Part 2
Important financial concepts

Chapters in this part

Chapter 4
Time value of money

■ Instructor's resources

Overview

This chapter introduces an important financial concept: the time value of money. The *PV* and *FV* of a sum, as well as the present and future values of an annuity, are explained. Special applications of the concepts include intra-year compounding, mixed cash flow streams, mixed cash flows with an embedded annuity, perpetuities, deposits to accumulate a future sum, as well as loan repayment. Numerous business and personal financial applications are used as examples. The chapter drives home the need to understand time value of money at the professional level because funding for new assets and programmes must be justified using these techniques. Decisions in a student's personal life should also be acceptable on the basis of applying time-value-of-money techniques to anticipated cash flows.

Study Guide

The following **Study Guide** examples are suggested for classroom presentation:

Example	Topic
5	More on annuities
6	Loan repayment
10	Effective rate

■ Suggested answer to chapter opening critical thinking question

From your knowledge of the cost of branded and generic drugs, how do you believe the market share of generics compares with the market share of branded prescription drugs in *dollar* terms?

Although generics accounted for 53% of prescriptions written in late 2006, they accounted for only 10% of the total market in dollar terms. This clearly demonstrates the price differential between generic and name-brand drugs.

■ Answers to Review Questions

1. *Future value* (*FV*), the value of a present amount at a future date, is calculated by applying compound interest over a specific time period. *Present value* (*PV*), represents the rand value today of a future amount, or the amount you would invest today at a given interest rate for a specified time period to equal the future amount. Financial managers prefer present value to future value because they typically make decisions at time zero, before the start of a project.

2. A *single amount* cash flow refers to an individual, stand alone, value occurring at one point in time. An *annuity* consists of an unbroken series of cash flows of equal rand amount occurring over more than one period. A *mixed stream* is a pattern of cash flows over more than one time period and the amount of cash associated with each period will vary.

3. *Compounding* of interest occurs when an amount is deposited into a savings account and the interest paid after the specified time period remains in the account, thereby becoming part of the principal for the following period. The general equation for future value in year *n* (FV_n) can be expressed using the specified notation as follows:

$$FV_n = PV \times (1 + i)^n$$

4. A decrease in the interest rate lowers the future amount of a deposit for a given holding period, since the deposit earns less at the lower rate. An increase in the holding period for a given interest rate would increase the future value. The increased holding period increases the *FV* since the deposit earns interest over a longer period of time.

5. The present value of a future amount indicates how much money today would be equivalent to the future amount if one could invest that amount at a specified rate of interest. Using the given notation, the present value of a future amount (FV_n) can be defined as follows:

$$PV = FV\left(\frac{1}{(1+i)^n}\right)$$

6. An increasing required rate of return would reduce the present value of a future amount, since future rands would be worth less today. Looking at the formula for present value in Question 5, it should be clear that by increasing the *i* value, which is the required return, the present value interest factor would decrease, thereby reducing the present value of the future sum.

7. Present value calculations are the exact inverse of compound interest calculations. Using compound interest, one attempts to find the future value of a present amount; using present value, one attempts to find the present value of an amount to be received in the future.

8. An *ordinary annuity* is one for which payments occur at the end of each period. An *annuity due* is one for which payments occur at the beginning of each period.

   The ordinary annuity is the more common. For otherwise identical annuities and interest rates, the annuity due results in a higher *FV* because cash flows occur earlier and have more time to compound.

9. The *PV* of an ordinary annuity, PVA_n, can be determined using the formula:

$$PVA_n = PMT \times (PVIFA_{i\%,n})$$

where:

PMT = the end of period cash inflows
$PVIFA_{i\%,n}$ = the *PV* interest factor of an annuity for interest rate i and n periods.

The PVIFA is related to the PVIF in that the annuity factor is the sum of the PVIFs over the number of periods for the annuity. For example, the PVIFA for 5% and 3 periods is 2.723, and the sum of the 5% PVIF for periods one through three is 2.723 (0.952 + 0.907 + 0.864).

10. The FVIFA factors for an ordinary annuity can be converted for use in calculating an annuity due by multiplying the $FVIFA_{i\%,n}$ by $1 + i$.

11. The PVIFA factors for an ordinary annuity can be converted for use in calculating an annuity due by multiplying the $PVIFA_{i\%,n}$ by $1 + i$.

12. A *perpetuity* is an infinite-lived annuity. The factor for finding the present value of a perpetuity can be found by dividing the discount rate into 1.0. The resulting quotient represents the factor for finding the present value of an infinite-lived stream of equal annual cash flows.

13. The future value of a mixed stream of cash flows is calculated by multiplying each year's cash flow by the appropriate future value interest factor. To find the present value of a mixed stream of cash flows multiply each year's cash flow by the appropriate present value interest factor. There will be at least as many calculations as the number of cash flows.

14. As interest is compounded more frequently than once a year, both (a) the future value for a given holding period and (b) the *effective annual rate* of interest will increase. This is due to the fact that the more frequently interest is compounded, the greater the quantity of money accumulated and reinvested as the principal value. In situations of intra-year compounding, the actual rate of interest is greater than the stated rate of interest.

15. *Continuous compounding* assumes interest will be compounded an infinite number of times per year, at intervals of microseconds. Continuous compounding of a given deposit at a given rate of interest results in the largest value when compared to any other compounding period.

16. The *nominal annual rate* is the contractual rate that is quoted to the borrower by the lender. The *effective annual rate*, sometimes called the *true rate*, is the actual rate that is paid by the borrower to the lender. The difference between the two rates is due to the compounding of interest at a frequency greater than once per year.

    APR is the *annual percentage rate* and is required by 'truth in lending laws' to be disclosed to consumers. This rate is calculated by multiplying the periodic rate by the number of periods in one year. The periodic rate is the nominal rate over the shortest time period in which interest is compounded. The APY, or *annual percentage yield*, is the effective rate of interest that must be disclosed to consumers by banks on their savings products as a result of the 'truth in savings laws'. These laws result in both favourable and unfavorable information to consumers. The good news is that rate quotes on both loans and savings are standardised among financial institutions. The negative is that the APR, or lending rate, is a nominal rate, while the APY, or saving rate, is an effective rate. These rates are the same when compounding occurs only once per year.

17. The size of the equal annual end-of-year deposits needed to accumulate a given amount over a certain time period at a specified rate can be found by dividing the interest factor for the future value of an annuity for the given interest rate and the number of years ($FVIFA_{i\%,n}$) into the desired future amount. The resulting quotient would be the amount of the equal annual end-of-year deposits required. The future value interest factor for an annuity is used in this calculation:

$$\text{PMT} = \frac{FV_n}{\text{FVIFA}_{i\%,n}}$$

18. Repaying a loan into equal annual payments involves finding the future payments whose *PV* at the loan interest rate just equals the amount of the initial principal borrowed. The formula is:

$$\text{PMT} = \frac{PV_n}{\text{PVIFA}_{i\%,n}}$$

19. a. Either the present value interest factor or the future value interest factor can be used to find the growth rate associated with a stream of cash flows.

    The growth rate associated with a stream of cash flows may be found by using the following equation, where the growth rate, g, is substituted for k.

$$PV = \frac{FV_n}{(1+g)}$$

    To find the rate at which growth has occurred, the amount received in the earliest year is divided by the amount received in the latest year. This quotient is the $PVIF_{i\%;n}$. The growth rate associated with this factor may be found in the PVIF table.

    b. To find the interest rate associated with an equal payment loan, the present value interest factors for a one-rand annuity table would be used.

    To determine the interest rate associated with an equal payment loan, the following equation may be used:

$$PV_n = \text{PMT} \times (\text{PVIFA}_{i\%,n})$$

    Solving the equation for $PVIFA_{i\%,n}$ we get:

$$\text{PVIFA}_{i\%,n} = \frac{PV_n}{\text{PMT}}$$

    Then substitute the values for PV_n and PMT into the formula, using the PVIFA table to find the interest rate most closely associated with the resulting PVIFA, which is the interest rate on the loan.

20. To find the number of periods it would take to compound a known present amount into a known future amount you can solve either the present value or future value equation for the interest factor as shown below using the present value:

$$PV = FV \times (\text{PVIF}_{i\%,n})$$

Solving the equation for $\text{PVIF}_{i\%,n}$ we get:

$$\text{PVIF}_{i\%,n} = \frac{PV}{FV}$$

Then substitute the values for PV and FV into the formula, using the PVIF table for the known interest rate find the number of periods most closely associated with the resulting PVIF.

The same approach would be used for finding the number of periods for an annuity except that the annuity factor and the PVIFA (or FVIFA) table would be used. This process is shown below.

$$PV_n = \text{PMT} \times (\text{PVIFA}_{i\%,n})$$

Solving the equation for $\text{PVIFA}_{i\%,n}$ we get:

$$\text{PVIFA}_{i\%,n} = \frac{PV_n}{\text{PMT}}$$

■ Suggested answer to critical thinking question for Focus on Practice

As a reaction to problems in the subprime area, lenders are already tightening lending standards. What effect will this have on the housing market?

The tightening of lending standards following the subprime fiasco will likely further depress home prices, which in 2007 were already undergoing their steepest, widest decline in history. If the housing-market slump persists, companies that are heavily involved with the manufacture and sale of durable household goods also will feel the impact of the slow housing market.

■ Suggested answer to critical thinking question for Focus on Ethics

What effect can a bank's order of process (cashing checks presented on the same day from smallest to largest or from largest to smallest) have on NSF fees?

If a customer experiences one nonsufficient funds (NSF) fee, he or she may encounter additional fees. The order of process can have a significant effect on the number of NSF fees paid. For example, suppose that you had R500 in your checking account and wrote seven checks totaling R630. The seven checks are for R400, R13, R50, R70, R25, R40, and R32. If they all arrive at the bank at the same time, the bank could clear the last six checks and bounce only the R400 check. In that case, you would pay one NSF. However, if the bank cleared the biggest ones first, the R400 check would clear and so would the R70 check. The R50 check, next in size, would bounce and so would the other smaller checks. In that sequence of checks, you would pay *five* NSF fees. In response to negative publicity on this issue, most major banks have adopted a "low to high" posting policy, but you should inquire as to your bank's specific policy.

■ Answers to Warm-up exercises

E4-1. Future value of a lump sum investment

Answer: FV = RR2,500 × (1 + 0.007) = RR2,517.50

E4-2. Finding the future value

Answer: Since the interest is compounded monthly, the number of periods is 4 × 12 = 48 and the monthly interest rate is 1/12th of the annual rate.

$FV_{48} = PV \times (1 + I)^{48}$ where I is the monthly interest rate

$I = 0.02 \div 12 = 0.00166667$

FV_{48} = (RR1,260 + RR975) × $(1 + 0.00166667)^{48}$

FV_{48} = (RR2,235) × 1.083215 = RR2,420.99

If using a financial calculator, set the calculator to 12 compounding periods per year and input the following:

PV = R2,235 I/year = 2

N = 48 (months) Solve for FV × FV = R2,420.99

Note**:** Not all financial calculators work in the same manner. Some require the user to use the CPT (Compute) button. Others require the user to calculate the monthly interest rate and input that amount rather than the annual rate. The steps shown in the solution manual will be the inputs needed to use the Hewlett Packard 10B or 10BII models. They are similar to the steps followed when using the Texas Instruments BAII calculators.

If using a spreadsheet, the solution is:

	Column A	Column B
Cell 1	Future value of a single amount	
Cell 2	Present value	R2,235
Cell 3	Interest rate, pct per year compounded monthly	= 2/12
Cell 4	Number of months	= 4 × 12
Cell 5	Future value	= *FV*(B3,B4,0,–B2,0)

Cell B5 = R2,420.99

E4-3. Comparing a lump sum with an annuity

Answer: This problem can be solved in either of two ways. Both alternatives can be compared as lump sums in net present value terms or both alternatives can be compared as a 25-year annuity. In each case, one of the alternatives needs to be converted.

Method 1: Perform a lump sum comparison. Compare R1.3 million now with the present value of the twenty-five payments of R100,000 per year. In this comparison, the present value of the R100,000 annuity must be found and compared with the R1.3 million. (Be sure to set the calculator to 1 compounding period per year.)

PMT = –R100,000
N = 25
I = 5%
Solve for *PV*

PV = R1,409,394 (greater than R1.3 million)

Choose the R100,000 annuity over the lump sum.

Method 2: Compare two annuities. Since the R100,000 per year is already an annuity, all that remains is to convert the R1.3 million into a 25-year annuity.

PV = –R1.3 million
N = 25 years
I = 5%
Solve for PMT

PMT = R92,238.19 (less than R100,000)

Choose the R100,000 annuity over the lump sum.

You may use the table method or a spreadsheet to do the same analysis.

E4-4. Comparing the present value of two alternatives

Answer: To solve this problem you must first find the present value of the expected savings over the 5-year life of the software.

Year	**Savings estimate**	**Present value of savings**
1	R35,000	R32,110
2	50,000	42,084
3	45,000	34,748
4	25,000	17,710
5	15,000	9,749
		R136,401

Since the R136,401 present value of the savings exceeds the R130,000 cost of the software, the firm should invest in the new software.

You may use a financial calculator, the table method or a spreadsheet to find the *PV* of the savings.

E4-5. Compounding more frequently than annually

Answer: Partners' Savings Bank:

$$FV_1 = PV \times \left(1 + \frac{i}{m}\right)^{m \times n}$$
$$FV_1 = \text{R}12{,}000 \times (1 + 0.03/2)^2$$
$$FV_1 = \text{R}12{,}000 \times (1 + 0.03/2)^2 = \text{R}12{,}000 \times 1.030225 = \text{R}12{,}362.70$$

Selwyn's:

$$FV_1 = PV \times (e^{i \times n}) = \text{R}12{,}000 \times (2.7183^{0.0275 \times 1})$$
$$= \text{R}12{,}0000 \times 1.027882 = \text{R}12{,}334.58$$

Joseph should choose the 3% rate with semiannual compounding.

E4-6. Determining deposits needed to accumulate a future sum

Answer: The financial calculator input is as follows:

$FV = –$R150,000 $N = 18$

$I = 6\%$

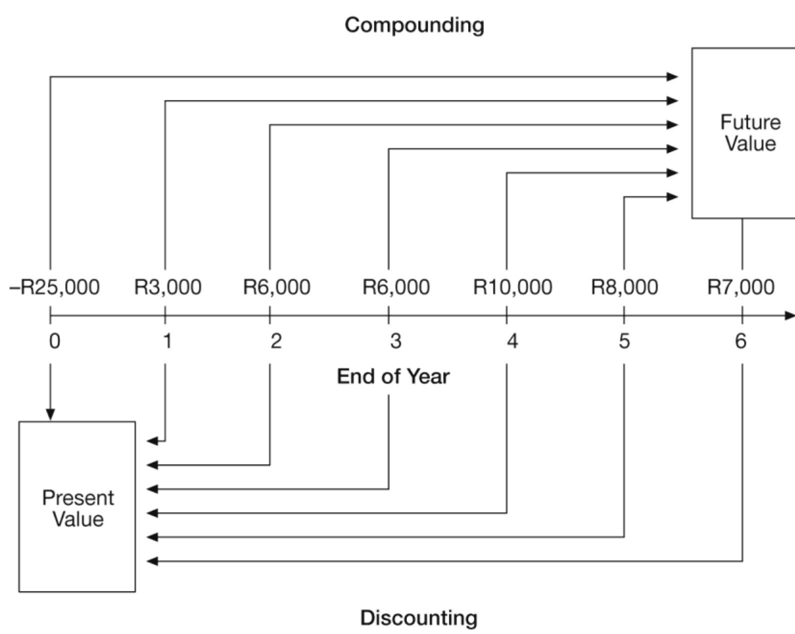
Solve for PMT. PMT = R4,853.48

■ Solutions to Problems

P4-1. LG 1: Using a time line

Basic

a, b, and c



d. Financial managers rely more on present value than future value because they typically make decisions before the start of a project, at time zero, as does the present value calculation.

P4-2. LG 2: Future value calculation: $FV_n = PV \times (1 + I)^n$

Basic

Case

A $FVIF_{12\%,2\text{ periods}} = (1 + 0.12)^2 = 1.254$

B $FVIF_{6\%,3\text{ periods}} = (1 + 0.06)^3 = 1.191$

C $FVIF_{9\%,2\text{ periods}} = (1 + 0.09)^2 = 1.188$

D $FVIF_{3\%,4\text{ periods}} = (1 + 0.03)^4 = 1.126$

P4-3. LG 2: Future value tables: $FV_n = PV \times (1 + \text{I})^n$

Basic

Case A

a. $2 = 1 \times (1 + 0.07)^n$
$2/1 = (1.07)^n$
$2 = FVIF_{7\%,\text{n}}$
10 years < n < 11 years
Nearest to 10 years

b. $4 = 1 \times (1 + 0.07)^n$
$4/1 = (1.07)^n$
$4 = FVIF_{7\%,n}$
20 years < n < 21 years
Nearest to 20 years

Case B

a. $2 = 1 \times (1 + 0.40)^n$
$2 = FVIF_{40\%,n}$
2 years < n < 3 years
Nearest to 2 years

b. $4 = (1 + 0.40)^n$
$4 = FVIF_{40\%,n}$
4 years < n < 5 years
Nearest to 4 years

Case C

a. $2 = 1 \times (1 + 0.20)^n$
$2 = FVIF_{20\%,n}$
3 years < n < 4 years
Nearest to 4 years

b. $4 = (1 + 0.20)^n$
$4 = FVIF_{20\%,n}$
7 years < n < 8 years
Nearest to 8 years

Case D

a. $2 = 1 \times (1 + 0.10)^n$
$2 = FVIF_{10\%,n}$
7 years < n < 8 years
Nearest to 7 years

b. $4 = (1 + 0.10)^n$
$4 = FVIF_{40\%,n}$
14 years < n <15 years
Nearest to 15 years

P4-4. LG 2: Future values: $FV_n = PV \times (1 + I)^n$ or $FV_n = PV \times (FVIF_{i\%,n})$

Intermediate

Case

A $FV_{20} = PV \times FVIF_{5\%,20\text{ yrs.}}$
$FV_{20} = \text{R}200 \times (2.653)$
$FV_{20} = \text{R}530.60$
Calculator solution: R530.66

Case

B $FV_7 = PV \times FVIF_{8\%,7\text{ yrs.}}$
$FV_7 = \text{R}4{,}500 \times (1.714)$
$FV_7 = \text{R}7{,}713$
Calculator solution: R7,712.21

C

$FV_{10} = PV \times \text{FVIF}_{9\%,10\text{ yrs.}}$
$FV_{10} = \text{R}10{,}000 \times (2.367)$
$FV_{10} = \text{R}23{,}670$
Calculator solution: R23,673.64

D

$FV_{12} = PV \times \text{FVIF}_{10\%,12\text{ yrs.}}$
$FV_{12} = \text{R}25{,}000 \times (3.138)$
$FV_{12} = \text{R}78{,}450$
Calculator solution: R78,460.71

E

$FV_5 = PV \times \text{FVIF}_{11\%,5\text{ yrs.}}$
$FV_5 = \text{R}37{,}000 \times (1.685)$
$FV_5 = \text{R}62{,}345$
Calculator solution: R62,347.15

F

$FV_9 = PV \times \text{FVIF}_{12\%,9\text{ yrs.}}$
$FV_9 = \text{R}40{,}000 \times (2.773)$
$FV_9 = \text{R}110{,}920$
Calculator solution: R110,923.15

P4-5. LG 2: Personal finance: Time value: $FV_n = PV \times (1 + I)^n$ or $FV_n = PV \times (\text{FVIF}_{i\%,n})$

Intermediate

a. (1) $FV_3 = PV \times (\text{FVIF}_{7\%,3})$
$FV_3 = \text{R}1{,}500 \times (1.225)$
$FV_3 = \text{R}1{,}837.50$
Calculator solution: R1,837.56

(2) $FV_6 = PV \times (\text{FVIF}_{7\%,6})$
$FV_6 = \text{R}1{,}500 \times (1.501)$
$FV_6 = \text{R}2{,}251.50$
Calculator solution: R2,251.10

(3) $FV_9 = PV \times (\text{FVIF}_{7\%,9})$
$FV_9 = \text{R}1{,}500 \times (1.838)$
$FV_9 = \text{R}2{,}757.00$
Calculator solution: R2,757.69

b. (1) Interest earned = $FV_3 - PV$
Interest earned = R1,837.50
–R1,500.00
R337.50

(2) Interest earned = $FV_6 - FV_3$
Interest earned = R2,251.50
–R1,837.50
R414.00

(3) Interest earned = $FV_9 - FV_6$
Interest earned = R2,757.00
–R2,251.50
R505.50

c. The fact that the longer the investment period is, the larger the total amount of interest collected will be, is not unexpected and is due to the greater length of time that the principal sum of R1,500 is invested. The most significant point is that the incremental interest earned per 3-year period increases with each subsequent 3 year period. The total interest for the first 3 years is R337.50; however, for the second 3 years (from year 3 to 6) the additional interest earned is R414.00. For the third 3-year period, the incremental interest is R505.50. This increasing change in interest earned is due to compounding, the earning of interest on previous interest earned. The greater the previous interest earned, the greater the impact of compounding.

P4-6. LG 2: Personal finance: Time value

Challenge

a. (1) $FV_5 = PV \times (\text{FVIF}_{2\%,5})$
$FV_5 = \text{R}14{,}000 \times (1.104)$
$FV_5 = \text{R}15{,}456.00$
Calculator solution: R15,457.13

(2) $FV_5 = PV \times (\text{FVIF}_{4\%,5})$
$FV_5 = \text{R}14{,}000 \times (1.217)$
$FV_5 = \text{R}17{,}038.00$
Calculator solution: R17,033.14

b. The car will cost R1,582 more with a 4% inflation rate than an inflation rate of 2%. This increase is 10.2% more (R1,582 ÷ R15,456) than would be paid with only a 2% rate of inflation.

P4-7. LG 2: Personal finance: Time value

Challenge

Deposit Now:	Deposit in 10 Years:
$FV_{40} = PV \times \text{FVIF}_{9\%,40}$	$FV_{30} = PV_{10} \times (\text{FVIF}_{9\%,30})$
$FV_{40} = \text{R}10{,}000 \times (1.09)^{40}$	$FV_{30} = PV_{10} \times (1.09)^{30}$
$FV_{40} = \text{R}10{,}000 \times (31.409)$	$FV_{30} = \text{R}10{,}000 \times (13.268)$
$FV_{40} = \text{R}314{,}090.00$	$FV_{30} = \text{R}132{,}680.00$
Calculator solution: R314,094.20	Calculator solution: R132,676.78

You would be better off by R181,410 (R314,090 – R132,680) by investing the R10,000 now instead of waiting for 10 years to make the investment.

P4-8. LG 2: Personal finance: Time value: $FV_n = PV \times \text{FVIF}_{i\%,n}$

Challenge

a. $\text{R}15{,}000 = \text{R}10{,}200 \times \text{FVIF}_{i\%,5}$
$\text{FVIF}_{i\%,5} = \text{R}15{,}000 \div \text{R}10{,}200 = 1.471$
$8\% < i < 9\%$
Calculator solution: 8.02%

b. $\text{R}15{,}000 = \text{R}8{,}150 \times \text{FVIF}_{i\%,5}$
$\text{FVIF}_{i\%,5} = \text{R}15{,}000 \div \text{R}8{,}150 =$ 1.840
$12\% < i < 13\%$
Calculator solution: 12.98%

c. $\text{R}15{,}000 = \text{R}7{,}150 \times \text{FVIF}_{i\%,5}$
$\text{FVIF}_{i\%,5} = \text{R}15{,}000 \div \text{R}7{,}150 = 2.098$
$15\% < i < 16\%$
Calculator solution: 15.97%

P4-9. LG 2: Personal finance: Single-payment loan repayment: $FV_n = PV \times \text{FVIF}_{i\%,n}$

Intermediate

a. $FV_1 = \text{PV} \times (\text{FVIF}_{14\%,1})$
$FV_1 = \text{R}200 \times (1.14)$
$FV_1 = \text{R}228$
Calculator solution: R228

b. $FV_4 = PV \times (\text{FVIF}_{14\%,4})$
$FV_4 = \text{R}200 \times (1.689)$
$FV_4 = \text{R}337.80$
Calculator solution: R337.79

c. $FV_8 = PV \times (\text{FVIF}_{14\%,8})$
$FV_8 = \text{R}200 \times (2.853)$
$FV_8 = \text{R}570.60$
Calculator solution: R570.52

P4-10. LG 2: Present value calculation: $\text{PVIF} = \dfrac{1}{(1+i)^n}$

Basic

Case

A $\text{PVIF} = 1 \div (1 + 0.02)^4 = 0.9238$
B $\text{PVIF} = 1 \div (1 + 0.10)^2 = 0.8264$
C $\text{PVIF} = 1 \div (1 + 0.05)^3 = 0.8638$
D $\text{PVIF} = 1 \div (1 + 0.13)^2 = 0.7831$

P4-11. LG 2: Present values: $PV = FV_n \times (\text{PVIF}_{i\%,n})$

Basic

Case		Calculator solution
A	$PV_{12\%,4\text{yrs}}$ = R7,000 × 0.636 = R4,452	R4,448.63
B	$PV_{8\%,\ 20\text{yrs}}$= R28,000 × 0.215 = R6,020	R6,007.35
C	$PV_{14\%,12\text{yrs}}$ = R10,000× 0.208 = R2,080	R2,075.59
D	$PV_{11\%,6\text{yrs}}$ = R150,000 × 0.535 = R80,250	R80,196.13
E	$PV_{20\%,8\text{yrs}}$ = R45,000 × 0.233 = R10,485	R10,465.56

P4-12. LG 2: Present value concept: $PV_n = FV_n \times (\text{PVIF}_{i\%,n})$

Intermediate

a. $PV = FV_6 \times (\text{PVIF}_{12\%,6})$
PV = R6,000 × (.507)
PV = R3,042.00
Calculator solution: R3,039.79

b. $PV = FV_6 \times (\text{PVIF}_{12\%,6})$
PV = R6,000 × (0.507)
PV = R3,042.00
Calculator solution: R3,039.79

c. $PV = FV_6 \times (\text{PVIF}_{12\%,6})$
PV = R6,000 × (0.507)
PV = R3,042.00
Calculator solution: R3,039.79

d. The answer to all three parts are the same. In each case the same questions is being asked but in a different way.

P4-13. LG 2: Personal finance: Time value: $PV = FV_n \times (\text{PVIF}_{i\%,n})$

Basic

Jim should be willing to pay no more than R408.00 for this future sum given that his opportunity cost is 7%.

PV = R500 × ($\text{PVIF}_{7\%,3}$)
PV = R500 × (0.816)
PV = R408.00
Calculator solution: R408.15

P4-14. LG 2: Time value: $PV = FV_n \times (\text{PVIF}_{i\%,n})$

Intermediate

PV = R100 × ($\text{PVIF}_{8\%,6}$)
PV = R100 × (0.630)
PV = R63.00
Calculator solution: R63.02

P4-15. LG 2: Personal finance: Time value and discount rates: $PV = FV_n \times (\text{PVIF}_{i\%,n})$

Intermediate

a. (1) PV = R1,000,000 × ($\text{PVIF}_{6\%,10}$)
PV = R1,000,000 × (0.558)
PV = R558,000.00
Calculator solution: R558,394.78

(2) PV = R1,000,000 × ($\text{PVIF}_{9\%,10}$)
PV = R1,000,000 × (0.422)
PV = R422,000.00
Calculator solution: R422,410.81

(3) PV = R1,000,000 × ($\text{PVIF}_{12\%,10}$)
PV = R1,000,000 × (0.322)
PV = R322,000.00
Calculator solution: R321,973.24

b. (1) PV = R1,000,000 × ($\text{PVIF}_{6\%,15}$)
PV = R1,000,000 × (0.417)
PV = R417,000.00
Calculator solution: R417,265.06

(2) PV = R1,000,000 × ($\text{PVIF}_{9\%,15}$)
PV = R1,000,000 × (0.275)
PV = R275,000.00
Calculator solution: R274,538.04

(3) PV = R1,000,000 × ($\text{PVIF}_{12\%,15}$)
PV = R1,000,000 × (0.183)
PV = R183,000.00
Calculator solution: R182,696.26

c. As the discount rate increases, the present value becomes smaller. This decrease is due to the higher opportunity cost associated with the higher rate. Also, the longer the time until the lottery payment is collected, the less the present value due to the greater time over which the opportunity cost applies. In other words, the larger the discount rate and the longer the time until the money is received, the smaller will be the present value of a future payment.

P4-16. Personal finance: LG 2: Time value comparisons of lump sums: $PV = FV_n \times (\text{PVIF}_{i\%,n})$

Intermediate

a. **A** PV = R28,500 × ($\text{PVIF}_{11\%,3}$)
PV = R28,500 × (0.731)
PV = R20,833.50
Calculator solution: R20,838.95

B PV = R54,000 × ($\text{PVIF}_{11\%,9}$)
PV = R54,000 × (0.391)
PV = R21,114.00
Calculator solution: R21,109.94

C PV = R160,000 × ($\text{PVIF}_{11\%,20}$)
PV = R160,000 × (0.124)
PV = R19,840.00
Calculator solution: R19,845.43

b. Alternatives A and B are both worth greater than R20,000 in term of the present value.

c. The best alternative is B because the present value of B is larger than either A or C and is also greater than the R20,000 offer.

P4-17. LG 2: Personal finance: cash flow investment decision: $PV = FV_n \times (\text{PVIF}_{i\%,n})$

Intermediate

A
PV = R30,000 × $(\text{PVIF}_{10\%,5})$
PV = R30,000 × (0.621)
PV = R18,630.00
Calculator solution: R18,627.64

B
PV = R3,000 × $(\text{PVIF}_{10\%,20})$
PV = R3,000 × (0.149)
PV = R447.00
Calculator solution: R445.93

C
PV = R10,000 × $(\text{PVIF}_{10\%,10})$
PV = R10,000 × (0.386)
PV = R3,860.00
Calculator solution: R3,855.43

D
PV = R15,000 × $(\text{PVIF}_{10\%,40})$
PV = R15,000 × (0.022)
PV = R330.00
Calculator solution: R331.42

Purchase	Do Not Purchase
A	B
C	D

P4-18. LG 3: Future value of an annuity

Intermediate

a. Future value of an ordinary annuity vs. annuity due

	(1) **Ordinary Annuity**	(2) **Annuity Due**
	$\text{FVA}_{k\%,n} = \text{PMT} \times (\text{FVIFA}_{k\%,n})$	$\text{FVA}_{\text{due}} = \text{PMT} \times [(\text{FVIFA}_{k\%,n} \times (1 + k)]$
A	$\text{FVA}_{8\%,10}$ = R2,500 × 14.487 $\text{FVA}_{8\%,10}$ = R36,217.50 Calculator solution: R36,216.41	FVA_{due} = R2,500 × (14.487 × 1.08) FVA_{due} = R39,114.90 Calculator solution: R39,113.72
B	$\text{FVA}_{12\%,6}$ = R500 × 8.115 $\text{FVA}_{12\%,6}$ = R4,057.50 Calculator solution: R4,057.59	FVA_{due} = R500 ×(8.115 × 1.12) FVA_{due} = R4,544.40 Calculator solution: R4,544.51
C	$\text{FVA}_{20\%,5}$ = R30,000 × 7.442 $\text{FVA}_{20\%,5}$ = R223,260 Calculator solution: R223,248	FVA_{due} = R30,000 × (7.442 × 1.20) FVA_{due} = R267,912 Calculator solution: R267,897.60
D	$\text{FVA}_{9\%,8}$ = R11,500 × 11.028 $\text{FVA}_{9\%,8}$ = R126,822 Calculator solution: R126,827.45	FVA_{due} = R11,500 × (11.028 × 1.09) FVA_{due} = R138,235.98 Calculator solution: R138,241.92
E	$\text{FVA}_{14\%,30}$ = R6,000 × 356.787 $\text{FVA}_{14\%,30}$ = R2,140,722 Calculator solution: R2,140,721.08	FVA_{due} = R6,000 × (356.787 × 1.14) FVA_{due} = R2,440,422.00 Calculator solution: R2,440,422.03

b. The annuity due results in a greater future value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one additional year of compounding.

P4-19. LG 3: Present value of an annuity: $PV_n = \text{PMT} \times (\text{PVIFA}_{i\%,n})$

Intermediate

a. Present value of an ordinary annuity vs. annuity due

	(1) **Ordinary Annuity**	(2) **Annuity Due**
	$\text{PVA}_{k\%,n} = \text{PMT} \times (\text{PVIFA}_{i\%,n})$	$\text{PVA}_{due} = \text{PMT} \times [(\text{PVIFA}_{i\%,n} \times (1 + k)]$
A	$\text{PVA}_{7\%,3} = \text{R}12{,}000 \times 2.624$ $\text{PVA}_{7\%,3} = \text{R}31{,}488$ Calculator solution: R31,491.79	$\text{PVA}_{due} = \text{R}12{,}000 \times (2.624 \times 1.07)$ $\text{PVA}_{due} = \text{R}33{,}692$ Calculator solution: R33,696.22
B	$\text{PVA}_{12\%15} = \text{R}55{,}000 \times 6.811$ $\text{PVA}_{12\%,15} = \text{R}374{,}605$ Calculator solution: R374,597.55	$\text{PVA}_{due} = \text{R}55{,}000 \times (6.811 \times 1.12)$ $\text{PVA}_{due} = \text{R}419{,}557.60$ Calculator solution: R419,549.25
C	$\text{PVA}_{20\%,9} = \text{R}700 \times 4.031$ $\text{PVA}_{20\%,9} = \text{R}2{,}821.70$ Calculator solution: R2,821.68	$\text{PVA}_{due} = \text{R}700 \times (4.031 \times 1.20)$ $\text{PVA}_{due} = \text{R}3{,}386.04$ Calculator solution: R3,386.01
D	$\text{PVA}_{5\%,7} = \text{R}140{,}000 \times 5.786$ $\text{PVA}_{5\%,7} = \text{R}810{,}040$ Calculator solution: R810,092.28	$\text{PVA}_{due} = \text{R}140{,}000 \times (5.786 \times 1.05)$ $\text{PVA}_{due} = \text{R}850{,}542$ Calculator solution: R850,596.89
E	$\text{PVA}_{10\%,5} = \text{R}22{,}500 \times 3.791$ $\text{PVA}_{10\%,5} = \text{R}85{,}297.50$ Calculator solution: R85,292.70	$\text{PVA}_{due} = \text{R}22{,}500 \times (2.791 \times 1.10)$ $\text{PVA}_{due} = \text{R}93{,}827.25$ Calculator solution: R93,821.97

b. The annuity due results in a greater present value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one less year to discount back.

P4-20. LG 3: Personal finance: Time value – annuities

Challenge

	a. **Annuity C (Ordinary)**	**Annuity D (Due)**
	$\text{FVA}_{i\%,n} = \text{PMT} \times (\text{FVIFA}_{i\%,n})$	$\text{FVA}_{due} = \text{PMT} \times [\text{FVIFA}_{i\%,n} \times (1 + i)]$
(1)	$\text{FVA}_{10\%,10} = \text{R}2{,}500 \times 15.937$ $\text{FVA}_{10\%,10} = \text{R}39{,}842.50$ Calculator solution: R39,843.56	$\text{FVA}_{due} = \text{R}2{,}200 \times (15.937 \times 1.10)$ $\text{FVA}_{due} = \text{R}38{,}567.54$ Calculator solution: R38,568.57
(2)	$\text{FVA}_{20\%,10} = \text{R}2{,}500 \times 25.959$ $\text{FVA}_{20\%,10} = \text{R}64{,}897.50$ Calculator solution: R64,896.71	$\text{FVA}_{due} = \text{R}2{,}200 \times (25.959 \times 1.20)$ $\text{FVA}_{due} = \text{R}68{,}531.76$ Calculator solution: R68,530.92

b. (1) At the end of year 10, at a rate of 10%, Annuity C has a greater value (R39,842.50 vs. R38,567.54).
   (2) At the end of year 10, at a rate of 20%, Annuity D has a greater value (R68,531.76 vs. R64,897.50).

c.

Annuity C (Ordinary)	**Annuity D (Due)**
$PVA_{i\%,n} = PMT \times (FVIFA_{i\%,n})$	$PVA_{due} = PMT \times [FVIFA_{i\%,n} \times (1 + i)]$
(1) $PVA_{10\%,10} = R2{,}500 \times 6.145$	$PVA_{due} = R2{,}200 \times (6.145 \times 1.10)$
$PVA_{10\%,10} = R15{,}362.50$	$PVA_{due} = R14{,}870.90$
Calculator solution: R15,361.42	Calculator solution: R14,869.85
(2) $PVA_{20\%,10} = R2{,}500 \times 4.192$	$PVA_{due} = R2{,}200 \times (4.192 \times 1.20)$
$PVA_{20\%,10} = R10{,}480$	$PVA_{due} = R11{,}066.88$
Calculator solution: R10,481.18	Calculator solution: R11,068.13

d. (1) At the beginning of the 10 years, at a rate of 10%, Annuity C has a greater value (R15,362.50 vs. R14,870.90).
   (2) At the beginning of the 10 years, at a rate of 20%, Annuity D has a greater value (R11,066.88 vs. R10,480.00).

e. Annuity C, with an annual payment of R2,500 made at the end of the year, has a higher present value at 10% than Annuity D with an annual payment of R2,200 made at the beginning of the year. When the rate is increased to 20%, the shorter period of time to discount at the higher rate results in a larger value for Annuity D, despite the lower payment.

P4-21. LG 3: Personal finance: Retirement planning

Challenge

a. $FVA_{40} = R2{,}000 \times (FVIFA_{10\%,40})$
   $FVA_{40} = R2{,}000 \times (442.593)$
   $FVA_{40} = R885{,}186$
   Calculator solution: R885,185.11

b. $FVA_{30} = R2{,}000 \times (FVIFA_{10\%,30})$
   $FVA_{30} = R2{,}000 \times (164.494)$
   $FVA_{30} = R328{,}988$
   Calculator solution: R328,988.05

c. By delaying the deposits by 10 years the total opportunity cost is R556,198. This difference is due to both the lost deposits of R20,000 (R2,000 × 10yrs.) and the lost compounding of interest on all of the money for 10 years.

d. **Annuity Due:**
   $FVA_{40} = R2{,}000 \times (FVIFA_{10\%,40}) \times (1 + 0.10)$
   $FVA_{40} = R2{,}000 \times (486.852)$
   $FVA_{40} = R973{,}704$
   Calculator solution: R973,703.62

FVA_{30} = R2,000 × ($FVIFA_{10\%,30}$) × (1.10)
FVA_{30} = R2,000 × (180.943)
FVA_{30} = R361,886
Calculator solution: R361,886.85

Both deposits increased due to the extra year of compounding from the beginning-of-year deposits instead of the end-of-year deposits. However, the incremental change in the 40 year annuity is much larger than the incremental compounding on the 30 year deposit (R88,518 versus R32,898) due to the larger sum on which the last year of compounding occurs.

P4-22. LG 3: Personal finance: Value of a retirement annuity

Intermediate

PVA = PMT × ($PVIFA_{9\%,25}$)
PVA = R12,000 × (9.823)
PVA = R117,876.00
Calculator solution: R117,870.96

P4-23. LG 3: Personal finance: Funding your retirement

Challenge

a. PVA = PMT × ($PVIFA_{11\%,30}$)
PVA = R20,000 × (8.694)
PVA = R173,880.00
Calculator solution: R173,875.85

b. $PV = FV$ × ($PVIF_{9\%,20}$)
PV = R173,880 × (0.178)
PV = R30,950.64
Calculator solution: R31,024.82

c. Both values would be lower. In other words, a smaller sum would be needed in 20 years for the annuity and a smaller amount would have to be put away today to accumulate the needed future sum.

P4-24. LG 2, 3: Personal finance: Value of an annuity versus a single amount

Intermediate

a. PVA_n = PMT × ($PVIFA_{i\%,n}$)
PVA_{25} = R40,000 × ($PVIFA_{5\%,25}$)
PVA_{25} = R40,000 × 14.094
PVA_{25} = R563,760
Calculator solution: R563,757.78

At 5%, taking the award as an annuity is better; the present value is R563,760, compared to receiving R500,000 as a lump sum.

b. $PVA_n = R40,000 \times (PVIFA_{7\%,25})$
$PVA_{25} = R40,000 \times (11.654)$
$PVA_{25} = R466,160$
Calculator solution: R466,143.33

At 7%, taking the award as a lump sum is better; the present value of the annuity is only R466,160, compared to the R500,000 lump sum payment.

c. Because the annuity is worth more than the lump sum at 5% and less at 7%, try 6%:
$PV_{25} = R40,000 \times (PVIFA_{6\%,25})$
$PV_{25} = R40,000 \times 12.783$
$PV_{25} = R511,320$

The rate at which you would be indifferent is greater than 6%; about 6.25% Calculator solution: 6.24%

P4-25. LG 3: Perpetuities: $PV_n = PMT \times (PVIFA_{i\%,\infty})$

Basic

a.

Case	*PV* Factor
A	1 ÷ 0.08 = 12.50
B	1 ÷ 0.10 = 10.00
C	1 ÷ 0.06 = 16.67
D	1 ÷ 0.05 = 20.00

b.

PMT × ($PVIFA_{i\%,\infty}$) = PMT × (1 ÷ *i*)			
R20,000	×	12.50	= R250,000
R100,000	×	10.00	= R1,000,000
R3,000	×	16.67	= R50,000
R60,000	×	20.00	= R1,200,000

P4-26. LG 3: Personal finance: creating an endowment

Intermediate

a. $PV = PMT \times (PVIFA_{i\%,\infty})$
$PV = (R6,000 \times 3) \times (1 \div i)$
$PV = R18,000 \times (1 \div 0.06)$
$PV = R18,000 \times (16.67)$
$PV = R300,060$
Calculator solution: R300,000

b. $PV = PMT \times (PVIFA_{i\%,\infty})$
$PV = (R6,000 \times 3) \times (1 \div i)$
$PV = R18,000 \times (1 \div 0.09)$
$PV = R18,000 \times (11.11)$
$PV = R199,980$
Calculator solution: R200,000

P4-27. LG 4: Value of a mixed stream

Challenge

a.

Cash flow stream	Year	Number of years to compound	$FV = CF \times \text{FVIF}_{12\%,n}$		Future value
A	1	3	R 900 × 1.405	=	R 1,264.50
	2	2	1,000 × 1.254	=	1,254.00
	3	1	1,200 × 1.120	=	1,344.00
					R 3,862.50
			Calculator solution:		R 3,862.84
B	1	5	R30,000 × 1.762	=	R 52,860.00
	2	4	25,000 × 1.574	=	39,350.00
	3	3	20,000 × 1.405	=	28,100.00
	4	2	10,000 × 1.254	=	12,540.00
	5	1	5,000 × 1.120	=	5,600.00
					R138,450.00
			Calculator solution:		R138,450.79
C	1	4	R 1,200 × 1.574	=	R 1,888.80
	2	3	1,200 × 1.405	=	1,686.00
	3	2	1,000 × 1.254	=	1,254.00
	4	1	1,900 × 1.120	=	2,128.00
					R 6,956.80
			Calculator solution:		R 6,956.54

b. If payments are made at the beginning of each period the present value of each of the end-of-period cash flow streams will be multiplied by $(1 + i)$ to get the present value of the beginning-of-period cash flows.

A R3,862.50 (1 + 0.12) = R4,326.00

B R138,450.00 (1 + 0.12) = R155,064.00

C R6,956.80 (1 + 0.12) = R7,791.62

P4-28. LG 4: Personal finance: Value of a single amount versus a mixed stream

Intermediate

Lump sum deposit

$FV_5 = PV \times (\text{FVIF}_{7\%,5})$

$FV_5 = \text{R}24{,}000 \times (1.403)$

$FV_5 = \text{R}33{,}672.00$

Calculator solution: R33,661.24

Mixed stream of payments

Beginning of year	Number of years to compound	$FV = CF \times \text{FVIF}_{7\%,n}$		Future value
1	5	R 2,000 × 1.403	=	R 2,805.00
2	4	R 4,000 × 1.311	=	R 5,243.00
3	3	R 6,000 × 1.225	=	R 7,350.00
4	2	R 8,000 × 1.145	=	R 9,159.00
5	1	R10,000 × 1.070	=	R10,700.00
				R35,257.00
		Calculator solution:		R35,257.75

Gina should select the stream of payments over the front-end lump sum payment. Her future wealth will be higher by R1,588.

P4-29. LG 4: Value of mixed stream

Basic

Cash flow stream	Year	CF	×	$\text{PVIF}_{12\%,n}$	=	Present value
A	1	–R2000	×	0.893	=	– R1,786
	2	3,000	×	0.797	=	2,391
	3	4,000	×	0.712	=	2,848
	4	6,000	×	0.636	=	3,816
	5	8,000	×	0.567	=	4,536
						R11,805
				Calculator solution:		R11,805.51
B	1	R10,000	×	0.893	=	R 8,930
	2–5	5,000	×	2.712[a]	=	13,560
	6	7,000	×	0.507	=	3,549
						R26,039
				Calculator solution:		R26,034.58
*Sum of *PV* factors for years 2–5						
C	1–5	R10,000	×	3.605[b]		R36,050
	6–10	8,000	×	2.045[c]		16,360
						R52,410
				Calculator solution:		R52,411.34

[a] PVIFA for 12% over years 2 through 5 = (PVIFA 12% 5 years) – (PVIFA 12% 1 year)

[b] PVIFA for 12% 5 years

[c] (PVIFA for 12%,10 years) – (PVIFA for 12%,5 years)

P4-30. LG 4: *PV*-mixed stream

Intermediate

a.

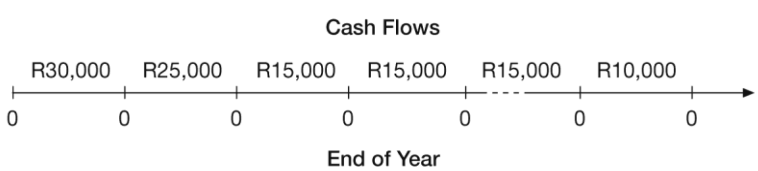
Cash flow stream	Year	CF	×	$PVIF_{15\%,n}$	=	Present value
A	1	R50,000	×	0.870	=	R43,500
	2	40,000	×	0.756	=	30,240
	3	30,000	×	0.658	=	19,740
	4	20,000	×	0.572	=	11,440
	5	10,000	×	0.497	=	4,970
						R109,890
				Calculator solution:		R109,856.33
B	1	R10,000	×	0.870	=	R 8,700
	2	20,000	×	0.756	=	15,120
	3	30,000	×	0.658	=	19,740
	4	40,000	×	0.572	=	22,880
	5	50,000	×	0.497	=	24,850
						R 91,290
				Calculator solution:		R 91,272.98

b. Cash flow stream A, with a present value of R109,890, is higher than cash flow stream B's present value of R91,290 because the larger cash inflows occur in A in the early years when their present value is greater, while the smaller cash flows are received further in the future.

P4-31. LG 1, 4: Value of a mixed stream

Intermediate

a.



b.

Cash flow stream	Year	CF	×	$PVIF_{12\%,n}$	=	Present value
A	1	R30,000	×	0.893	=	R 26,790
	2	25,000	×	0.797	=	19,925
	3–9	15,000	×	3.639*	=	54,585
	10	10,000	×	0.322	=	3,220
						R104,520
				Calculator solution:		R104,508.28

* The PVIF for this 7-year annuity is obtained by summing together the PVIFs of 12% for periods 3 through 9. This factor can also be calculated by taking the $PVIFA_{12\%,7}$ and multiplying by the $PVIF_{12\%,2}$. Alternatively, one could subtract $PVIFA_{12\%,2}$ from $PVIFA_{12\%,9}$.

c. Harte should accept the series of payments offer. The present value of that mixed stream of payments is greater than the R100,000 immediate payment.

P4-32. LG 5: Personal finance: Funding budget shortfalls

Intermediate

a.

Year	Budget shortfall	×	$PVIF_{8\%,n}$	=	Present value
1	R5,000	×	0.926	=	R 4,630
2	4,000	×	0.857	=	3,428
3	6,000	×	0.794	=	4,764
4	10,000	×	0.735	=	7,350
5	3,000	×	0.681	=	2,043
					R22,215
			Calculator solution:		R22,214.03

A deposit of R22,215 would be needed to fund the shortfall for the pattern shown in the table.

b. An increase in the earnings rate would reduce the amount calculated in part (a). The higher rate would lead to a larger interest being earned each year on the investment. The larger interest amounts will permit a decrease in the initial investment to obtain the same future value available for covering the shortfall.

P4-33. LG 4: Relationship between future value and present value-mixed stream

Intermediate

a. **Present value**

Year	***CF***	×	**PVIF$_{5\%,n}$**	=	**Present value**
1	R800	×	0.952	=	R 761.60
2	900	×	0.907	=	816.30
3	1,000	×	0.864	=	864.00
4	1,500	×	0.822	=	1,233.00
5	2,000	×	0.784	=	1,568.00
					R5,242.90
			Calculator solution:		R5,243.17

b. The maximum you should pay is R5,242.90.

c. A higher 7% discount rate will cause the present value of the cash flow stream to be lower than R5,242.90.

P4-34. LG 5: Changing compounding frequency

Intermediate

a. Compounding frequency: $FV_n = PV \times \text{FVIF}_{i\%/m, n \times m}$

(1) **Annual**

12%, 5 years

FV_5 = R5,000 × (1.762)

FV_5 = R8,810

Calculator solution: R8,811.71

Semiannual

12% ÷ 2 = 6%, 5 × 2 = 10 periods

FV_5 = R5,000 × (1.791)

FV_5 = R8,955

Calculator solution: R8,954.24

Quarterly

12% ÷ 4 = 3%, 5 × 4 = 20 periods

FV_5 = R5,000 (1.806)

FV_5 = R9,030

Calculator solution: R9,030.56

(2) **Annual**

16%, 6 years

FV_6 = R5,000 (2.436)

FV_6 = R12,180

Calculator solution: R12,181.98

Semiannual

16% ÷ 2 = 8%, 6 × 2 = 12 periods

FV_6 = R5,000 (2.518)

FV_6 = R12,590

Calculator solution: R12,590.85

Quarterly
16% ÷ 4 = 4%, 6 × 4 = 24 periods
FV_6 = R5,000 (2.563)
FV_6 = R12,815
Calculator solution: R12,816.52

(3) **Annual**
20%, 10 years
FV_{10} = R5,000 × (6.192)
FV_{10} = R30,960
Calculator solution: R30,958.68

Semiannual
20% ÷ 2 = 10%, 10 × 2 = 20 periods
FV_{10} = R5,000 × (6.727)
FV_{10} = R33,635
Calculator solution: R33,637.50

Quarterly
20% ÷ 4 = 5%, 10 × 4 = 40 periods
FV_{10} = R5,000 × (7.040)
FV_{10} = R35,200
Calculator solution: R35,199.94

b. Effective interest rate: $i_{\text{eff}} = (1 + i/m)^m - 1$

(1) **Annual**
$i_{\text{eff}} = (1 + 0.12/1)^1 - 1$
$i_{\text{eff}} = (1.12)^1 - 1$
$i_{\text{eff}} = (1.12) - 1$
$i_{\text{eff}} = 0.12 = 12\%$

Semiannual
$i_{\text{eff}} = (1 + 12/2)^2 - 1$
$i_{\text{eff}} = (1.06)^2 - 1$
$i_{\text{eff}} = (1.124) - 1$
$i_{\text{eff}} = 0.124 = 12.4\%$

Quarterly
$i_{\text{eff}} = (1 + 12/4)^4 - 1$
$i_{\text{eff}} = (1.03)^4 - 1$
$i_{\text{eff}} = (1.126) - 1$
$i_{\text{eff}} = 0.126 = 12.6\%$

(2) **Annual**
$i_{\text{eff}} = (1 + 0.16/1)^1 - 1$
$i_{\text{eff}} = (1.16)^1 - 1$
$i_{\text{eff}} = (1.16) - 1$
$i_{\text{eff}} = 0.16 = 16\%$

Semiannual
$i_{\text{eff}} = (1 + 0.16/2)^2 - 1$
$i_{\text{eff}} = (1.08)^2 - 1$
$i_{\text{eff}} = (1.166) - 1$
$i_{\text{eff}} = 0.166 = 16.6\%$

Quarterly
$i_{\text{eff}} = (1 + 0.16/4)^4 - 1$
$i_{\text{eff}} = (1.04)^4 - 1$
$i_{\text{eff}} = (1.170) - 1$
$i_{\text{eff}} = 0.170 = 17\%$

(3) **Annual**

$i_{eff} = (1 + 0.20/1)^1 - 1$
$i_{eff} = (1.20)^1 - 1$
$i_{eff} = (1.20) - 1$
$i_{eff} = 0.20 = 20\%$

Semiannual

$i_{eff} = (1 + 0.20/2)^2 - 1$
$i_{eff} = (1.10)^2 - 1$
$i_{eff} = (1.210) - 1$
$i_{eff} = 0.210 = 21\%$

Quarterly

$I_{eff} = (1 + 0.20/4)^4 - 1$
$I_{eff} = (1.05)^4 - 1$
$I_{eff} = (1.216) - 1$
$I_{eff} = 0.216 = 21.6\%$

P4-35. LG 5: Compounding frequency, time value, and effective annual rates

Intermediate

a. Compounding frequency: $FV_n = PV \times \text{FVIF}_{i\%,n}$

A
$FV_5 = \text{R}2{,}500 \times (\text{FVIF}_{3\%,10})$
$FV_5 = \text{R}2{,}500 \times (1.344)$
$FV_5 = \text{R}3{,}360$
Calculator solution: R3,359.79

B
$FV_3 = \text{R}50{,}000 \times (\text{FVIF}_{2\%,18})$
$FV_3 = \text{R}50{,}000 \times (1.428)$
$FV_3 = \text{R}71{,}400$
Calculator solution: R71,412.31

C
$FV_{10} = \text{R}1{,}000 \times (\text{FVIF}_{5\%,10})$
$FV_{10} = \text{R}1{,}000 \times (1.629)$
$FV_{10} = \text{R}16{,}290$
Calculator solution: R1,628.89

D
$\text{FV}_6 = \text{R}20{,}000 \times (\text{FVIF}_{4\%,24})$
$FV_6 = \text{R}20{,}000 \times (2.563)$
$FV_6 = \text{R}51{,}260$
Calculator solution: R51,266.08

b. Effective interest rate: $i_{eff\,=} (1 + i\%/m)^m - 1$

A
$i_{eff} = (1 + 0.06/2)^2 - 1$
$i_{eff\ f} = (1 + 0.03)^2 - 1$
$i_{eff} = (1.061) - 1$
$i_{eff} = 0.061 = 06.1\%$

B
$i_{eff} = (1 + 0.12/6)^6 - 1$
$i_{eff} = (1 + 0.02)^6 - 1$
$i_{eff} = (1.126) - 1$
$i_{eff} = 0.126 = 12.6\%$

C
$i_{eff} = (1 + 0.05/1)^1 - 1$
$i_{eff} = (1 + 0.05)^1 - 1$
$i_{eff} = (1.05) - 1$
$i_{eff} = 0.05 = 5\%$

D
$i_{eff} = (1 + 0.16/4)^4 - 1$
$i_{eff} = (1 + 0.04)^4 - 1$
$i_{eff} = (1.170) - 1$
$i_{eff} = 0.17 = 17\%$

c. The effective rates of interest rise relative to the stated nominal rate with increasing compounding frequency.

P4-36. LG 5: Continuous compounding: FVcont. = $PV \times e^x$ (e = 2.7183)

Intermediate

A $FV_{cont.}$ = R1,000 × $e^{0.18}$ = R1,197.22

B $FV_{cont.}$ = R 600 × e^1 = R1,630.97

C $FV_{cont.}$ = R4,000 × $e^{0.56}$ = R7,002.69

D $FV_{cont.}$ = R2,500 × $e^{0.48}$ = R4,040.19

Note: If calculator doesn't have e^x key, use y^x key, substituting 2.7183 for y.

P4-37. LG 5: Personal finance: Compounding frequency and time value

Challenge

a. (1) FV_{10} = R2,000 × ($FVIF_{8\%,10}$)
FV_{10} = R2,000 × (2.159)
FV_{10} = R4,318
Calculator solution: R4,317.85

(2) FV_{10} = R2,000 × ($FVIF_{4\%,20}$)
FV_{10} = R2,000 × (2.191)
FV_{10} = R4,382
Calculator solution: R4,382.25

(3) FV_{10} = R2,000 × ($FVIF_{0.022\%,3650}$)
FV_{10} = R2,000 × (2.232)
FV_{10} = R4,464
Calculator solution: R4,450.69

(4) FV_{10} = R2,000 × ($e^{0.8}$)
FV_{10} = R2,000 × (2.226)
FV_{10} = R4,452
Calculator solution: R4,451.08

b. (1) $i_{eff} = (1 + 0.08/1)^1 - 1$
$i_{eff} = (1 + 0.08)^1 - 1$
$i_{eff} = (1.08) - 1$
$i_{eff} = 0.08 = 8\%$

(2) $i_{eff} = (1 + 0.08/2)^2 - 1$
$i_{eff} = (1 + 0.04)^2 - 1$
$i_{eff} = (1.082) - 1$
$i_{eff} = 0.082 = 8.2\%$

(3) $i_{eff} = (1 + 0.08/365)^{365} - 1$
$i_{eff} = (1 + 0.00022)^{365} - 1$
$i_{eff} = (1.0833) - 1$
$i_{eff} = 0.0833 = 8.33\%$

(4) $i_{eff} = (e^k - 1)$
$i_{eff} = (e^{0.08} - 1)$
$i_{eff} = (1.0833 - 1)$
$i_{eff} = 0.0833 = 8.33\%$

c. Compounding continuously will result in R134 more rands at the end of the 10 year period than compounding annually.

d. The more frequent the compounding the larger the future value. This result is shown in part a by the fact that the future value becomes larger as the compounding period moves from annually to continuously. Since the future value is larger for a given fixed amount invested, the effective return also increases directly with the frequency of compounding. In part **b** we see this fact as the effective rate moved from 8% to 8.33% as compounding frequency moved from annually to continuously.

P4-38. LG 5: Personal finance: Comparing compounding periods

Challenge

a. $FV_n = PV \times \text{FVIF}_{i\%,n}$

(1) **Annually:** $FV = PV \times \text{FVIF}_{12\%,2}$ = R15,000 × (1.254) = R18,810

Calculator solution: R18,816

(2) **Quarterly:** $FV = PV \times \text{FVIF}_{3\%,8}$ = R15,000 × (1.267) = R19,005

Calculator solution: R19,001.55

(3) **Monthly:** $FV = PV \times \text{FVIF}_{1\%,24}$ = R15,000 × (1.270) = R19,050

Calculator solution: R19,046.02

(4) **Continuously:** $FV_{\text{cont.}} = PV \times e^{xt}$

$FV = PV \times 2.7183^{0.24}$ = R15,000 × 1.27125 = R19,068.77

Calculator solution: R19,068.74

b. The future value of the deposit increases from R18,810 with annual compounding to R19,068.77 with continuous compounding, demonstrating that future value increases as compounding frequency increases.

c. The maximum future value for this deposit is R19,068.77, resulting from continuous compounding, which assumes compounding at every possible interval.

P4-39. LG 3, 5: Personal finance: Annuities and compounding: $\text{FVA}_n = \text{PMT} \times (\text{FVIFA}_{i\%,n})$

Intermediate

a.

(1) **Annual**

FVA_{10} = R300 × ($\text{FVIFA}_{8\%,10}$)

FVA_{10} = R300 × (14.487)

FVA_{10} = R4,346.10

Calculator solution: R4,345.97

(2) **Semiannual**

FVA_{10} = R150 × ($\text{FVIFA}_{4\%,20}$)

FVA_{10} = R150 × (29.778)

FVA_{10} = R4,466.70

Calculator solution: R4,466.71

(3) **Quarterly**

FVA_{10} = R75 ×.($\text{FVIFA}_{2\%,40}$)

FVA_{10} = R75 × (60.402)

FVA_{10} = R4,530.15

Calculator solution: R4,530.15

b. The sooner a deposit is made the sooner the funds will be available to earn interest and contribute to compounding. Thus, the sooner the deposit and the more frequent the compounding, the larger the future sum will be.

P4-40. LG 6: Deposits to accumulate growing future sum: $PMT = \dfrac{FVA_n}{FVIFA_{i\%,n}}$

Basic

Case	**Terms**	**Calculation**		**Payment**
A	12%, 3 yrs.	PMT = R5,000 ÷ 3.374	=	R1,481.92
		Calculator solution:		R1,481.74
B	7%, 20 yrs.	PMT = R100,000 ÷ 40.995	=	R2,439.32
		Calculator solution:		R2,439.29
C	10%, 8 yrs.	PMT = R30,000 ÷ 11.436	=	R2,623.29
		Calculator solution:		R2,623.32
D	8%, 12 yrs.	PMT = R15,000 ÷ 18.977	=	R 790.43
		Calculator solution:		R 790.43

P4-41. LG 6: Personal finance: Creating a retirement fund

Intermediate

a. $PMT = FVA_{42} \div (FVIFA_{8\%,42})$
$PMT = R220,000 \div (304.244)$
$PMT = R723.10$
Calculator solution: R723.10

b. $FVA_{42} = PMT \times (FVIFA_{8\%,42})$
$FVA_{42} = R600 \times (304.244)$
$FVA_{42} = R182,546.40$
Calculator solution: R182,546.11

P4-42. LG 6: Personal finance: Accumulating a growing future sum

Intermediate

$FV_n = PV \times (FVIF_{i\%,n})$
$FV_{20} = R185,000 \times (FVIF_{6\%,20})$
$FV_{20} = R185,000 \times (3.207)$
$FV_{20} = R593,295$ = Future value of retirement home in 20 years.
Calculator solution: R593,320.06

$PMT = FV \div (FVIFA_{i\%,n})$
$PMT = R593,295 \div (FVIFA_{10\%,20})$
$PMT = R593,295 \div (57.274)$
$PMT = R10,358.89$
Calculator solution: R10,359.15 = annual payment required.

P4-43. LG 3, 6: Personal finance: Deposits to create a perpetuity

Intermediate

a. Present value of a perpetuity $= \text{PMT} \times (1 \div i)$

$= \text{R}6{,}000 \times (1 \div 0.10)$

$= \text{R}6{,}000 \times 10$

$= \text{R}60{,}000$

b. $\text{PMT} = \text{FVA} \div (\text{FVIFA}_{10\%,10})$

$\text{PMT} = \text{R}60{,}000 \div (15.937)$

$\text{PMT} = \text{R}3{,}764.82$

Calculator solution: R3,764.72

P4-44. LG 2, 3, 6: Personal finance: Inflation, time value, and annual deposits

Challenge

a. $FV_n = \text{PV} \times (\text{FVIF}_{i\%,n})$

$FV_{20} = \text{R}200{,}000 \times (\text{FVIF}_{5\%,25})$

$FV_{20} = \text{R}200{,}000 \times (3.386)$

$FV_{20} = \text{R}677{,}200$ = Future value of retirement home in 25 years.

Calculator solution: R677,270.99

b. $\text{PMT} = FV \div (\text{FVIFA}_{i\%,n})$

$\text{PMT} = \text{R}677{,}270.99 \div (\text{FVIFA}_{9\%,25})$

$\text{PMT} = \text{R}677{,}270.99 \div (84.699)$

$\text{PMT} = \text{R}7{,}995.37$

Calculator solution: R7,996.03 = annual payment required.

c. Since John will have an additional year on which to earn interest at the end of the 25 years his annuity deposit will be smaller each year. To determine the annuity amount John will first discount back the R677,200 one period.

$PV_{24} = \text{R}677{,}200 \times 0.9174 = \text{R}621{,}263.28$

This is the amount John must accumulate over the 25 years. John can solve for his annuity amount using the same calculation as in part **b**.

$\text{PMT} = FV \div (\text{FVIFA}_{i\%,n})$

$\text{PMT} = \text{R}621{,}263.28 \div (\text{FVIFA}_{9\%,25})$

$\text{PMT} = \text{R}621{,}263.28 \div (84.699)$

$\text{PMT} = \text{R}7{,}334.95$

Calculator solution: R7,335.81 = annual payment required.

To check this value, multiply the annual payment by 1 plus the 9% discount rate.

R7,335.81 (1.09) = R7996.03

P4-45. LG 6: Loan payment: $PMT = \dfrac{PVA}{PVIFA_{i\%,n}}$

Basic

Loan

A PMT = R12,000 ÷ ($PVIFA_{8\%,3}$)
PMT = R12,000 ÷ 2.577
PMT = R4,656.58
Calculator solution: R4,656.40

B PMT = R60,000 ÷ ($PVIFA_{12\%,10}$)
PMT = R60,000 ÷ 5.650
PMT = R10,619.47
Calculator solution: R10,619.05

C PMT = R75,000 ÷ ($PVIFA_{10\%,30}$)
PMT = R75,000 ÷ 9.427
PMT = R7,955.87
Calculator solution: R7,955.94

D PMT = R4,000 ÷ ($PVIFA_{15\%,5}$)
PMT = R4,000 ÷ 3.352
PMT = R1,193.32
Calculator solution: R1,193.26

P4-46. LG 6: Personal finance: Loan repayment schedule

Intermediate

a. PMT = R15,000 ÷ ($PVIFA_{14\%,3}$)
PMT = R15,000 ÷ 2.322
PMT = R6,459.95
Calculator solution: R6,460.97

b.

End of year	**Loan payment**	**Beginning of year principal**	**Payments**		**End of year principal**
			Interest	**Principal**	
1	R6,459.95	R15,000.00	R2,100.00	R4,359.95	R10,640.05
2	6,459.95	10,640.05	1,489.61	4,970.34	5,669.71
3	6,459.95	5,669.71	793.76	5,666.19	0

(The difference in the last year's beginning and ending principal is due to rounding.)

c. Through annual end-of-the-year payments, the principal balance of the loan is declining, causing less interest to be accrued on the balance.

P4-47. LG 6: Loan interest deductions

Challenge

a. PMT = R10,000 ÷ ($PVIFA_{13\%,3}$)
PMT = R10,000 ÷ (2.361)
PMT = R4,235.49
Calculator solution: R4,235.22

b.

End of Year	Loan payment	Beginning of year principal	Payments		End of year principal
			Interest	**Principal**	
1	R4,235.49	R10,000.00	R1,300.00	R2,935.49	R7,064.51
2	4,235.49	7,064.51	918.39	3,317.10	3,747.41
3	4,235.49	3,747.41	487.16	3,748.33	0

(The difference in the last year's beginning and ending principal is due to rounding.)

P4-48. LG 6: Personal finance: Monthly loan payments

Challenge

a. PMT = R40,000 ÷ ($\text{PVIFA}_{1\%,24}$)
PMT = R40,000 ÷ (21.243)
PMT = R1882,80
Calculator solution: R1882,90

b. PMT = R40,000 ÷ ($\text{PVIFA}_{0.75\%,24}$)
PMT = R40,000 ÷ (21.889)
PMT = R1827,40
Calculator solution: R1827,40

P4-49. LG 6: Growth rates

Basic

a. $PV = FV_n \times \text{PVIF}_{i\%,n}$

Case

A
$PV = FV_4 \times \text{PVIF}_{k\%,4\text{yrs.}}$
$\text{R}500 = \text{R}800 \times \text{PVIF}_{k\%,4\text{yrs}}$
$0.625 = \text{PVIF}_{k\%,4\text{yrs}}$
$12\% < k < 13\%$
Calculator solution: 12.47%

B
$PV = FV_9 \times \text{PVIF}_{i\%,9\text{yrs.}}$
$\text{R}1{,}500 = \text{R}2{,}280 \times \text{PVIF}_{k\%,9\text{yrs.}}$
$0.658 = \text{PVIF}_{k\%,9\text{yrs.}}$
$4\% < k < 5\%$
Calculator solution: 4.76%

C
$PV = FV_6 \times \text{PVIF}_{i\%,6}$
$\text{R}2{,}500 = \text{R}2{,}900 \times \text{PVIF}_{k\%,6\text{ yrs.}}$
$0.862 = \text{PVIF}_{k\%,6\text{yrs.}}$
$2\% < k < 3\%$
Calculator solution: 2.50%

b.

Case

A Same as in **a**
B Same as in **a**
C Same as in **a**

c. The growth rate and the interest rate should be equal, since they represent the same thing.

P4-50. LG 6: Personal finance: Rate of return: $PV_n = FV_n \times (\text{PVIF}_{i\%,n})$

Intermediate

a. $PV = \text{R}2{,}000 \times (\text{PVIF}_{i\%,3\text{yrs.}})$

$\text{R}1{,}500 = \text{R}2{,}000 \times (\text{PVIF}_{i\%,3\text{ yrs.}})$

$0.75 = \text{PVIF}_{i\%,3\text{ yrs.}}$

$10\% < i < 11\%$

Calculator solution: 10.06%

b. Mr. Singh should accept the investment that will return R2,000 because it has a higher return for the same amount of risk.

P4-51. LG 6: Personal finance: Rate of return and investment choice

Intermediate

a. **A**

$PV = \text{R}8{,}400 \times (\text{PVIF}_{i\%,6\text{yrs.}})$

$\text{R}5{,}000 = \text{R}8{,}400 \times (\text{PVIF}_{i\%,6\text{ yrs.}})$

$0.595 = \text{PVIF}_{i\%,6\text{ yrs.}}$

$9\% < i < 10\%$

Calculator solution: 9.03%

B

$PV = \text{R}15{,}900 \times (\text{PVIF}_{i\%,15\text{yrs.}})$

$\text{R}5{,}000 = \text{R}15{,}900 \times (\text{PVIF}_{i\%,15\text{yrs.}})$

$0.314 = \text{PVIF}_{i\%,15\text{yrs.}}$

$8\% < i < 9\%$

Calculator solution: 8.02%

C

$\text{PV} = \text{R}7{,}600 \times (\text{PVIF}_{i\%,4\text{yrs.}})$

$\text{R}5{,}000 = \text{R}7{,}600 \times (\text{PVIF}_{i\%,4\text{ yrs.}})$

$0.658 = \text{PVIF}_{i\%,4\text{ yrs.}}$

$11\% < i < 12\%$

Calculator solution: 11.04%

D

$PV = \text{R}13{,}000 \times (\text{PVIF}_{i\%,10\text{ yrs.}})$

$\text{R}5{,}000 = \text{R}13{,}000 \times (\text{PVIF}_{i\%,10\text{ yrs.}})$

$0.385 = \text{PVIF}_{i\%,10\text{ yrs..}}$

$10\% < i < 11\%$

Calculator solution: 10.03%

b. Investment C provides the highest return of the four alternatives. Assuming equal risk for the alternatives, Clare should choose C.

P4-52. LG 6: Rate of return-annuity: $\text{PVA}_n = \text{PMT} \times (\text{PVIFA}_{i\%,n})$

Basic

$\text{R}10{,}606 = \text{R}2{,}000 \times (\text{PVIFA}_{i\%,10\text{ yrs.}})$

$5.303 = \text{PVIFA}_{i\%,10\text{ yrs.}}$

$13\% < i < 14\%$

Calculator solution: 13.58%

P4-53. LG 6: Personal finance: Choosing the best annuity: $\text{PVA}_n = \text{PMT} \times (\text{PVIFA}_{i\%,n})$

Intermediate

a. **Annuity A**

$\text{R}30{,}000 = \text{R}3{,}100 \times (\text{PVIFA}_{i\%,20\text{ yrs.}})$

$9.677 = \text{PVIFA}_{i\%,20\text{ yrs.}}$

$8\% < i < 9\%$

Calculator solution: 8.19%

Annuity B

$\text{R}25{,}000 = \text{R}3{,}900 \times (\text{PVIFA}_{i\%,10\text{ yrs.}})$

$6.410 = \text{PVIFA}_{i\%,10\text{ yrs.}}$

$9\% < i < 10\%$

Calculator solution: 9.03%

Annuity C

$R40,000 = R4,200 \times (PVIFA_{i\%,15\ yrs.})$

$9.524 = PVFA_{i\%,15\ yrs.}$

$6\% < i < 7\%$

Calculator solution: 6.3%

Annuity D

$R35,000 = R4,000 \times (PVIFA_{i\%,12\ yrs.})$

$8.75 = PVIFA_{i\%,12\ yrs.}$

$5\% < i < 6\%$

Calculator solution: 5.23%

b. Annuity B gives the highest rate of return at 9% and would be the one selected based upon Raina's criteria.

P4-54. LG 6: Personal finance: Interest rate for an annuity

Challenge

a. **Defendants interest rate assumption**

$R2,000,000 = R156,000 \times (PVIFA_{i\%,25\ yrs.})$

$12.821 = PVFA_{i\%,25\ yrs.}$

$5\% < i < 6\%$

Calculator solution: 5.97%

b. **Prosecution interest rate assumption**

$R2,000,000 = R255,000 \times (PVIFA_{i\%,25\ yrs.})$

$7.843 = PVFA_{i\%,25\ yrs.}$

$i = 12\%$

Calculator solution: 12.0%

c. $R2,000,000 = PMT \times (PVIFA_{9\%,25yrs.})$

$R2,000,000 = PMT\ (9.823)$

$PMT = R203,603.79$

Calculator solution: R203,612.50

P4-55. LG 6: Personal finance: Loan rates of interest: $PVA_n = PMT \times (PVIFA_{i\%,n})$

Intermediate

a. **Loan A**

$R5,000 = R1,352.81 \times (PVIFA_{i\%,5\ yrs.})$

$3.696 = PVIFA_{i\%,5\ yrs.}$

$i = 11\%$

Loan B

$R5,000 = R1,543.21 \times (PVIFA_{i\%,4\ yrs.})$

$3.24 = PVIFA_{i\%,\ 4\ yrs.}$

$i = 9\%$

Loan C

$R5,000 = R2,010.45 \times (PVIFA_{i\%,3\ yrs.})$

$2.487 = PVIFA_{k\%,3\ yrs.}$

$i = 10\%$

Calculator solutions are identical.

b. Mr Fleming should choose Loan B, which has the lowest interest rate.

P4-56. LG 6: Number of years to equal future amount

Intermediate

A

$FV = PV \times (\text{FVIF}_{7\%,n\text{ yrs.}})$

$\text{R}1{,}000 = \text{R}300 \times (\text{FVIF}_{7\%,n\text{ yrs.}})$

$3.333 = \text{FVIF}_{7\%,n\text{ yrs.}}$

$17 < n < 18$

Calculator solution: 17.79 years

B

$FV = \text{R}12{,}000 \times (\text{FVIF}_{5\%,n\text{ yrs.}})$

$\text{R}15{,}000 = \text{R}12{,}000 \times (\text{FVIF}_{5\%,n\text{ yrs.}})$

$1.250 = \text{FVIF}_{5\%,n\text{ yrs.}}$

$4 < n < 5$

Calculator solution: 4.573 years

C

$FV = PV \times (\text{FVIF}_{10\%,n\text{ yrs.}})$

$\text{R}20{,}000 = \text{R}9{,}000 \times (\text{FVIF}_{10\%,\text{n yrs.}})$

$2.222 = \text{FVIF}_{10\%,n\text{ yrs.}}$

$8 < \underline{n} < 9$

Calculator solution: 8.38 years

D

$FV = \text{R}100 \times (\text{FVIF}_{9\%,n\text{ yrs.}})$

$\text{R}500 = \text{R}100 \times (\text{FVIF}_{9\%,n\text{ yrs.}})$

$5.00 = \text{FVIF}_{9\%,n\text{ yrs.}}$

$18 < n < 19$

Calculator solution: 18.68 years

E

$FV = PV \times (\text{FVIF}_{15\%,}n\text{ yrs.})$

$\text{R}30{,}000 = \text{R}7{,}500 \times (\text{FVIF}_{15\%,n\text{ yrs.}})$

$4.000 = \text{FVIF}_{15\%,n\text{ yrs.}}$

$9 < n < 10$

Calculator solution: 9.92 years

P4-57. LG 6: Personal finance: Time to accumulate a given sum

Intermediate

a. $20{,}000 = \text{R}10{,}000 \times (\text{FVIF}_{10\%,n\text{ yrs.}})$

$2.000 = \text{FVIF}_{10\%,n\text{ yrs.}}$

$7 < n < 8$

Calculator solution: 7.27 years

b. $20{,}000 = \text{R}10{,}000 \times (\text{FVIF}_{7\%,n\text{ yrs.}})$

$2.000 = \text{FVIF}_{7\%,n\text{ yrs.}}$

$10 < n < 11$

Calculator solution: 10.24 years

c. $20{,}000 = \text{R}10{,}000 \times (\text{FVIF}_{12\%,n\text{ yrs.}})$

$2.000 = \text{FVIF}_{12\%,n\text{ yrs.}}$

$6 < n < 7$

Calculator solution: 6.12 years

d. The higher the rate of interest the less time is required to accumulate a given future sum.

P4-58. LG 6: Number of years to provide a given return

Intermediate

A

$\text{PVA} = \text{PMT} \times (\text{PVIFA}_{11\%,n\text{ yrs.}})$

$\text{R}1{,}000 = \text{R}250 \times (\text{PVIFA}_{11\%,n\text{ yrs.}})$

$4.000 = \text{PVIFA}_{11\%,n\text{ yrs.}}$

$5 < n < 6$

Calculator solution: 5.56 years

B

$\text{PVA} = \text{PMT} \times (\text{PVIFA}_{15\%,n\text{ yrs.}})$

$\text{R}150{,}000 = \text{R}30{,}000 \times (\text{PVIFA}_{15\%,n\text{ yrs.}})$

$5.000 = \text{PVIFA}_{15\%,n\text{ yrs.}}$

$9 < n < 10$

Calculator solution: 9.92 years

C

PVA $= PMT \times (PVIFA_{10\%,n \text{ yrs.}})$

R80,000 = R10,000 × $(PVIFA_{10\%,n \text{ yrs.}})$

8 $= PVIFA_{10\%,n \text{ yrs.}}$

$16 < n < 17$

Calculator solution: 16.89 years

D

PVA $= PMT \times (PVIFA_{9\%,n \text{ yrs.}})$

R600 = R275 × $(PVIFA_{9\%,n \text{ yrs.}})$

2.182 $= PVIFA_{9\%,n \text{ yrs.}}$

$2 < n < 3$

Calculator solution: 2.54 years

E

PVA $= PMT \times (PVIFA_{6\%,n \text{ yrs.}})$

R17,000 = R3,500 × $(PVIFA_{6\%,n \text{ yrs.}})$

4.857 $= PVIFA_{6\%,n \text{ yrs.}}$

$5 < n < 6$

Calculator solution: 5.91 years

P4-59. LG 6: Personal finance: Time to repay installment loan

Intermediate

a. R14,000 = R2,450 × $(PVIFA_{12\%,n \text{ yrs.}})$

5.714 $= PVIFA_{12\%,n \text{ yrs.}}$

$10 < n < 11$

Calculator solution: 10.21 years

b. R14,000 = R2,450 × $(PVIFA_{9\%,n \text{ yrs.}})$

5.714 $= PVIFA_{9\%,n \text{ yrs.}}$

$8 < n < 9$

Calculator solution: 8.38 years

c. R14,000 = R2,450 × $(PVIFA_{15\%,n \text{ yrs.}})$

5.714 $= PVIFA_{15\%,n \text{ yrs.}}$

$13 < n < 14$

Calculator solution: 13.92 years

d. The higher the interest rate the greater the number of time periods needed to repay the loan fully.

P4-60. Ethics problem

Intermediate

This is a tough issue. Even back in the Middle Ages, scholars debated the idea of a ‘just price’. The ethical debate hinges on (1) the basis for usury laws, (2) whether full disclosure is made of the true cost of the advance, and (3) whether customers understand the disclosures. Usury laws are premised on the notion that there is such a thing as an interest rate (price of credit) that is ‘too high’. A centuries-old fairness notion guides us into not taking advantage of someone in duress or facing an emergency situation. One must ask, too, why there are not market-supplied credit sources for borrowers, which would charge lower interest rates and receive an acceptable risk-adjusted return. On issues #2 and #3, there is no assurance that borrowers comprehend or are given adequate disclosures. See the box for the key ethics issues on which to refocus attention (some would view the objection cited as a smokescreen to take our attention off the true ethical issues in this credit offer).

■ Group exercises

This set of deliverables concern each group's fictitious firm. The first scenario involves the replacement of a copy machine. The first decision pertains to a choice between competing leases, while the second is choosing among purchase plans to buy the machine outright. In the first case leasing information is provided, while for the second option students are asked to get pricing information. This information is readily available on the Web, as is the needed information regarding interest rates for both the possible savings plans regarding the copy machine, and the computer upgrade scenario.

For the savings plan the groups are asked to look at several deposit options while for the computer upgrade purchase an amortisation schedule must be developed. Modifications or even elimination of one of these scenarios is perfectly allowable and shouldn't affect future work. The same can be said of the final deliverable involving a simple calculation of the present value of a court-ordered settlement of a patent-infringement case.

■ Integrative Case 4: Tomlinsen Leather Company

Integrative Case 4, Tomlinsen Leather Company, is an exercise in evaluating the cost of capital and available investment opportunities. The student must calculate the component costs of financing, long-term debt, preference shares, and ordinary shareholders' equity, and determine the weighted average cost of capital (WACC). Investment decisions must be made between competing projects. Finally, the student must reanalyse the case given a new, more highly leveraged capital structure.

1. Cost of financing sources

Debt:

R0 – R700,000

$$r_d = \frac{I + \frac{R1{,}000 - N_d}{n}}{\frac{N_d + R1{,}000}{2}}$$

$$r_d = \frac{R120 + \frac{R1{,}000 - R970}{10}}{\frac{R970 + R1{,}000}{2}} = \frac{R123}{R985} = 12.5\%$$

$$r_i = r_d \times (1 - t)$$
$$r_i = 0.125 \times (1 - 0.4)$$
$$r_i = 0.075 \text{ or } 7.5\%$$

Above R700,000:

$$r_j = 0.18 \times (1 - t)$$
$$r_j = 0.18 \times (1 - 0.4)$$
$$r_j = 0.108 \text{ or } 10.8\%$$

Preference shares:

$$r_p = \frac{D_p}{N_p}$$

$$r_p = \frac{R10.20}{R57.00} = 17.9\%$$

Ordinary shareholders' equity:

R0 – R1,300,000

$$r_s = \frac{D_1}{P_0} + g$$

$$r_s = \frac{R1.76}{R20.00} + 0.15 = 23.8\%$$

Above R1,300,000

$$r_s = \frac{D_1}{N_n} + g$$

$$r_s = \frac{R1.76}{R16.00} + 0.15 = 26\%$$