Chapter 11

11.1 H_0: The drug is not safe and effective

H_1: The drug is safe and effective

11.2 H_0: I will complete the Ph.D.

H_1: I will not be able to complete the Ph.D.

11.3 H_0: The batter will hit one deep

H_1: The batter will not hit one deep

11.4 H_0: Risky investment is more successful

H_1: Risky investment is not more successful

11.5 H_1: The plane is on fire

H_1: The plane is not on fire

11.6 The defendant in both cases was O. J. Simpson. The verdicts were logical because in the criminal trial the amount of evidence to convict is greater than the amount of evidence required in a civil trial. The two juries concluded that there was enough (preponderance of) evidence in the civil trial, but not enough evidence (beyond a reasonable doubt) in the criminal trial.

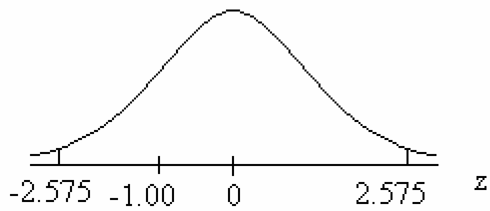
All p-values and probabilities of Type II errors were calculated manually using Table 3 in Appendix B.

11.7 Rejection region: $z < -z_{.005} = -2.575$ or $z > z_{.005} = 2.575$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{980 - 1000}{200 / \sqrt{100}} = -1.00$$

p-value = 2P(Z < –1.00) = 2(.1587) = .3174

There is not enough evidence to infer that $\mu \neq 1000$.

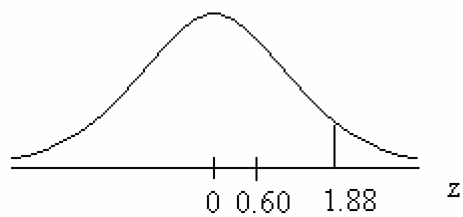


11.8 Rejection region: $z > z_{.03} = 1.88$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{51 - 50}{5 / \sqrt{9}} = .60$$

p-value = $P(Z > .60) = 1 - .7257 = .2743$

There is not enough evidence to infer that $\mu > 50$.

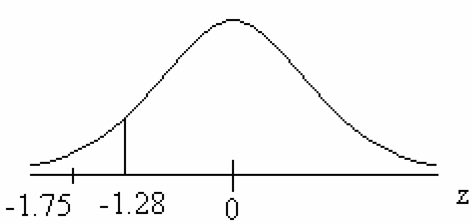


11.9 Rejection region: $z < -z_{.10} = -1.28$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{14.3 - 15}{2 / \sqrt{25}} = -1.75$$

p-value = $P(Z < -1.75) = .0401$

There is enough evidence to infer that $\mu < 15$.

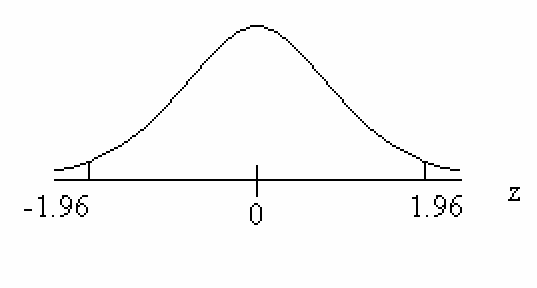


11.10 Rejection region: $z < -z_{.025} = -1.96$ or $z > z_{.025} = 1.96$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{100 - 100}{10 / \sqrt{100}} = 0$$

p-value = $2P(Z > 0) = 2(.5) = 1.00$

There is not enough evidence to infer that $\mu \neq 100$.

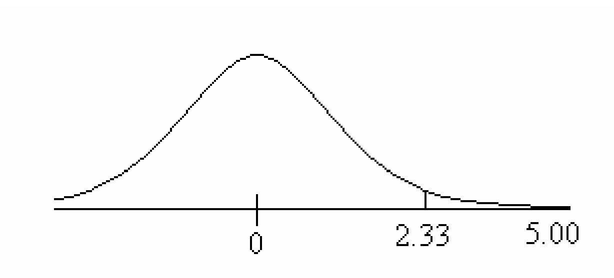


11.11 Rejection region: $z > z_{.01} = 2.33$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{80 - 70}{20 / \sqrt{100}} = 5.00$$

p-value = p(z > 5.00) = 0

There is enough evidence to infer that $\mu > 70$.

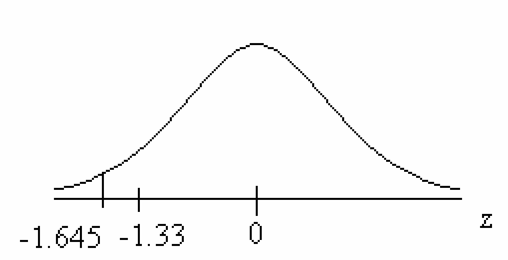


11.12 Rejection region: $z < -z_{.05} = -1.645$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{48 - 50}{15 / \sqrt{100}} = -1.33$$

p-value = P(Z < –1.33) = .0918

There is not enough evidence to infer that $\mu < 50$.



11.13a. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{52 - 50}{5 / \sqrt{9}} = 1.20$

p-value = P(Z > 1.20) = 1 – .8849 = .1151

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{52 - 50}{5 / \sqrt{25}} = 2.00$

p-value = P(Z > 2.00) = .5 – .4772 = .0228.

c. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{52 - 50}{5 / \sqrt{100}} = 4.00$

p-value = P(Z > 4.00) = 0.

d. The value of the test statistic increases and the p-value decreases.

11.14a. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{190 - 200}{50 / \sqrt{9}} = -.60$

p-value = P(Z < –.60) = .5 – .2257 = .2743

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{190 - 200}{30 / \sqrt{9}} = -1.00$

p-value = P(Z < –1.00) = .1587

c $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{190 - 200}{10 / \sqrt{9}} = -3.00$

p-value = P(Z < –3.00) = .0013

d. The value of the test statistic decreases and the p-value decreases.

11.15 a. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{21 - 20}{5 / \sqrt{25}} = 1.00$

p-value = 2P(Z > 1.00) = 2(1 – .8413) = .3174

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{22 - 20}{5 / \sqrt{25}} = 2.00$

p-value = 2P(Z > 2.00) = 2(1 – .9772) = .0456

c. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{23 - 20}{5 / \sqrt{25}} = 3.00$

p-value = 2P(Z > 3.00) = 2(1 – .9987) = .0026

d. The value of the test statistic increases and the p-value decreases.

11.16 a. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{99 - 100}{8 / \sqrt{100}} = -1.25$

p-value = 2P(Z < –1.25) = 2(.1056) = .2112

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{99 - 100}{8 / \sqrt{50}} = -.88$

p-value = 2P(Z < –.88) = 2(.1894) = .3788

c. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{99 - 100}{8 / \sqrt{20}} = -.56$

p-value = 2P(Z < –.56) = 2(.2877) = .5754

d. The value of the test statistic increases and the p-value increases.

11.17 a. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{990 - 1000}{25 / \sqrt{100}} = -4.00$

p-value = P(Z < –4.00) = 0

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{990 - 1000}{50 / \sqrt{100}} = -2.00$

p-value = P(Z < –2.00) = .0228

c. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{990 - 1000}{100 / \sqrt{100}} = -1.00$

p-value = P(Z < –1.00) = .1587

d. d. The value of the test statistic increases and the p-value increases.

11.18 a. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{72 - 60}{20 / \sqrt{25}} = 3.00$

p-value = P(Z > 3.00) = 1 – .9987 = .0013

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{68 - 60}{20 / \sqrt{25}} = 2.00$

p-value = P(Z > 2.00) = 1 – .9772 = .0228

c. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{64 - 60}{20 / \sqrt{25}} = 1.00$

p-value = P(Z > 1.00) = 1 – .8413 = .1587

d. The value of the test statistic decreases and the p-value increases.

11.19 a $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{178 - 170}{65 / \sqrt{200}} = 1.74$

p-valu.e = P(Z > 1.74) = 1 – .9591 = .0409

b. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{178 - 170}{65 / \sqrt{100}} = 1.23$

p-value = P(Z > 1.23) = 1 – .8907 = .1093

c. The value of the test statistic increases and the p-value decreases.

11.20 a $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{178 - 170}{35 / \sqrt{400}} = 4.57$

p-value = P(Z > 4.57) = 0.

b $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{178 - 170}{100 / \sqrt{400}} = 1.60$

p-value = P(Z > 1.60) = 1 – .9452 = .0548

The value of the test statistic decreases and the p-value increases.

11.21 See Table 11.1 in the book.

11.22 a $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{21.63 - 22}{6 / \sqrt{100}} = -.62$

p-value = P(Z < –.62) = .2676

b $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{21.63 - 22}{6 / \sqrt{500}} = -1.38$

p-value = P(Z < –1.38) = .0838

The value of the test statistic decreases and the p-value decreases.

11.23 a $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{21.63 - 22}{3 / \sqrt{220}} = -1.83$

p-value = P(Z < –1.83) = .0336

b $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{21.63 - 22}{12 / \sqrt{220}} = -.46$

p-value = P(Z < –.46) = .3228

The value of the test statistic increases and the p-value increases.

11.24

$\bar{x}$	$z = \frac{\bar{x} - 22}{6 / \sqrt{220}}$	p-value
22.0	0	.5
21.8	–.49	.3121
21.6	–.99	.1611
21.4	–1.48	.0694
21.2	–1.98	.0239
21.0	–2.47	.0068
20.8	–2.97	.0015
20.6	–3.46	0
20.4	–3.96	0

11.25 a $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{17.55 - 17.09}{3.87 / \sqrt{50}} = .84$

p-value = 2P(Z > .84) = 2(1 – .7995) = 2(.2005) = .4010

b $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{17.55 - 17.09}{3.87 / \sqrt{400}} = 2.38$

p-value = 2P(Z > 2.38) = 2(1 – .9913) = 2(.0087) = .0174

The value of the test statistic increases and the p-value decreases.

11.26 a $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{17.55-17.09}{2/\sqrt{100}} = 2.30$

p-value = 2P(Z > 2.30) = 2(1 – .9893) = 2(.0107) = .0214

b $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{17.55-17.09}{10/\sqrt{100}} = .46$

p-value = 2P(Z > .46) = 2(1 – .6772) = 2(.3228) = .6456

The value of the test statistic decreases and the p-value increases.

11.27a	$\bar{x}$	$z = \frac{\bar{x}-17.09}{3.87/\sqrt{100}}$	p-value
	15.0	–5.40	0
	15.5	–4.11	0
	16.0	–2.82	.0048
	16.5	–1.52	.1286
	17.0	–.23	.8180
	17.5	1.06	.2892
	18.0	2.35	.0188
	18.5	3.64	0
	19.0	4.94	0

11.28 $H_0: \mu = 5$

$H_1: \mu > 5$

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{6-5}{1.5/\sqrt{10}} = 2.11$

p-value = P(Z > 2.11) = 1 – .9826 = .0174

There is enough evidence to infer that the mean is greater than 5 cases.

11.29 $H_0: \mu = 50$

$H_1: \mu > 50$

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{59.17-50}{10/\sqrt{18}} = 3.89$

p-value = P(Z > 3.89) = 0

There is enough evidence to infer that the mean is greater than 50 minutes.

11.30 $H_0: \mu = 12$

$H_1: \mu < 12$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{11.00 - 12}{3 / \sqrt{15}} = -1.29$$

p-value = P(Z < –1.29) = .0985

There is enough evidence to infer that the average number of golf balls lost is less than 12.

11.31 $H_0 : \mu = 36$

$H_1 : \mu < 36$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{34.25 - 36}{8 / \sqrt{12}} = -.76$$

p-value = P(Z < –.76) = .2236

There is not enough evidence to infer that the average student spent less time than recommended.

11.32 $H_0 : \mu = 6$

$H_1 : \mu > 6$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{6.60 - 6}{2 / \sqrt{10}} = .95$$

p-value = P(Z > .95) = 1 – .8289 = .1711

There is not enough evidence to infer that the mean time spent putting on the 18^{th} green is greater than 6 minutes.

11.33 $H_0 : \mu = .50$

$H_1 : \mu \neq .50$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{.493 - .50}{.05 / \sqrt{10}} = -.44$$

p-value = 2P(Z < –.44) = 2(.3300) = .6600

There is not enough evidence to infer that the mean diameter is not .50 inch.

11.34 $H_0 : \mu = 25$

$H_1 : \mu > 25$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{30.22 - 25}{12 / \sqrt{18}} = 1.85$$

p-value = P(Z > 1.85) = 1 – .9678 =.0322

There is not enough evidence to conclude that the manager is correct.

11.35 $H_0 : \mu = 5{,}000$

$H_1 : \mu > 5{,}000$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{5{,}065 - 5{,}000}{400 / \sqrt{100}} = 1.62$$

p-value = P(Z > 1.62) = 1 – .9474 =.0526

There is not enough evidence to conclude that the claim is true.

11.36 $H_0 : \mu = 30{,}000$

$H_1 : \mu < 30{,}000$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{29{,}120 - 30{,}000}{8{,}000 / \sqrt{350}} = -2.06$$

p-value = (P(Z < –2.06) = .0197

There is enough evidence to infer that the president is correct

11.37 $H_0 : \mu = 560$

$H_1 : \mu > 560$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{569.0 - 560}{50 / \sqrt{20}} = .80$$

p-value = P(Z > .80) = 1 – .7881 = .2119

There is not enough evidence to conclude that the dean's claim is true.

11.38a $H_0 : \mu = 17.85$

$H_1 : \mu > 17.85$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{19.13 - 17.85}{3.87 / \sqrt{25}} = 1.65$$

p-value = P(Z > 1.65) = 1 – .9505 = .0495

There is enough evidence to infer that the campaign was successful.

b We must assume that the population standard deviation is unchanged.

11.39 $H_0 : \mu = 0$

$H_1 : \mu < 0$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{-1.20 - 0}{6 / \sqrt{50}} = -1.41$$

p-value = P(Z < –1.70) = .0793

There is not enough evidence to conclude that the safety equipment is effective.

11.40 $H_0 : \mu = 55$

$H_1: \mu > 55$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{55.80 - 55}{5 / \sqrt{200}} = 2.26$$

p-value = P(Z > 2.26) = 1 – .9881 = .0119

There is not enough evidence to support the officer's belief.

11.41 $H_0: \mu = 4$

$H_1: \mu > 4$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{5.04 - 4}{1.5 / \sqrt{50}} = 4.90$$

p-value = P(Z > 4.90) = 0

There is enough evidence to infer that the expert is correct.

11.42 $H_0: \mu = 20$

$H_1: \mu < 20$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{19.39 - 20}{3 / \sqrt{36}} = -1.22$$

p-value = P(Z < –1.22) = .1112

There is not enough evidence to infer that the manager is correct.

11.43 $H_0: \mu = 100$

$H_1: \mu > 100$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{105.7 - 100}{16 / \sqrt{40}} = 2.25$$

p-value = P(Z > 2.25) = 1 – .9878 = .0122

There is not enough evidence to infer that the site is acceptable.

11.44 $H_0: \mu = 4$

$H_1: \mu \neq 4$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{4.84 - 4}{2 / \sqrt{63}} = 3.33$$

p-value = 2P(Z > 3.33) = 0

There is enough evidence to infer that the average Alpine skier does not ski 4 times per year.

11.45 $H_0: \mu = 5$

$H_1: \mu > 5$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{5.64 - 5}{2 / \sqrt{25}} = 1.60$$

p-value = P(Z > 1.60) = 1 – .9452 = .0548

There is enough evidence to infer that the golf professional's claim is true.

11.46 $H_0: \mu = 32$

$H_1: \mu < 32$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{29.92 - 32}{8 / \sqrt{110}} = -2.73$$

p-value = P(Z < –2.73) = 1– .9968 = .0032

There is enough evidence to infer that there has been a decrease in the mean time away from desks. A type I error occurs when we conclude that the plan decreases the mean time away from desks when it actually does not. This error is quite expensive. Consequently we demand a low p-value. The p-value is small enough to infer that there has been a decrease.

11.47 $H_0: \mu = 230$

$H_1: \mu > 230$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{231.56 - 230}{10 / \sqrt{100}} = 1.56$$

p-value = P(Z > 1.56) = 1 – .9406 = .0594

There is not enough evidence to infer that Nike is correct.

11.48 a. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha}$

$\frac{\bar{x}-200}{30/\sqrt{25}} < -z_{.10} = -1.28$

$\bar{x} < 192.31$

$\beta = P(\bar{x} > 192.31 \text{ given } \mu = 196) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{192.31-196}{30/\sqrt{25}}\right) = P(z > -.62) = 1 - .2676 = .7324$

b. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha}$

$\frac{\bar{x}-200}{30/\sqrt{100}} < -z_{.10} = -1.28$

$\bar{x} < 196.16$

$\beta = P(\bar{x} > 196.16 \text{ given } \mu = 196) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{196.16-196}{30/\sqrt{100}}\right) = P(z > .05) = 1 - .5199 = .4801$

c. β decreases.

11.49 $H_0: \mu = 170$

$H_1: \mu < 170$

A Type I error occurs when we conclude that the new system is not cost effective when it actually is. A Type II error occurs when we conclude that the new system is cost effective when it actually is not.

The test statistic is the same. However, the p-value equals 1 minus the p-value calculated Example 11.1. That is,

p-value = 1 – .0069 = .9931

We conclude that there is no evidence to infer that the mean is less than 170. That is, there is no evidence to infer that the new system will not be cost effective.

11.50 a. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha}$

$\frac{\bar{x}-100}{20/\sqrt{100}} > z_{.10} = 1.28$

$\bar{x} > 102.56$

$\beta = P(\bar{x} < 102.56 \text{ given } \mu = 102) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{102.56-102}{20/\sqrt{100}}\right) = P(z < .28) = .6103$

b. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha}$

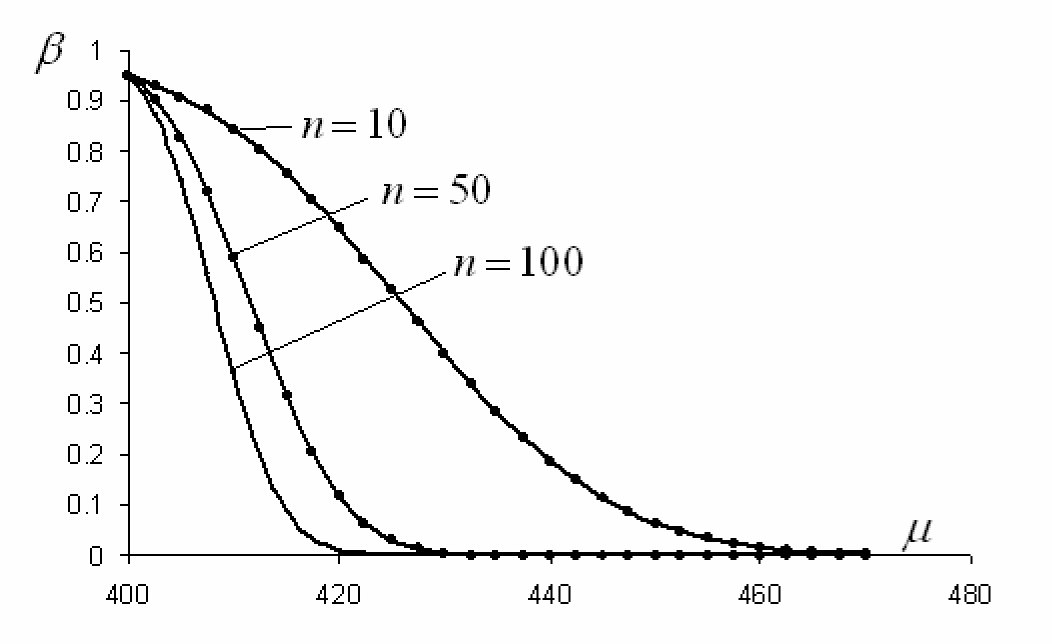
$\frac{\bar{x}-100}{20/\sqrt{100}} > z_{.02} = 2.55$

$\bar{x} > 104.11$

$\beta = P(\bar{x} < 104.11 \text{ given } \mu = 102) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{104.11-102}{20/\sqrt{100}}\right) = P(z < 1.06) = .8554$

c. β increases.

11.51



11.52 a. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$\frac{\bar{x}-40}{5/\sqrt{25}} < -z_{.05} = -1.645$

$\bar{x} < 38.36$

$\beta = P(\bar{x} > 38.36 \text{ given } \mu = 37) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{38.36-37}{5/\sqrt{25}}\right) = P(z > 1.36) = 1 - .9131 = .0869$

b. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$\frac{\bar{x}-40}{5/\sqrt{25}} < -z_{.15} = -1.04$

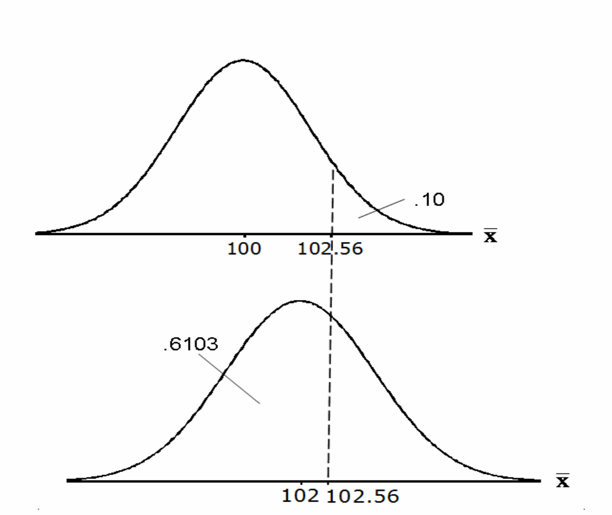
$\bar{x} < 38.96$

$\beta = P(\bar{x} > 38.96 \text{ given } \mu = 37) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{38.96-37}{5/\sqrt{25}}\right) = P(z > 1.96) = 1 - .9750 = .0250$

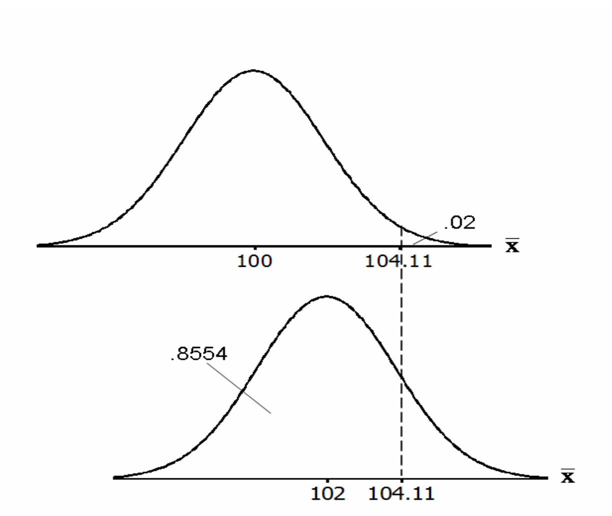
c. β decreases.

11.53

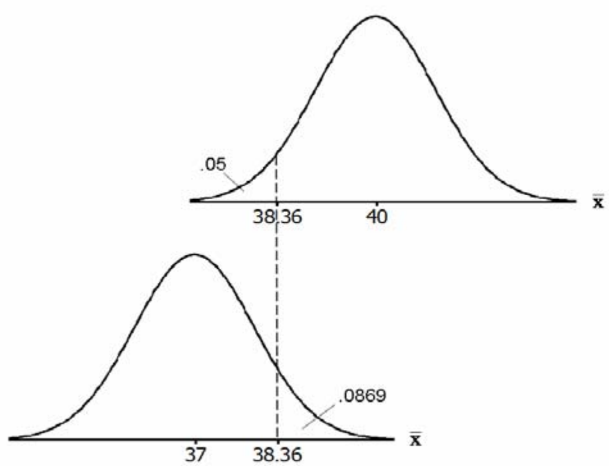
Exercise 11.50 a



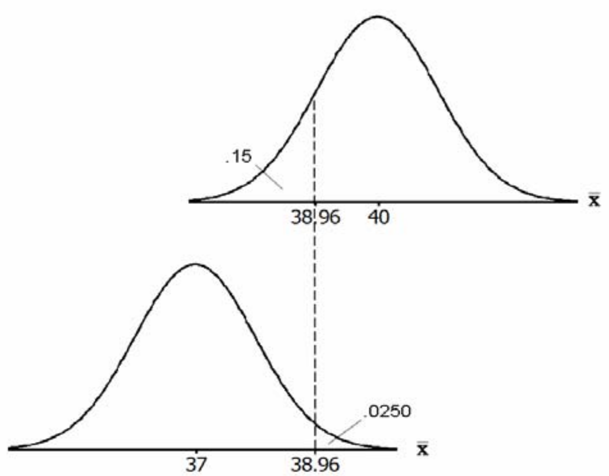
Exercise 11.50 b



Exercise 11.52 a

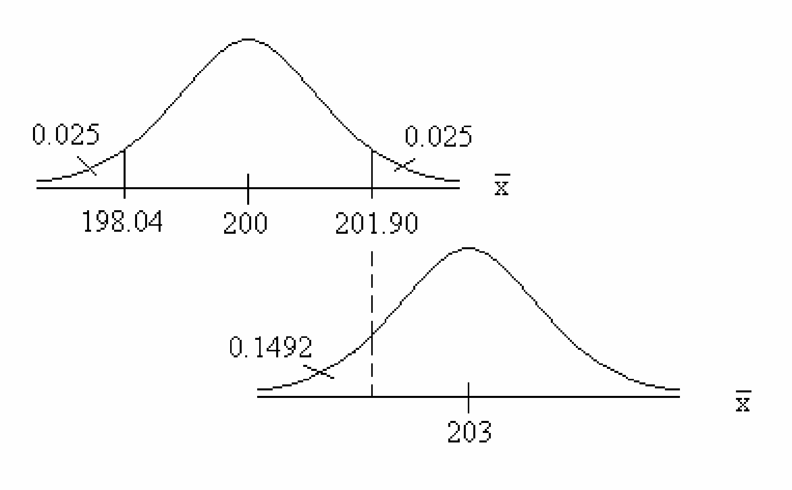


Exercise 11.52 b

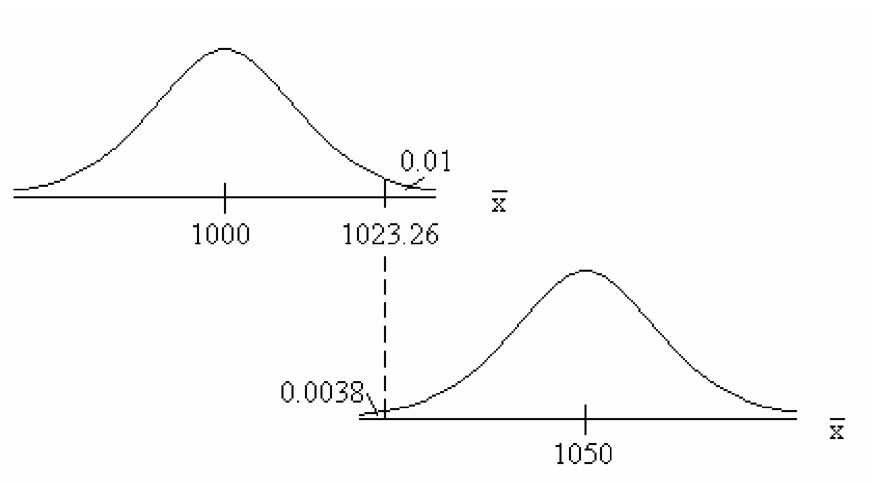


11.54

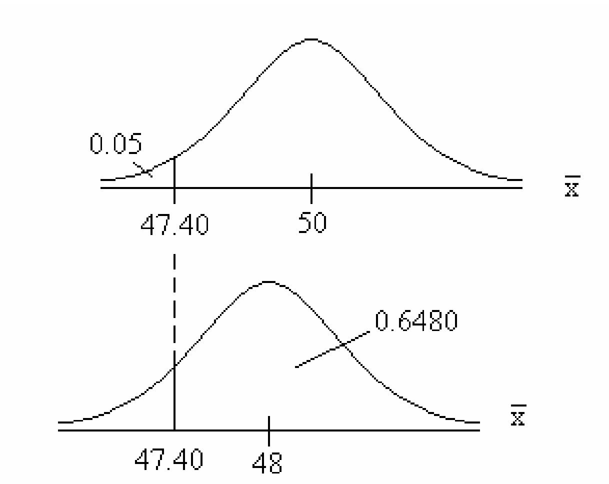
Exercise 11.56



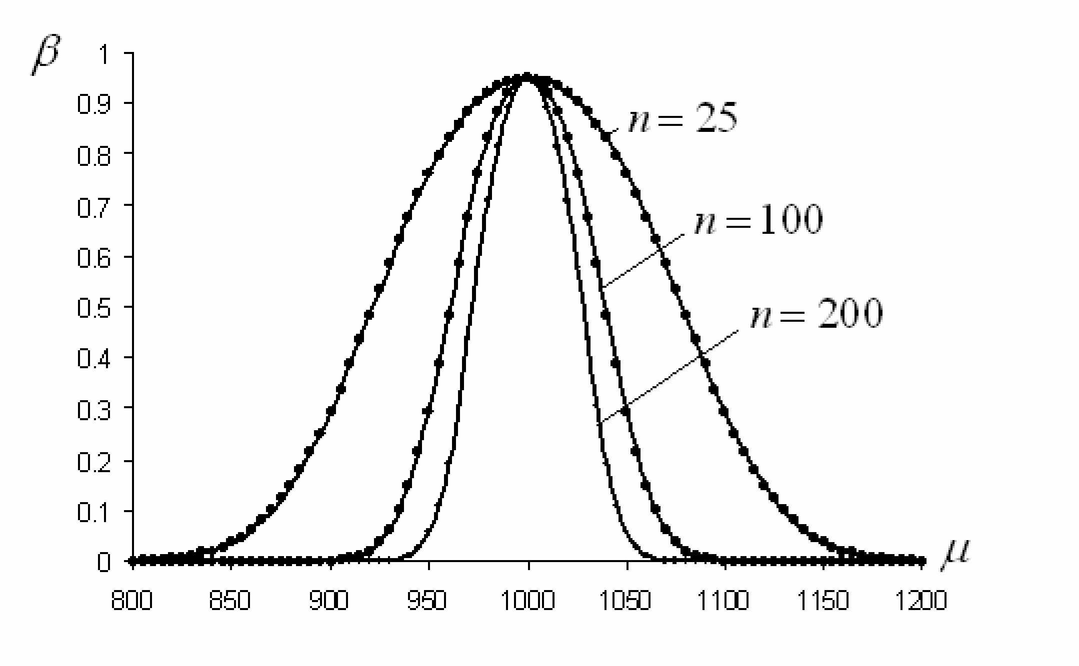
Exercise 11.56



Exercise 11.58



11.55



11.56 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha}$

$\frac{\bar{x}-50}{10/\sqrt{40}} < -z_{.05} = -1.645$

$\bar{x} < 47.40$

$\beta = P(\bar{x} > 47.40 \text{ given } \mu = 48) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{47.40-48}{10/\sqrt{40}}\right) = P(z > -.38) = 1 - .3520 = .6480$

11.57 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha/2}$ or $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha/2}$

$\frac{\bar{x}-200}{10/\sqrt{100}} > z_{.025} = 1.96$ or $\frac{\bar{x}-200}{10/\sqrt{100}} < -1.96$

$\bar{x} > 201.96$ or $\bar{x} < 198.04$

$\beta = P(198.04 < \bar{x} < 201.96 \text{ given } \mu = 203)$

$= P\left(\frac{198.04-203}{10/\sqrt{100}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{201.96-203}{10/\sqrt{100}}\right) = P(-4.96 < z < -1.04) = .1492 - 0 = .1492$

11.58 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha}$

$\frac{\bar{x}-1000}{50/\sqrt{25}} > z_{.01} = 2.33$

$\bar{x} > 1023.3$

$\beta = P(\bar{x} < 1023.3 \text{ given } \mu = 1050) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{1023.3-1050}{50/\sqrt{25}}\right) = P(z < -2.67) = .0038$

11.59 a. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha}$

$\frac{\bar{x}-300}{50/\sqrt{81}} > z_{.05} = 1.645$

$\bar{x} > 309.14$

$\beta = P(\bar{x} < 309.14 \text{ given } \mu = 310) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{309.14-310}{50/\sqrt{81}}\right) = P(z < -.15) = .4404$

b. Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha}$

$\frac{\bar{x}-300}{50/\sqrt{36}} > z_{.05} = 1.645$

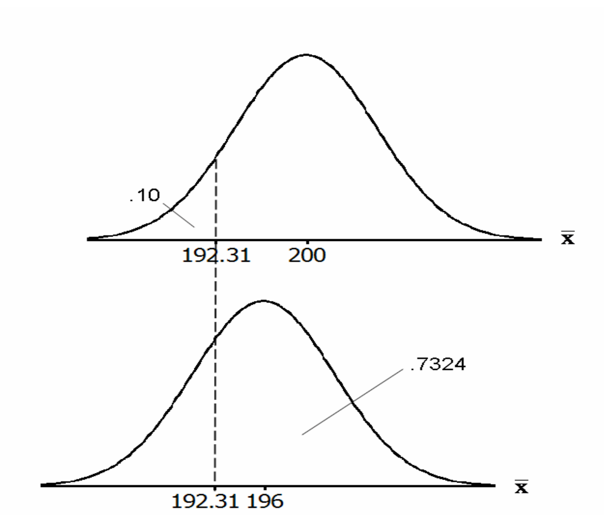
$\bar{x} > 313.71$

$\beta = P(\bar{x} < 313.71 \text{ given } \mu = 310) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{313.71-310}{50/\sqrt{36}}\right) = P(z < .45) = .6736$

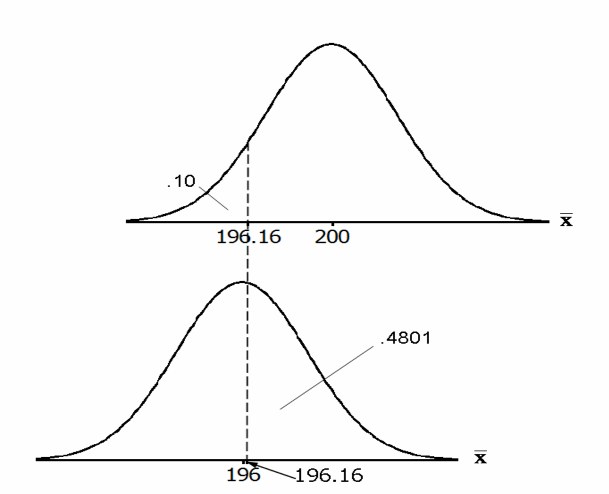
c. β increases.

11.60

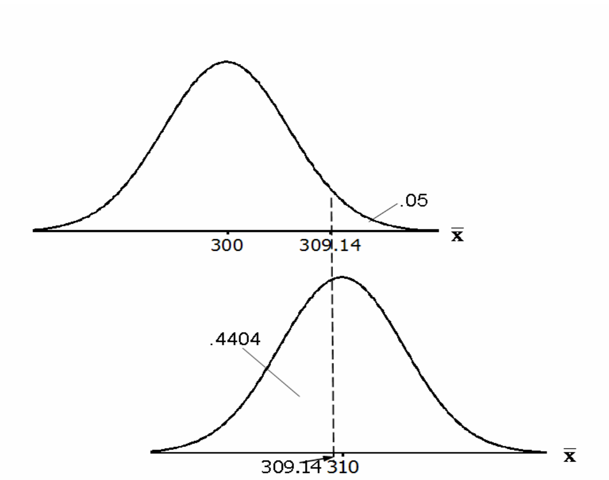
Exercise 11.48 a



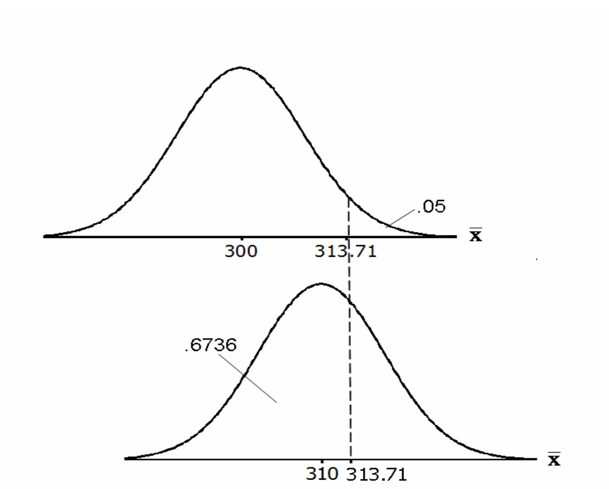
Exercise 11.48 b



Exercise 11.59 a



Exercise 11.59 b



11.61 A Type I error occurs when we conclude that the site is feasible when it is not. The consequence of this decision is to conduct further testing. A Type II error occurs when we do not conclude that a site is feasible when it actually is. We will do no further testing on this site, and as a result we will not build on a good site. If there are few other possible sits, this could be an expensive mistake.

11.62 $H_0: \mu = 20$

$H_1: \mu > 25$

Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_\alpha$

$\frac{\bar{x}-20}{8/\sqrt{25}} > z_{.01} = 2.33$

$\bar{x} > 23.72$

$\beta = P(\bar{x} < 23.72 \text{ given } \mu = 25) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{23.72-25}{8/\sqrt{25}}\right) = P(z < -.80) = .2119$

The process can be improved by increasing the sample size.

11.63 i Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$\frac{\bar{x}-10}{3/\sqrt{100}} < -z_{.01} = -2.33$

$\bar{x} < 9.30$

$\beta = P(\bar{x} > 9.30 \text{ given } \mu = 9) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{9.30-9}{3/\sqrt{100}}\right) = P(z > 1) = 1 - .8413 = .1587$

ii Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$\frac{\bar{x}-10}{3/\sqrt{75}} < -z_{.05} = -1.645$

$\bar{x} < 9.43$

$\beta = P(\bar{x} > 9.43 \text{ given } \mu = 9) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{9.43-9}{3/\sqrt{75}}\right) = P(z > 1.24) = 1 - .8925 = .1075$

iii Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_\alpha$

$\frac{\bar{x}-10}{3/\sqrt{50}} < -z_{.10} = -1.28$

$\bar{x} < 9.46$

$\beta = P(\bar{x} > 9.46 \text{ given } \mu = 9) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{9.46-9}{3/\sqrt{50}}\right) = P(z > 1.08) = 1 – .8599 = .1401$

Plan ii has the lowest probability of a type II error.

11.64 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > z_{\alpha}$

$\frac{\bar{x}-100}{16/\sqrt{40}} > z_{.01} = 2.33$

$\bar{x} > 105.89$

$\beta = P(\bar{x} < 105.89 \text{ given } \mu = 104) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{105.89-104}{16/\sqrt{40}}\right) = P(z < .75) = .7734$

11.65 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha}$

$\frac{\bar{x}-22}{6/\sqrt{220}} < -z_{.10} = -1.28$

$\bar{x} < 21.48$

$\beta = P(\bar{x} > 21.48 \text{ given } \mu = 21) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{21.48-21}{6/\sqrt{220}}\right) = P(z > 1.19) = 1 – .8830 = .1170$

The company can decide whether the sample size and significance level are appropriate.

11.66 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha}$

$\frac{\bar{x}-32}{8/\sqrt{110}} < -z_{.05} = -1.645$

$\bar{x} < 30.75$

$\beta = P(\bar{x} > 30.75 \text{ given } \mu = 30) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{30.75-30)}{8/\sqrt{110}}\right) = P(z > .98) = 1 – .8365 = .1635$

β can be decreased by increasing α and/or increasing the sample size.

11.67 Rejection region: $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < -z_{\alpha}$

$$\frac{\bar{x}-0}{6/\sqrt{50}} < -z_{.10} = -1.28$$

$\bar{x} < -1.09$

$\beta = P(\bar{x} > -1.09 \text{ given } \mu = -2) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{-1.09-(-2)}{6/\sqrt{50}}\right) = P(z > 1.07) = 1 - .8577 = .1423$

β can be decreased by increasing α and/or increasing the sample size.