .0114. There is evidence to indicate that the second commercial is viable.

A14.11 t-test of μ_D

$$H_0: \mu_D = 0$$

$$H_1: \mu_D < 0$$

$$t = \frac{\bar{x}_D - \mu_D}{s_D / \sqrt{n_D}}$$

	A	B	C
1	t-Test: Paired Two Sample for Means		
2			
3		*Prior*	*After*
4	Mean	24.91	26.24
5	Variance	48.65	87.88
6	Observations	100	100
7	Pearson Correlation	0.79	
8	Hypothesized Mean Difference	0	
9	df	99	
10	t Stat	-2.29	
11	P(T<=t) one-tail	0.0121	
12	t Critical one-tail	1.6604	
13	P(T<=t) two-tail	0.0242	
14	t Critical two-tail	1.9842	

t = –2.29, p-value = .0121. There is enough evidence to conclude that company should proceed to stage 2.

A14.12 Two-factor analysis of variance

	A	B	C	D	E	F	G
29	ANOVA						
30	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
31	Sample	427.61	2	213.81	39.97	0.0000	3.06
32	Columns	20.17	1	20.17	3.77	0.0541	3.91
33	Interaction	17.77	2	8.89	1.66	0.1935	3.06
34	Within	770.32	144	5.35			
35							
36	Total	1235.87	149				

Interaction: F = 1.66, p-value = .1935. There is no evidence of interaction.

Gender (Columns) : F = 3.77, p-value = .0541. There is not enough evidence of a difference between men and women.

Fitness (Sample): F = 39.97, p-value = 0. There is overwhelming evidence of differences among the three levels of fitness.

A14.13 Equal-variances t-test of $\mu_1 - \mu_2$

$$H_0: (\mu_1 - \mu_2) = 0$$

$$H_1: (\mu_1 - \mu_2) \neq 0$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

	A	B	C
1	t-Test: Two-Sample Assuming Equal Variances		
2			
3		*ABS speed*	*No ABS speed*
4	Mean	34.72	33.94
5	Variance	25.27	25.63
6	Observations	100	100
7	Pooled Variance	25.45	
8	Hypothesized Mean Difference	0	
9	df	198	
10	t Stat	1.09	
11	P(T<=t) one-tail	0.1394	
12	t Critical one-tail	1.6526	
13	P(T<=t) two-tail	0.2788	
14	t Critical two-tail	1.9720	

t = 1.09, p-value = .2788. There is not enough evidence that operating an ABS-equipped car changes a driver's behavior.

A14.14 One-way analysis of variance

	A	B	C	D	E	F	G
10	ANOVA						
11	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
12	Between Groups	1813.7	2	906.87	6.46	0.0030	3.16
13	Within Groups	7998.0	57	140.32			
14							
15	Total	9811.7	59				

Multiple comparisons

	A	B	C	D	E
1	**Multiple Comparisons**				
2					
3				LSD	Omega
4	Treatment	Treatment	Difference	Alpha = 0.0167	Alpha = 0.05
5	*Price: $34*	*Price: $39*	-12.9	9.24	9.01
6		*Price: $44*	-3.1	9.24	9.01
7	*Price: $39*	*Price: $44*	9.8	9.24	9.01

Sales with $34 and $44 dollar prices do not differ. Sales with $39 differ from sales with $34 and $44 prices.

A14.15 Equal-variances t-test of $\mu_1 - \mu_2$

$$H_0: (\mu_1 - \mu_2) = 0$$

$$H_1: (\mu_1 - \mu_2) < 0$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

	A	B	C
1	t-Test: Two-Sample Assuming Equal Variances		
2			
3		*Echinacea*	*Placebo*
4	Mean	7.02	7.06
5	Variance	2.51	2.42
6	Observations	262	262
7	Pooled Variance	2.47	
8	Hypothesized Mean Difference	0	
9	df	522	
10	t Stat	-0.31	
11	P(T<=t) one-tail	0.3799	
12	t Critical one-tail	1.6478	
13	P(T<=t) two-tail	0.7598	
14	t Critical two-tail	1.9645	

t = –.31, p-value = .3799. There is not enough evidence to conclude that Echinacea is effective.

A14.16 Two-way analysis of variance

	A	B	C	D	E	F	G
24	ANOVA						
25	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
26	Rows	335.17	13	25.78	16.86	0.0000	2.12
27	Columns	10.90	2	5.45	3.57	0.0428	3.37
28	Error	39.76	26	1.53			
29							
30	Total	385.83	41				

a $H_0: \mu_1 = \mu_2 = \mu_3$

H_1: At least two means differ.

F = 3.57, p-value = .0428. There is enough evidence to conclude that there are differences in waiting times between the three resorts.

b The waiting times are required to be normally distributed with the same variance at all three resorts.

c Histograms are used to check the normality requirement.

A14.17a There are 4 levels of ranks and 4 levels of faculties for a total of 16 treatments.

b

	A	B	C	D	E	F	G
23	ANOVA						
24	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
25	Between Groups	1091.15	15	72.74	2.84	0.0019	1.83
26	Within Groups	1638.40	64	25.60			
27							
28	Total	2729.55	79				

F = 2.84, p-value = .0019. There is enough evidence to infer that at least two treatment means differ.

c Factor A (columns) is the faculty. The levels are business, engineering, arts, and science. Factor B (samples) is the rank. The levels are professor, associate professor, assistant professor, and lecturer.

	A	B	C	D	E	F	G
35	ANOVA						
36	*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
37	Sample	46.85	3	15.62	0.61	0.6109	2.75
38	Columns	344.65	3	114.88	4.49	0.0064	2.75
39	Interaction	699.65	9	77.74	3.04	0.0044	2.03
40	Within	1638.40	64	25.60			
41							
42	Total	2729.55	79				

d F = 3.04, p-value = .0044. There is evidence to conclude that ranks and faculties interact.

e F =.61, p-value = .6109. There is not enough evidence to conclude that differences exist between the ranks. The answer to Part d indicates that this test is irrelevant.

f F = 4.49, p-value = .0064. There is enough evidence to conclude that differences exist between the faculties. The answer to Part d indicates that this test is irrelevant.

A14.18 a z-estimate of p

	A	B	C	D	E
1	**z-Estimate of a Proportion**				
2					
3	**Sample proportion**	0.3569	**Confidence Interval Estimate**		
4	**Sample size**	1328	**0.357**	±	**0.026**
5	**Confidence level**	0.95	**Lower confidence limit**		**0.331**
6			**Upper confidence limit**		**0.383**

Estimate of the number of households with at least one dog

LCL = 112 million × .331 = 37.072 million

UCL = 112 million × .383 = 42.896 million

b

	A	B	C	D	E
1	**z-Estimate of a Proportion**				
2					
3	**Sample proportion**	0.316	**Confidence Interval Estimate**		
4	**Sample size**	1328	**0.316**	±	**0.025**
5	**Confidence level**	0.95	**Lower confidence limit**		**0.291**
6			**Upper confidence limit**		**0.341**

Number of households with at least one cat

LCL = 112 million × .291 = 32.592 million

UCL = 112 million × .341 = 38.192million

c t-estimate of μ

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

	A	B	C	D
1	**t-Estimate: Mean**			
2				
3				*Dogs*
4	Mean			247.19
5	Standard Deviation			133.16
6	LCL			235.17
7	UCL			259.20

Estimate of the total amount spent on dogs

LCL = 40 million ×235.17 = $9.407 billion

UCL = 40 million ×259.29 = $10.368 billion,

d t-estimate of μ

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

	A	B	C	D
1	**t-Estimate: Mean**			
2				
3				*Cats*
4	Mean			158.07
5	Standard Deviation			88.94
6	LCL			149.53
7	UCL			166.61

Estimate of the total amount spent on cats

LCL = 35 million ×149.53 = $5.234 billion

UCL = 35 million ×166.61 = $5.831 billion