Chapter 6
Interest rates and bond valuation

■ Instructor's resources

Overview

This chapter begins with a thorough discussion of interest rates, yield curves and their relationship to required returns. Features of the major types of bond issues are presented along with their legal issues, risk characteristics and indenture convents. The chapter then introduces students to the important concept of valuation and demonstrates the impact of cash flows, timing, and risk on value. It explains models for valuing bonds and the calculation of yield-to-maturity using either the trial-and-error approach or the approximate yield formula. Students learn how interest rates may affect their ability to borrow and expand business operations or assets under personal control.

Study Guide

Suggested **Study Guide** examples for classroom presentation:

Example	Topic
1	Valuation of any asset
4	Bond valuation
9	Yield to call

■ Suggested answer to chapter opening critical thinking question

How might the issuance of large amounts of public debt affect the corporate debt market?

Because corporate bonds are related to the risk-free interest rate as denoted by T-bills, as T-bill rates rise, so do newly issued corporate bonds. As the government's demand for funds increases, the Treasury will need to issue more debt unless tax revenues increase to meet that demand. Issuing more public debt can have a negative effect on *all* debt instruments because the increased supply may require higher interest rates to attract buyers with sufficient capital to buy up all of the new debt. However, potential buyers can come from all over the world, and S.A. debt instruments are an attractive investment for many foreign investors because of their relatively high yields.

■   Answers to Review Questions

1. The *real rate of interest* is the rate that creates an equilibrium between the supply of savings and demand for investment funds. The nominal rate of interest is the actual rate of interest charged by the supplier and paid by the demander. The nominal rate of interest differs from the real rate of interest due to two factors: (1) a premium due to inflationary expectations (*IP*) and (2) a premium due to issuer and issue characteristic risks (*RP*). The nominal rate of interest for a security can be defined as $r_1 = r^* + IP + RP$. For a three-month Treasury bill, the nominal rate of interest can be stated as $r_1 = r^* + IP$. The default risk premium, *RP*, is assumed to be zero since the security is backed by the government; this security is commonly considered the risk-free asset.

2. The *term structure of interest rates* is the relationship of the rate of return to the time to maturity for any class of similar-risk securities. The graphic presentation of this relationship is the yield curve.

3. For a given class of securities, the slope of the curve reflects an expectation about the movement of interest rates over time. The most commonly used class of securities is government Treasury securities.

    a. Downward sloping: long-term borrowing costs are lower than short-term borrowing costs.
    b. Upward sloping: Short-term borrowing costs are lower than long-term borrowing costs.
    c. Flat: Borrowing costs are relatively similar for short- and long-term loans.

    The upward-sloping yield curve has been the most prevalent historically.

4. a. According to the *expectations theory*, the yield curve reflects investor expectations about future interest rates, with the differences based on inflation expectations. The curve can take any of the three forms. An upward-sloping curve is the result of increasing inflationary expectations, and vice versa.
    b. The *liquidity preference theory* is an explanation for the upward-sloping yield curve. This theory states that long-term rates are generally higher than short-term rates due to the desire of investors for greater liquidity, and thus a premium must be offered to attract adequate long-term investment.
    c. The *market segmentation theory* is another theory that can explain any of the three curve shapes. Since the market for loans can be segmented based on maturity, sources of supply and demand for loans within each segment determine the prevailing interest rate. If supply is greater than demand for short-term funds at a time when demand for long-term loans is higher than the supply of funding, the yield curve would be upward sloping. Obviously, the reverse also holds true.

5. In the Fisher equation, $r = r^* + IP + RP$, the risk premium, *RP*, consists of the following issuer- and issue-related components:

    *   *Default risk*: The possibility that the issuer will not pay the contractual interest or principal as scheduled.
    *   *Maturity* (*interest rate*) *risk*: The possibility that changes in the interest rates on similar securities will cause the value of the security to change by a greater amount the longer its maturity, and vice versa.
    *   *Liquidity risk*: The ease with which securities can be converted to cash without a loss in value.
    *   *Contractual provisions*: Covenants included in a debt agreement or share issue defining the rights and restrictions of the issuer and the purchaser. These can increase or reduce the risk of a security.

The risks that are debt specific are default, maturity and contractual provisions.

6. Most corporate bonds are issued in denominations of R1,000 with maturities of 10 to 30 years. The *stated interest rate* on a bond represents the percentage of the bond's par value that will be paid out annually, although the actual payments may be divided up and made quarterly or semiannually.

    Both *bond indentures* and *trustees* are means of protecting the bondholders. The bond indenture is a complex and lengthy legal document stating the conditions under which a bond is issued. The trustee may be a paid individual, corporation, or commercial bank trust department that acts as a third-party 'watch dog' on behalf of the bondholders to ensure that the issuer does not default on its contractual commitment to the bondholders.

7. Long-term lenders include *restrictive covenants* in loan agreements in order to place certain operating and/or financial constraints on the borrower. These constraints are intended to assure the lender that the borrowing firm will maintain a specified financial condition and managerial structure during the term of the loan. Since the lender is committing funds for a long period of time, he seeks to protect himself against adverse financial developments that may affect the borrower. The restrictive provisions (also called *negative covenants*) differ from the so-called *standard debt provisions* in that they place certain constraints on the firm's operations, whereas the standard provisions (also called *affirmative covenants*) require the firm to operate in a respectable and businesslike manner. Standard provisions include such requirements as providing audited financial statements on a regular schedule, paying taxes and liabilities when due, maintaining all facilities in good working order, and keeping accounting records in accordance with generally accepted accounting procedures (GAAP).

    Violation of any of the standard or restrictive loan provisions gives the lender the right to demand immediate repayment of both accrued interest and principal of the loan. However, the lender does not normally demand immediate repayment but instead evaluates the situation in order to determine if the violation is serious enough to jeopardise the loan. The lender's options are: Waive the violation, waive the violation and renegotiate terms of the original agreement, or demand repayment.

8. *Short-term borrowing* is normally less expensive than *long-term borrowing* due to the greater uncertainty associated with longer maturity loans. The major factors affecting the cost of long-term debt (or the interest rate), in addition to loan maturity, are loan size, borrower risk and the basic cost of money.

9. If a bond has a *conversion feature*, the bondholders have the option of converting the bond into a certain number of ordinary shares within a certain period of time. A *call feature* gives the issuer the opportunity to repurchase, or call, bonds at a stated price prior to maturity. It provides extra compensation to bondholders for the potential opportunity losses that would result if the bond were called due to declining interest rates. This feature allows the issuer to retire outstanding debt prior to maturity and, in the case of convertibles, to force conversion. *Share purchase warrants*, which are sometimes included as part of a bond issue, give the holder the right to purchase a certain number of ordinary shares at a specified price.

    Bonds are rated by independent rating agencies such as Moody's and Standard & Poor's with respect to their overall quality, as measured by the safety of repayment of principal and interest. Ratings are the result of detailed financial ratio and cash flow analyses of the issuing firm. The bond rating affects the rate of return on the bond. The higher the rating, the less risk and the lower the rate.

10. *Current yields* are calculated by dividing the annual interest payment by the current price. Bonds are quoted in percentage of par terms, to the thousandths place. Hence, corporate bond prices are effectively quoted in rands and cents. A quote of 98.621 means the bond is priced at 98.621% of par, or R986.21. Bonds are often also quoted in terms of their yield to maturity from which one can calculate its market value.

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11. *Eurobonds* are bonds issued by an international borrower and sold to investors in countries with currencies other than that in which the bond is denominated. For example, a dollar-denominated Eurobond issued by an American corporation can be sold to French, German, Swiss, or Japanese investors. A *foreign bond*, on the other hand, is issued by a foreign borrower in a host country's capital market and denominated in the host currency. An example is a French-franc denominated bond issued in France by an English company.

12. A financial manager must understand the valuation process in order to judge the value of benefits received from shares, bonds, and other assets in view of their risk, return, and combined impact on share value.

13. Three key inputs to the valuation process are:
    a. *Cash flows* – the cash generated from ownership of the asset;
    b. *Timing* – the time period(s) in which cash flows are received; and
    c. *Required return* – the interest rate used to discount the future cash flows to a *PV*. The selection of the required return allows the level of risk to be adjusted; the higher the risk, the higher the required return (discount rate).

14. The valuation process applies to assets that provide an intermittent cash flow or even a single cash flow over any time period.

15. The value of any asset is the *PV* of future cash flows expected from the asset over the relevant time period. The three key inputs in the valuation process are cash flows, the required rate of return, and the timing of cash flows. The equation for value is:

$$V_0 = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots \frac{CF_n}{(1+r)^n}$$

where:

V_0 = value of the asset at time zero
CF_1 = cash flow expected at the end of year t
r = appropriate required return (discount rate)
n = relevant time period

16. The basic *bond valuation equation* for a bond that pays annual interest is:

$$V_0 = I \times \left[\sum_{t=1}^{n} \frac{1}{(1+r_d)^t} \right] + M \times \left[\frac{1}{(1+r_d)^n} \right]$$

where:

V_0 = value of a bond that pays annual interest
I = interest
n = years to maturity
M = dollar par value
r_d = required return on the bond

To find the value of bonds paying interest semiannually, the basic bond valuation equation is adjusted as follows to account for the more frequent payment of interest:

a.   The annual interest must be converted to semiannual interest by dividing by two.

b.   The number of years to maturity must be multiplied by two.

c.   The required return must be converted to a semiannual rate by dividing it by two.

17. A bond sells at a *discount* when the required return exceeds the coupon rate. A bond sells at a *premium* when the required return is less than the coupon rate. A bond sells at par value when the required return equals the coupon rate. The coupon rate is generally a fixed rate of interest, whereas the required return fluctuates with shifts in the cost of long-term funds due to economic conditions and/or risk of the issuing firm. The disparity between the required rate and the coupon rate will cause the bond to be sold at a discount or premium.

18. If the required return on a bond is constant until maturity and different from the coupon interest rate, the bond's value approaches its R1,000 par value as the time to maturity declines.

19. To protect against the impact of rising interest rates, a risk-averse investor would prefer bonds with short periods until maturity. The responsiveness of the bond's market value to interest rate fluctuations is an increasing function of the time to maturity.

20. The *yield-to-maturity* (*YTM*) on a bond is the rate investors earn if they buy the bond at a specific price and hold it until maturity. The YTM can be found precisely by using a hand-held financial calculator and using the time value functions. Enter the B_0 as the *PV*, and the I as the annual payment, and the n as the number of periods until maturity. Have the calculator solve for the interest rate. This interest value is the YTM. Many calculators are already programmed to solve for the internal rate of return (IRR). Using this feature will also obtain the YTM since the YTM and IRR are determined the same way. Spreadsheets include formula for computing the yield to maturity.

The trial-and-error approach to calculating the YTM requires finding the value of the bond at various rates to determine the rate causing the calculated bond value to equal its current value. The approximate approach for calculating YTM uses the following equation:

$$\text{Approximate Yield} = \frac{I + [(M - B_0)/n]}{(M + B_0)/2}$$

where:

I = annual interest
M = maturity value
B_0 = market value
n = periods to maturity

◼ Suggested answer to critical thinking question for Focus on Practice

What effect do you think the inflation-adjusted interest rate has on the cost of an ILB-bond in comparison with similar bonds with no allowance for inflation?

The cost of the ILB-bond when issued is the face value the bond has an inflation protection feature, the Treasury can issue the ILB-bond at slightly lower interest rates than comparable bonds.

◼ Suggested answer to critical thinking question for Focus on Ethics

What is the role of ratings agencies in facilitating financial intermediation? How will it affect the bond market if investors lose faith in ratings agencies?

Ratings agencies provide investors with a signal of the default risk of a security. This allows investors to more accurately assess the risks of purchasing securities like bonds. Without this information investors would be less inclined to purchase such instruments thus making it harder for firms needing to raise capital to do so. Ratings agencies therefore facilitate financial intermediation by providing investors with the necessary information to allow them to make informed decisions about lending money to firms that need it.

If investors lose faith in ratings agencies it will make it harder for investors to assess the default risk of securities. This will make them more cautious before purchasing bond instruments. As a result the demand for bonds will decrease and as a result bonds will have to offer higher yields to compensate investors for this greater uncertainty. As a result the cost of debt for firms issuing bonds will increase.
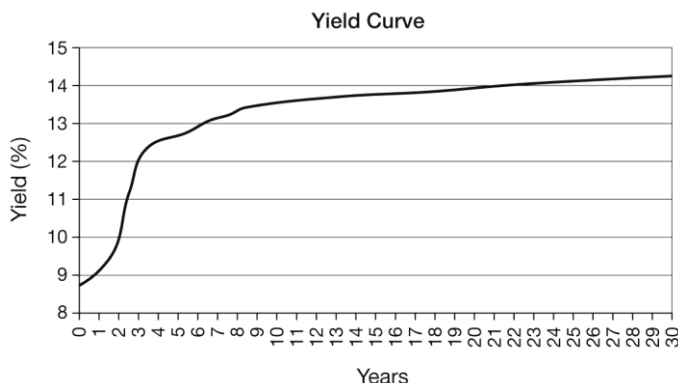
◼ Answers to Warm-up exercises

E6-1.   Finding the real rate of interest

Answer: $r^* = R_F - IP$

$0.8\% = 7.23\% - IP$

$IP = 7.23\% - 0.8\% = 6.53\%$

E6-2.   Yield curve[B1]

E6-3.   Calculating inflation expectation

Answer: The inflation expectation for a specific maturity is the difference between the yield and the real interest rate at that maturity.

Maturity	Yield	Real Rate of Interest	Inflation Expectation
3 months	8.41%	0.80%	7.61%
6 months	8.71	0.80	7.91
2 years	9.68	0.80	8.88
3 years	10.01	0.80	9.21
5 years	10.70	0.80	9.90
10 years	11.51	0.80	10.71
30 years	12.25	0.80	11.45

E6-4.   Real returns

Answer: A T-bill can experience a negative real return if its interest rate is less than the inflation rate as measured by the CPI. The real return would be zero if the T-bill rate was 6.3% exactly matching the CPI rate. To obtain a minimum 2% real return, the T-bill rate would have to be at least 8.3%.

E6-5.   Calculating risk premium

Answer: We calculate the risk premium of other securities by subtracting the risk-free rate, 4.51%, from each nominal interest rate.

Security	Nominal Interest Rate	Risk Premium
AAA	12.12%	12.12% – 114.51% = 0.61%
BBB	12.78	12.78% – 114.51% = 1.27%
B	14.82	14.82% – 114.51% = 3.31%

E6-6.   The basic valuation model

Answer: Find the *PV* of the cash flow stream for each asset by discounting the expected cash flows using the respective required return.

Asset 1: $PV = R500 \div 0.15 = R3,333.33$

Asset 2: $PV = \dfrac{R1,200}{1.10} + \dfrac{R1,500}{(1.10)^2} + \dfrac{R850}{(1.10)^3} = R2,969.20$

E6-7.   Calculating the *PV* of a bond when the required return exceeds the coupon rate

Answer: The *PV* of a bond is the *PV* of its future cash flows. In the case of the 5-year bond, the expected cash flows are R1,200 at the end of each year for 5 years, plus the face value of the bond that will be received at the maturity of the bond (end of year 5). You may use the bond valuation formula found in your text or you may use a financial calculator. The solution presented below is derived using a financial calculator. Set the calculator on 1 period/year.

PV of interest: $PMT = -1,200$

$I = 8\%/\text{year}$

$N = 5 \text{ periods}$

Solve for $PV = R4,791.25$

PV of the bond's face value: $FV = R20,000$

$N = 5 \text{ periods}$

$I = 8\%/\text{year}$

Solve for $PV = R13,611.66$

The *PV* of this bond is R4,791.25 + R13,611.66 = R18,402.91.

This answer is consistent with the knowledge that when interest rates rise, the values of previously issued bonds fall.

E6-8.   Bond valuations using required rates of return

Answer: a.   Student answers will vary but any required rate of return **above** the coupon rate will cause the bond to sell at a discount, while at a required return of 4.5% the bond will sell at par. Any required rate of return **below** the coupon rate will cause the bond to sell at a premium.

b.   Student answers will vary but should be consistent with their answers to part **a**.

■ Solutions to Problems

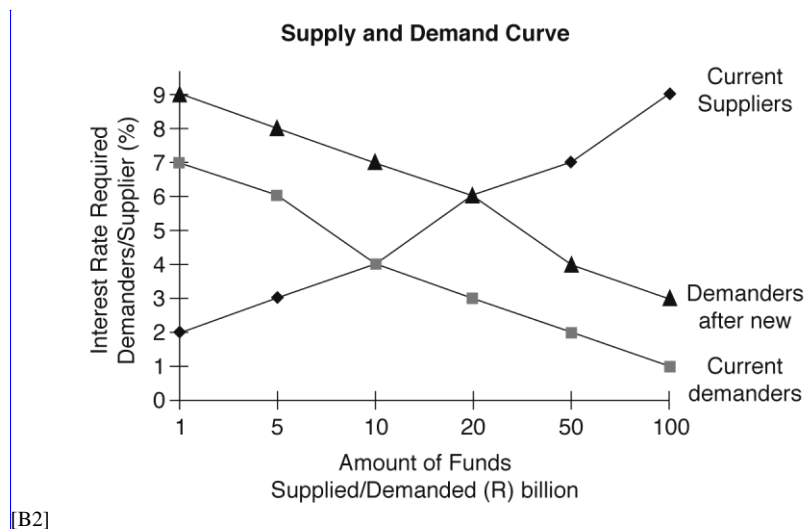P6-1.   LG 1: Interest rate fundamentals: The real rate of return

Basic

Real rate of return = 8.5% − 6.0% = 2.5%

P6-2.   LG 1: Real rate of interest

Intermediate

a.

[B2]

b.   The real rate of interest creates an equilibrium between the supply of savings and the demand for funds, which is shown on the graph as the intersection of lines for current suppliers and current demanders. $r = 4\%$

c.   See graph.

d.   A change in the tax law causes an upward shift in the demand curve, causing the equilibrium point between the supply curve and the demand curve (the real rate of interest) to rise from $r_0 = 4\%$ to $r_0 = 6\%$ (intersection of lines for current suppliers and demanders after new law).

P6-3.   LG 1: Personal finance: Real and nominal rates of interest

Intermediate

a.   4 shirts

b. $R300 + (R300 \times 0.09) = R327$

c. $R75 + (R75 \times 0.05) = R78.75$

d.   The number of polo shirts in one year = $R327 \div R78.75 = 4.1524$. He can buy 3.8% more shirts ($4.1524 \div 4 = 0.0381$).

e.   The real rate of return is $9\% - 5\% = 4\%$. The change in the number of shirts that can be purchased is determined by the real rate of return since the portion of the nominal return for expected inflation (5%) is available just to maintain the ability to purchase the same number of shirts.

P6-4.   LG 1: Yield curve
Intermediate

a.

Yield Curve

[B3]
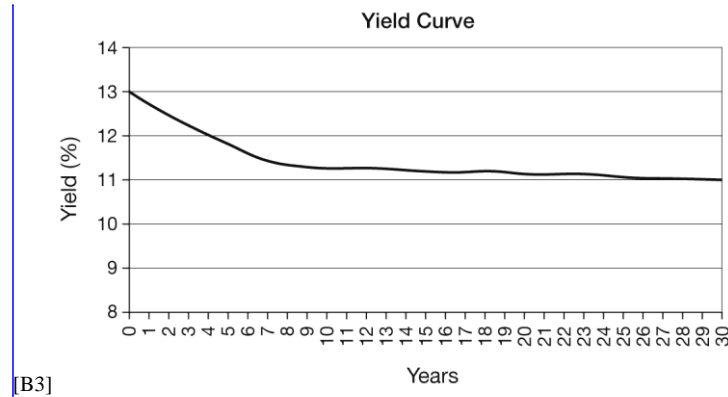
b.   The yield curve is slightly downward sloping, reflecting lower expected future rates of interest. The curve may reflect a general expectation for an economic recovery due to inflation coming under control and a stimulating impact on the economy from the lower rates. However, a slowing economy may diminish the perceived need for funds and the resulting interest rate being paid for cash. Obviously, the second scenario is not good for business and highlights the challenge of forecasting the future based on the term structure of interest rates.

P6-5.   LG 1: Nominal interest rates and yield curves
Challenge

a. $r_1 = r^* + IP + RP_1$

For Treasury issues, $RP = 0$

$r_F = r^* + IP$
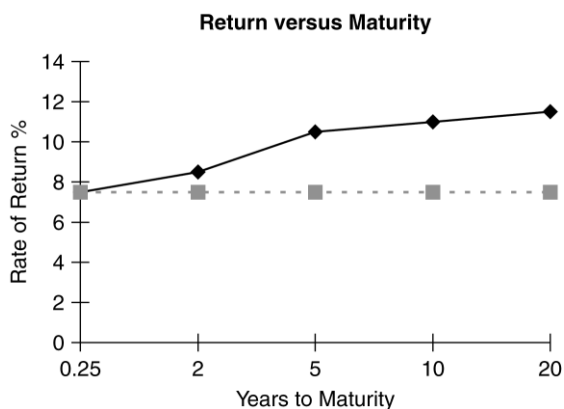
20 year bond: $R_F = 2.5 + 9\% = 11.5\%$
3 month bill: $R_F = 2.5 + 5\% = 7.5\%$
2 year note: $R_F = 2.5 + 6\% = 8.5\%$
5 year bond: $R_F = 2.5 + 8\% = 10.5\%$

b.   If the real rate of interest (r^*) drops to 2.0%, the nominal interest rate in each case would decrease by 0.5% point.

c.

Return versus Maturity

The yield curve for Treasury issues is upward sloping, reflecting the prevailing expectation of higher future inflation rates.

d. Followers of the liquidity preference theory would state that the upward sloping shape of the curve is due to the desire by lenders to lend short term and the desire by business to borrow long term. The dashed line in the part (**c**) graph shows what the curve would look like without the existence of liquidity preference, ignoring the other yield curve theories.

e. Market segmentation theorists would argue that the upward slope is due to the fact that under current economic conditions there is greater demand for long-term loans for items such as real estate than for short-term loans such as seasonal needs.

P6-6.   LG 1: Nominal and real rates and yield curves

Challenge

Real rate of interest (r^*):

$r_i = r^* + IP + RP$

$RP = 0$ for Treasury issues

$r^* = r_i - IP$

a.

Security	Nominal Rate (r_j)	−	IP	=	Real Rate of Interest (r^*)
A	12.6%	−	9.5%	=	3.1%
B	11.2%	−	8.2%	=	3.0%
C	13.0%	−	10.0%	=	3.0%
D	11.0%	−	8.1%	=	2.9%
E	11.4%	−	8.3%	=	3.1%

b. The real rate of interest decreased from January to March, remained stable from March through August, and finally increased in December. Forces that may be responsible for a change in the real rate of interest include changing economic conditions such as the international trade balance, a federal government budget deficit, or changes in tax legislation.

c.

Yield Curve of U.S. Treasury Securities
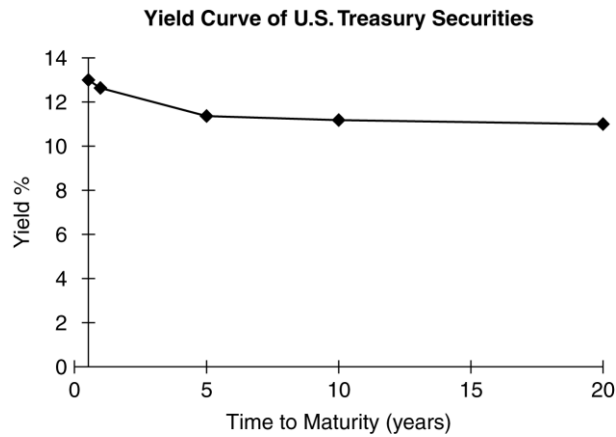
d. The yield curve is slightly downward sloping, reflecting lower expected future rates of interest. The curve may reflect a current, general expectation for an economic recovery due to inflation coming under control and a stimulating impact on the economy from the lower rates.

P6-7.   LG 1: Term structure of interest rates

Intermediate

a.

Yield Curve of High-Quality Corporate Bonds
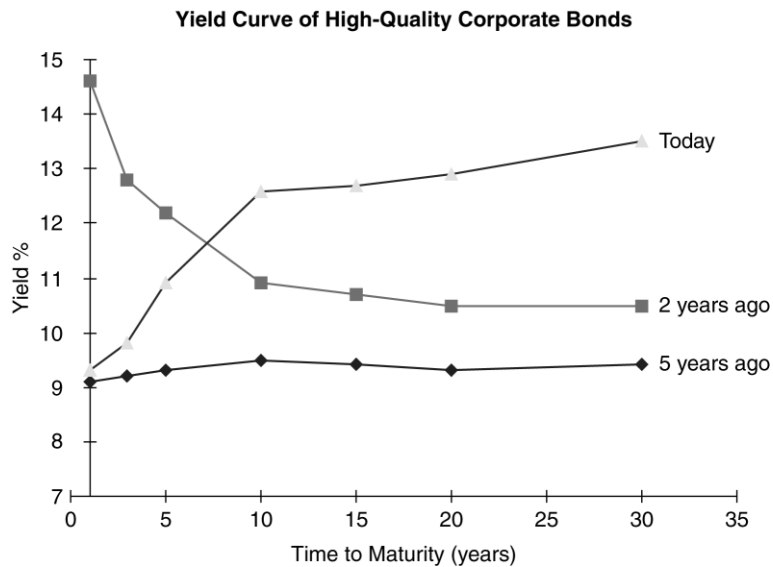
b. and c.

Five years ago, the yield curve was relatively flat, reflecting expectations of stable interest rates and stable inflation. Two years ago, the yield curve was downward sloping, reflecting lower expected interest rates due to a decline in the expected level of inflation. Today, the yield curve is upward sloping, reflecting higher expected inflation and higher future rates of interest.

P6-8.   LG 1: Risk-free rate and risk premiums
Basic

a.   Risk-free rate: $R_F = r^* + IP$

Security	r^*	+	IP	=	R_F
A	3%	+	6%	=	9%
B	3%	+	9%	=	12%
C	3%	+	8%	=	11%
D	3%	+	5%	=	8%
E	3%	+	11%	=	14%

b.   Since the expected inflation rates differ, it is probable that the maturity of each security differs.

c.   Nominal rate: $r = r^* + IP + RP$

Security	r^*	+	IP	+	RP	=	r
A	3%	+	6%	+	3%	=	12%
B	3%	+	9%	+	2%	=	14%
C	3%	+	8%	+	2%	=	13%
D	3%	+	5%	+	4%	=	12%
E	3%	+	11%	+	1%	=	15%

P6-9.   LG 1: Risk premiums
Intermediate

a. $R_{Ft} = r^* + IP_t$
Security A: $R_{F3} = 2\% + 9\% = 11\%$
Security B: $R_{F15} = 2\% + 7\% = 9\%$

b.   Risk premium:
RP = default risk + maturity risk + liquidity risk + other risk
Security A: $RP = 1\% + 0.5\% + 1\% + 0.5\% = 3\%$
Security B: $RP = 2\% + 1.5\% + 1\% + 1.5\% = 6\%$

c. $r_i = r^* + IP + RP$ or $r_1 = r_F +$ risk premium
Security A: $r_1 = 11\% + 3\% = 14\%$
Security B: $r_1 = 9\% + 6\% = 15\%$

Security A has a higher risk-free rate of return than Security B due to expectations of higher near-term inflation rates. The issue characteristics of Security A in comparison to Security B indicate that Security A is less risky.

P6-10. LG 2: Bond interest payments before and after taxes

Intermediate

a.   Yearly interest = [(R2,500,000/2500) x 0.11] = (R1,000 × 0.11) = R110.00

b.   Total interest expense = R110.00 per bond × 2,500 bonds = R275,000

c.   Total before tax interest   R275,000
     Interest expense tax savings (0.30 × R275,000)   82,500
     Net after-tax interest expense   R192,500

P6-11. LG 4: Bond prices and yields

Basic

a.   0.97708 × R1,000 = R977.08

b.   (0.05700 × R1,000) ÷ R977.08 = R57.000 ÷ R977.08 = 0.0583 = 5.83%

c.   The bond is selling at a discount to its R1,000 par value.

d.   The yield to maturity is higher than the current yield, because the former includes R22.92 in price appreciation between today and the May 15, 2017 bond maturity.

P6-12. LG 4: Personal finance: Valuation fundamentals

Basic

a.   Cash flows: CF_{1-5} R12,00
                          CF_5 R50,000

     Required return: 6%

b. $$V_0 = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \frac{CF_5}{(1+r)^5}$$

     $$V_0 = \frac{R12,000}{(1+0.06)^1} + \frac{R12,000}{(1+0.06)^2} + \frac{R12,000}{(1+0.06)^3} + \frac{R12,000}{(1+0.06)^4} + \frac{R62,000}{(1+0.06)^5}$$

     $V_0 = R87,911$

Using PVIF formula:

$V_0 = [(CF_1 \times PVIF_{6\%,1}) + (CF_2 \times PVIF_{6\%,2}) \cdots (CF_5 \times PVIF_{6\%,5})]$

$V_0 = [(R12,000 \times 0.943) + (R12,00 \times 0.890) + (R12,000 \times 0.840) + (R12,000 \times 0.792) + (R62,000 \times 0.747)]$

$V_0 = R11,316 + R10,680 + R10,080 + R9504 + R46,314$

$V_0 = R87,894$

The maximum price you should be willing to pay for the car is R87,894, since if you paid more than that amount, you would be receiving less than your required 6% return.

P6-13. LG 4: Valuation of assets

Basic

Asset	End of Year	Amount	PVIF or PVIFA$_{r\%,n}$	*PV* of Cash Flow
A	1	R 5000		
	2	R 5000	2.174	
	3	R 5000		R10,870.00
			Calculator solution:	R10,871.36
B	1–∞	R 300	1 ÷ 0.15	R2,000
C	1	0		
	2	0		
	3	0		
	4	0		
	5	R35,000	0.476	R16,660.0
			Calculator solution:	R16,663.96
D	1–5	R 1,500	3.605	R 5,407.50
	6	8,500	0.507	4,309.50
				R 9,717.00
			Calculator solution:	R 9,713.53
E	1	R 2,000	0.877	R 1,754.00
	2	3,000	0.769	2,307.00
	3	5,000	0.675	3,375.00
	4	7,000	0.592	4,144.00
	5	4,000	0.519	2,076.00
	6	1,000	0.456	456.00
				R14,112.00
			Calculator solution:	R14,115.27

P6-14. LG 4: Personal finance: Asset valuation and risk

Intermediate

a.

		10% Low Risk		15% Average Risk		22% High Risk	
		PVIFA	*PV* of *CF*	PVIFA	*PV* of *CF*	PVIFA	*PV* of *CF*
CF_{1-4}	R3,000	3.170	R 9,510	2.855	R 8,565	2.494	R 7,482
CF_5	15,000	0.621	9,315	0.497	7,455	0.370	5,550
PV of *CF*:			R18,825		R16,020		R13,032
Calculator solutions:			R18,823.42		R16,022.59		R13,030.9 1

b. The maximum price Laura should pay is R13,032. Unable to assess the risk, Laura would use the most conservative price, therefore assuming the highest risk.

c. By increasing the risk of receiving cash flow from an asset, the required rate of return increases, which reduces the value of the asset.

P6-15. LG 5: Basic bond valuation

Intermediate

a. $B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

$B_0 = 120 \times (\text{PVIFA}_{10\%,16}) + M \times (\text{PVIF}_{10\%,16})$

$B_0 = R120 \times (7.824) + R1{,}000 \times (0.218)$

$B_0 = R938.88 + R218$

$B_0 = R1{,}156.88$

Calculator solution: R1,156.47

b. Since Complex Systems' bonds were issued, there may have been a shift in the supply-demand relationship for money or a change in the risk of the firm.

c. $B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

$B_0 = 120 \times (\text{PVIFA}_{12\%,16}) + M \times (\text{PVIF}_{12\%,16})$

$B_0 = R120 \times (6.974) + R1{,}000 \times (0.163)$

$B_0 = R836.88 + R163$

$B_0 = R999.88$

Calculator solution: R1,000

When the required return is equal to the coupon rate, the bond value is equal to the par value. In contrast to **a** above, if the required return is less than the coupon rate, the bond will sell at a premium (its value will be greater than par).

P6-16. LG 5: Bond valuation–annual interest

Basic

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

Bond	Table Values	Calculator Solution
A	$B_0 = R140 \times (7.469) + R1{,}000 \times (0.104) = R1{,}149.66$	R1,149.39
B	$B_0 = R80 \times (8.851) + R1{,}000 \times (0.292) = R1{,}000.00$	R1,000.00
C	$B_0 = R10 \times (4.799) + R100 \times (0.376) = R85.59$	R 85.60
D	$B_0 = R80 \times (4.910) + R500 \times (0.116) = R\ 450.80$	R 450.90
E	$B_0 = R120 \times (6.145) + R1{,}000 \times (0.386) = R1{,}123.40$	R1,122.89

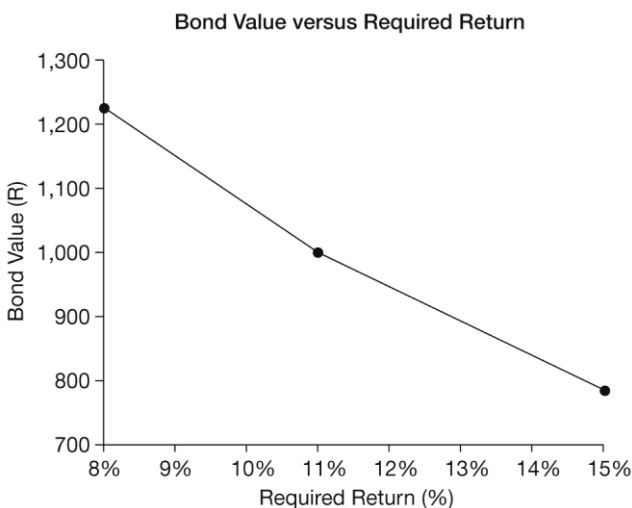P6-17.   LG 5: Bond value and changing required returns

Intermediate

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

a.

Bond	Table Values	Calculator Solution
(1)	$B_0 = \text{R}110 \times (6.492) + \text{R}1{,}000 \times (0.286) = \text{R}1{,}000.00$	R1,000.00
(2)	$B_0 = \text{R}110 \times (5.421) + \text{R}1{,}000 \times (0.187) = \text{R}\ 783.31$	R 783.18
(3)	$B_0 = \text{R}110 \times (7.536) + \text{R}1{,}000 \times (0.397) = \text{R}1{,}225.96$	R1,226.08

b.

Bond Value versus Required Return

c.   When the required return is less than the coupon rate, the market value is greater than the par value and the bond sells at a premium. When the required return is greater than the coupon rate, the market value is less than the par value; the bond therefore sells at a discount.

d.   The required return on the bond is likely to differ from the coupon interest rate because either (1) economic conditions have changed, causing a shift in the basic cost of long-term funds, or (2) the firm's risk has changed.

P6-18.   LG 5: Bond value and time–constant required returns

Intermediate

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

a.

Bond	Table Values	Calculator Solution
(1)	$B_0 = \text{R}120 \times (6.142) + \text{R}1{,}000 \times (0.140) = \text{R}877.04$	R877.16
(2)	$B_0 = \text{R}120 \times (5.660) + \text{R}1{,}000 \times (0.208) = \text{R}887.20$	R886.79
(3)	$B_0 = \text{R}120 \times (4.946) + \text{R}1{,}000 \times (0.308) = \text{R}901.52$	R901.07
(4)	$B_0 = \text{R}120 \times (3.889) + \text{R}1{,}000 \times (0.456) = \text{R}922.68$	R922.23

(5)	$B_0 = R120 \times (2.322) + R1{,}000 \times (0.675) = R953.64$	R953.57
(6)	$B_0 = R120 \times (0.877) + R1{,}000 \times (0.877) = R982.24$	R982.46

b.

Bond Value versus Years to Maturity
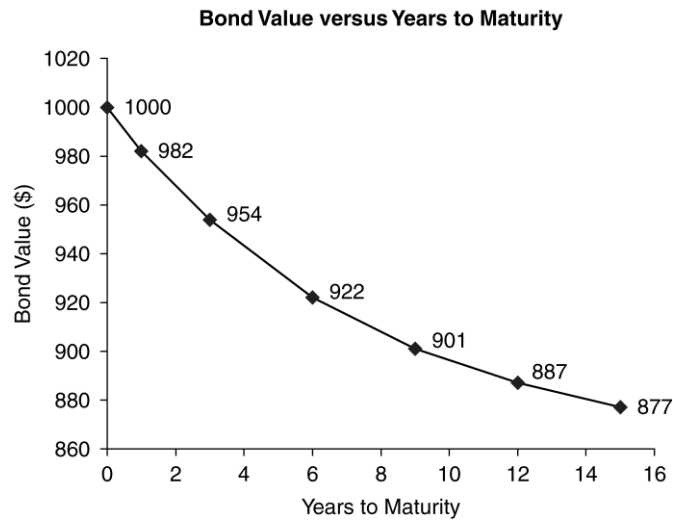
c.   The bond value approaches the par value.

P6-19. LG 5: Personal finance: Bond value and time–changing required returns

Challenge

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

a.

Bond	Table Values	Calculator Solution
(1)	$B_0 = R110 \times (3.993) + R1{,}000 \times (0.681) = R1{,}120.23$	R1,119.78
(2)	$B_0 = R110 \times (3.696) + R1{,}000 \times (0.593) = R1{,}000.00$	R1,000.00
(3)	$B_0 = R110 \times (3.433) + R1{,}000 \times (0.519) = R\ \ 896.63$	R 897.01

b.

Bond	Table Values	Calculator Solution
(1)	$B_0 = R110 \times (8.560) + R1{,}000 \times (0.315) = R1{,}256.60$	R1,256.78
(2)	$B_0 = R110 \times (7.191) + R1{,}000 \times (0.209) = R1{,}000.00$	R1,000.00
(3)	$B_0 = R110 \times (6.142) + R1{,}000 \times (0.140) = R\ \ 815.62$	R 815.73

c.

	Value	
Required Return	**Bond A**	**Bond B**
8%	R1,120.23	R1,256.60
11%	1,000.00	1,000.00
14%	896.63	815.62

The greater the length of time to maturity, the more responsive the market value of the bond to changing required returns, and vice versa.

d. If Lynn wants to minimise interest rate risk in the future, she would choose Bond A with the shorter maturity. Any change in interest rates will impact the market value of Bond A less than if she held Bond B.

P6-20. LG 6: Yield to maturity

Basic

Bond A is selling at a discount to par.
Bond B is selling at par value.
Bond C is selling at a premium to par.
Bond D is selling at a discount to par.
Bond E is selling at a premium to par.

P6-21. LG 6: Yield to maturity

Intermediate

a. Using a financial calculator the YTM is 12.685%. The correctness of this number is proven by putting the YTM in the bond valuation model. This proof is as follows:

$B_0 = 120 \times (PVIFA_{12.685\%,15}) + 1,000 \times (PVIF_{12.685\%,15})$
$B_0 = R120 \times (6.569) + R1,000 \times (0.167)$
$B_0 = R788.28 + 167$
$B_0 = R955.28$

Since B_0 is R955.28 and the market value of the bond is R955, the YTM is equal to the rate derived on the financial calculator.

b. The market value of the bond approaches its par value as the time to maturity declines. The yield to maturity approaches the coupon interest rate as the time to maturity declines.

P6-22. LG 6: Yield to maturity

Intermediate

a.

Bond	Approximate YTM	Trial-and-Error YTM Approach	Error (%)	Calculator Solution
A	$= \dfrac{R90 + [(R1,000 - R820) \div 8]}{[(R1,000 + R820) \div 2]}$			
	= 12.36%	12.71%	−0.35	12.71%
B	= 12.00%	12.00%	0.00	12.00%
C	$= \dfrac{R60 + [(R500 - R560) \div 12]}{[(R500 + R560) \div 2]}$			
	= 10.38%	10.22%	+0.15	10.22%
D	$= \dfrac{R150 + [(R1,000 - R1,120) \div 10]}{[(R1,000 + R1,120 \div 2]}$			
	= 13.02%	12.81%	+0.21	12.81%

E $\quad = \dfrac{\text{R}50 + [(\text{R}1,000 - \text{R}900) \div 3]}{[(\text{R}1,000 + \text{R}900) \div 2]}$

$\quad = 8.77\%$ $\qquad\qquad\qquad$ 8.94% $\qquad\qquad$ −0.017 $\qquad\quad$ 8.95%

b.   The market value of the bond approaches its par value as the time to maturity declines. The yield-to-maturity approaches the coupon interest rate as the time to maturity declines. Case B highlights the fact that if the current price equals the par value, the coupon interest rate equals the yield to maturity (regardless of the number of years to maturity).

P6-23. LG 2, 5, 6: Personal finance: Bond valuation and yield to maturity

Challenge

a. $B_A = R60(PVIFA_{12\%,5}) + R1{,}000(PVIF_{12\%,5})$
$B_A = R60(3.605) + R1{,}000(0.567)$
$B_A = R216.30 + 567$
$B_A = R783.30$
Calculator solution: R783.71

$B_B = R140(PVIFA_{12\%,5}) + R1{,}000(PVIF_{12\%,5})$
$B_B = R140(3.605) + R1{,}000(0.567)$
$B_B = R504.70 + 567$
$B_B = R1{,}071.70$
Calculator solution: R1,072.10

b.   Number of bonds $= \dfrac{R20{,}000}{R783.30} = 25.533$ of Bond A

Number of bonds $= \dfrac{R20{,}000}{R1{,}071.70} = 18.662$ of Bond B [B4]

c.   Interest income of A = 25.533 bonds × R60 = R1,531.98
Interest income of B = 18.66194 bonds × R140 = R2,612.67

d.   At the end of the 5 years both bonds mature and will sell for par of R1,000.
$FV_A = R60(FVIFA_{10\%,5}) + R1{,}000$
$FV_A = R60(6.105) + R1{,}000$
$FV_A = R366.30 + R1{,}000 = R1{,}366.30$
Calculator solution: R1,366.31

$FV_B = R140(FVIFA_{10\%,5}) + R1{,}000$
$FV_B = R140(6.105) + R1{,}000$
$FV_B = R854.70 + R1{,}000 = R1{,}854.70$
Calculator solution: R1,854.71

e.   The difference is due to the differences in interest payments received each year. The principal payments at maturity will be the same for both bonds. Using the calculator, the yield to maturity of Bond A is 11.77% and the yield to maturity of Bond B is 11.59% with the 10% reinvestment rate for the interest payments. Mark would be better off investing in Bond A. The reasoning behind this result is that for both bonds the principal is priced to yield the YTM of 12%. However, Bond B is more dependent upon the reinvestment of the large coupon payment at the YTM to earn the 12% than is the lower coupon payment of Bond A.

P6-24.   LG 6: Bond valuation–semiannual interest

Intermediate

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

$B_0 = \text{R}50 \times (\text{PVIFA}_{7\%,12}) + M \times (\text{PVIF}_{7\%,12})$

$B_0 = \text{R}50 \times (7.943) + \text{R}1,000 \times (0.444)$

$B_0 = \text{R}397.15 + \text{R}444$

$B_0 = \text{R}841.15$

Calculator solution: R841.15

P6-25.   LG 6: Bond valuation–semiannual interest

Intermediate

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

Bond	Table Values	Calculator Solution
A	$B_0 = \text{R}50 \times (15.247) + \text{R}1,000 \times (0.390) = \text{R}1,152.35$	R1,152.47
B	$B_0 = \text{R}60 \times (15.046) + \text{R}1,000 \times (0.097) = \text{R}1,000.00$	R1,000.00
C	$B_0 = \text{R}30 \times (7.024) + \text{R}500 \times (0.508) = \text{R}\ \ 464.72$	R 464.88
D	$B_0 = \text{R}70 \times (12.462) + \text{R}1,000 \times (0.377) = \text{R}1,249.34$	R1,249.24
E	$B_0 = \text{R}3 \times (5.971) + \text{R}100 \times (0.582) = \text{R}\ \ \ 76.11$	R 76.11

P6-26.   LG 6: Bond valuation–quarterly interest

Challenge

$B_0 = I \times (\text{PVIFA}_{rd\%,n}) + M \times (\text{PVIF}_{rd\%,n})$

$B_0 = \text{R}125 \times (\text{PVIFA}_{3\%,40}) + \text{R}5,000 \times (\text{PVIF}_{3\%,40})$

$B_0 = \text{R}125 \times (23.115) + \text{R}5,000 \times (0.307)$

$B_0 = \text{R}2,889.38 + \text{R}1,535$

$B_0 = \text{R}4,424.38$

Calculator solution: R4,422.13

P6-27.   Ethics problem

Intermediate

Student answers will vary. Some students may argue that such a policy decreases the reliability of the rating agency's bond ratings since the rating is not purely based on the quantitative and non-quantitative factors that should be considered. Other students may argue that, like a loss leader, ratings are a way to generate additional business for the rating firm. In light of recent events the potential conflict of interest that arises from ratings agencies using their ratings to generate revenue from the firms they are rating is likely to be the subject of much debate.

■ **Case**

Evaluating Annie Hadebe's proposed investment in bonds of Ateljeer Industries

This case demonstrates how a risky investment can affect a firm's value. First, students must calculate the current value of Ateljeer's bonds, rework the calculations assuming that the firm makes the risky investment, and then draw some conclusions about the value of the firm in this situation. In addition to gaining experience in valuation of bonds, students will see the relationship between risk and valuation.

1.   Annie should convert the bonds. The value of the shares if the bond is converted is:

50 shares × R30 per share = R1,500

while if the bond was allowed to be called in the value would be on R1,080

2.   Current value of bond under different required returns – annual interest

a. $B_0 = I \times (\text{PVIFA}_{6\%,25 \text{ yrs.}}) + M \times (\text{PVIF}_{6\%,25 \text{ yrs.}})$
    $B_0 = R80 \times (12.783) + R1,000 \times (0.233)$
    $B_0 = R1,022.64 + R233$
    $B_0 = R1,255.64$
    Calculator solution: R1,255.67
    The bond would be at a premium.

b. $B_0 = I \times (\text{PVIFA}_{8\%,25 \text{ yrs.}}) + M \times (\text{PVIF}_{8\%,25 \text{ yrs.}})$
    $B_0 = R80 \times (10.674) + R1,000 \times (0.146)$
    $B_0 = R853.92 + R146$
    $B_0 = R999.92$
    Calculator solution: R1,000.00
    The bond would be at par value.

c. $B_0 = I \times (\text{PVIFA}_{10\%,25 \text{ yrs.}}) + M \times (\text{PVIF}_{10\%,25 \text{ yrs.}})$
    $B_0 = R80 \times (9.077) + R1,000 \times (0.092)$
    $B_0 = R726.16 + R92$
    $B_0 = R818.16$
    Calculator solution: R818.46
    The bond would be at a discount.

3.   Current value of bond under different required returns – semiannual interest

a. $B_0 = I \times (\text{PVIFA}_{3\%,50 \text{ yrs.}}) + M \times (\text{PVIF}_{3\%,50 \text{ yrs.}})$
    $B_0 = R40 \times (25.730) + R1,000 \times (0.228)$
    $B_0 = R1,029.20 + R228$
    $B_0 = R1,257.20$
    Calculator solution: R1,257.30
    The bond would be at a premium.

b. $B_0 = I \times (\text{PVIFA}_{4\%, 50 \text{ yrs.}}) + M \times (\text{PVI}_{4\%, 50 \text{ yrs.}})$
    $B_0 = \text{R}40 \times (21.482) + \text{R}1{,}000 \times (0.141)$
    $B_0 = \text{R}859.28 + \text{R}146$
    $B_0 = \text{R}1005.28$
    Calculator solution: R1,000.00
    The bond would be at par value.

c. $B_0 = I \times (\text{PVIFA}_{5\%, 50 \text{ yrs.}}) + M \times (\text{PVIF}_{5\%, 50 \text{ yrs.}})$
    $B_0 = \text{R}40 \times (18.256) + \text{R}1{,}000 \times (0.087)$
    $B_0 = \text{R}730.24 + \text{R}87$
    $B_0 = \text{R}817.24$
    Calculator solution: R817.44
    The bond would be at a discount.

Under all three required returns for both annual and semiannual interest payments the bonds are consistent in their direction of pricing. When the required return is above (below) the coupon the bond sells at a discount (premium). When the required return and coupon are equal the bond sells at par. When the change is made from annual to semiannual payments the value of the premium and par value bonds increase while the value of the discount bond decreases. This difference is due to the higher effective return associated with compounding frequency more often than annual.

4.   If expected inflation increases by 1% the required return will increase from 8% to 9%, and the bond price would drop to R901.84. This amount is the maximum Annie should pay for the bond.

   $B_0 = I \times (\text{PVIFA}_{9\%, 25 \text{ yrs.}}) + M \times (\text{PVIF}_{9\%, 25 \text{ yrs.}})$
   $B_0 = \text{R}80 \times (9.823) + \text{R}1{,}000 \times (0.116)$
   $B_0 = \text{R}785.84 + \text{R}116$
   $B_0 = \text{R}901.84$
   Calculator solution: R901.77

5.   The value of the bond would decline to R925.00 due to the higher required return and the inverse relationship between bond yields and bond values.

   $B_0 = I \times (\text{PVIFA}_{8.75\%, 25 \text{ yrs.}}) + M \times (\text{PVIF}_{8.75\%, 25 \text{ yrs.}})$
   $B_0 = \text{R}80 \times (10.025) + \text{R}1{,}000 \times (0.123)$
   $B_0 = \text{R}802.00 + \text{R}123$
   $B_0 = \text{R}925.00$
   Calculator solution: R924.81

6.   The bond would increase in value and a gain of R110.88 would be earned by Annie.

   Bond value at 7% and 22 years to maturity.
   $B_0 = I \times (\text{PVIFA}_{7\%, 22 \text{ yrs.}}) + M \times (\text{PVIF}_{7\%, 22 \text{ yrs.}})$
   $B_0 = \text{R}80 \times (11.061) + \text{R}1{,}000 \times (0.226)$
   $B_0 = \text{R}884.88 + \text{R}226$
   $B_0 = \text{R}1{,}110.88$
   Calculator solution: R1,110.61

7.   The bond would increase in value and a gain of R90.64 would be earned by Annie.

Bond value at 7% and 15 years to maturity.

$B_0 = I \times (\text{PVIFA}_{7\%,15 \text{ yrs.}}) + M \times (\text{PVIF}_{7\%,15 \text{ yrs.}})$

$B_0 = \text{R}80 \times (9.108) + \text{R}1{,}000 \times (0.362)$

$B_0 = \text{R}728.64 + \text{R}362$

$B_0 = \text{R}1{,}090.64$

Calculator solution: R1,091.08

The bond is more sensitive to interest rate changes when the time to maturity is longer (22 years) than when the time to maturity is shorter (15 years). Maturity risk decreases as the bond gets closer to maturity.

8.   Ateljeer Industries provides a yield of 8% (R80), and is priced at R983.80 (0.98380 × 1000). Hence, the current yield is 80/983.80 = .0813 or about 8.13% of par. Using the calculator the YTM on this bond assuming annual interest payments of R80, 25 years to maturity, and a current price of R983.80 would be 8.15%.

9.   Annie should probably not invest in the Ateljeer bond. There are several reasons for this conclusion.
   a.   The term to maturity is long and thus the maturity risk is high.
   b.   An increase in interest rates is likely due to the potential downgrading of the bond thus driving the price down.
   c.   An increase in interest rates is likely due to the possibility of higher inflation thus driving the price down.
   d.   The price of R983.75 is well above her minimum price of R901.84 assuming an increase in interest rates of 1%.

■   Spreadsheet Exercise

The answer to Chapter 6's CSM Corporation spreadsheet problem is located in the Instructor's Resource Center at *www.prenhall.com/irc*.

■   Group exercises

This chapter is concerned with credit ratings. Each group is asked to use current information from their shadow firm to flesh out the details for their fictitious firm. The first lesson students will learn is the lack of transparency in the bond market, particularly when compared to the stock market. Updated information is not as easily compared across multiple sites and details are often sketchy.

The steps for the assignment are very straight forward. Each group is asked to retrieve the interest rate of a recent debt issuance. For many firms there will be multiple offerings; any recent filing will suffice. This information on rates is then combined with the credit rating of the offering. Students should realise the same firm can be given different ratings on different offerings according to each offering's covenants. Using the current yield on a comparable Treasury, the risk premium can then be calculated.

The final step for the group is to address a potential capital investment. The interest rate will be derived from the information of the shadow firm; however the details of the project are entirely up to the discretion of the group. Students should be encouraged to get creative as this is the firm they will be living with for another two months.

■ Integrative Case 6: Organic Solutions

Integrative Case 6, Organic Solutions, asks students to evaluate a proposed acquisition by means of either a cash transaction or a share swap. The effects on the short- and long- term earnings per share (EPS) should be calculated and other proposals to achieve the merger discussed. The students must also consider the qualitative implications of acquiring a non-U.S.-based company.

1.   Price for cash acquisition of GTI:

Year	Incremental cash flow	$PVIF_{16, n\%}$	Present value of GTI
1	R18,750,000	0.862	R 16,162,500
2	18,750,000	0.743	13,931,250
3	20,500,000	0.641	13,140,500
4	21,750,000	0.552	12,006,000
5	24,000,000	0.476	11,424,000
6–30	25,000,000	(6.177 – 3.274)	72,575,000
		Total	R139,239,250
		Calculator solution:	R139,243,245

The maximum price Organic Solutions (OS) should offer GTI for a cash acquisition is R139,239,250. Net of the R8,400,000 liabilities, GTI's owners would receive R130,839,250.

2. a. *Straight bonds* – Financing such a large portion of the acquisition with straight bonds will dramatically increase the financial risk of the firm. The management of OS must be very comfortable that the combined firm is able to generate adequate cash to service this debt. The coupon rate on these bonds could also be quite high. The potential benefit to the OS owners is the magnified return on equity that could result from the leverage.

   b. *Convertible bonds* – Initially convertibles will provide much of the same concern as straight bonds since financial leverage will increase. There are two benefits to convertible bonds not available with straight bonds. First is that the coupon rate will be lower. Investors will value the conversion feature and will be willing to pay more, thus reducing the cost, for the convertible bond. The second advantage is that the leverage will decrease as conversion occurs, assuming the benefits of the acquisition ultimately proves favourable, and the value of the firm increases by the merger. The drawback is the potential dissolution of ownership that will occur if and when the bonds are converted.

   c. *Bonds with share purchase warrants attached* – The benefits and disadvantages of this security mix are similar to those of a convertible bond. However, there is one major difference. The attached warrants may eventually be used to supply the firm with additional equity capital. This inflow of capital will lower the financial risk of the firm and generate additional funds. There will still be the dissolution of ownership potential.

3. a. Ratio of exchange: R30 ÷ R50 = 0.60

   OS must exchange 0.60 of its shares for each of GTI's shares to acquire the firm in a share swap.