Chapter 1: Time Value of Money

Study Unit 1: Time Value of Money Concepts

Basic Concepts

Cash Flows

A cash flow has 2 components:

1. The receipt or payment of money: This differs from the accounting concepts of "income" and "expenses" – these accounting concepts arise from the accrual basis of accounting, in which accounting entries are detached from the associated cash flows and recognized in accounting periods that may differ from those in which the cash flows occurs.

2. Defined point in time at which it occurs: It is the separation in time from a reference point that causes the cash flow to be affected by interest and makes the application of time value of money essential.

   Time lines

   One of the most important tools in time value analysis is the time line, which is used by analysts to help visualize what is happening in a particular problem and then to help set up the problem for solution.

   Example:

   Time: 0   1   2   3   4   5
            |-----|-----|-----|-----|-----|
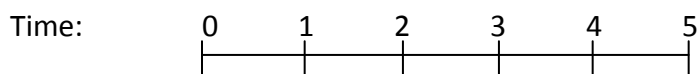
   Time 0 is today; Time 1 is one period from today, or the end of period 1; Time 2 is two periods from today, or the end of period 2; and so on.

   Cash flows are placed directly below the tick marks.
   Interest rates are shown directly above the line.
   Unknown cash flows, which you are trying to find in the analysis, are indicated by question marks.

Time: 0   5% 1   2   3

Cash Flows: -100   ?

Above: Here the interest rate for each of the 3 periods is 5%, a single amount (or lump sum) cash outflow is made at Time 0, and the Time 3 value is an unknown inflow. Since the initial $100 is an outflow (investment), it has a minus sign. The end value implies a plus sign. No cash flows occur at Times 1 and 2.
Do not show $ on time-lines.

Now consider a different situation where a $100 cash outflow is made today and we will receive an unknown amount at the end of Time 2.

0   5%   1   10%   2

-100   ?

Above: Here the interest rate is 5% during the first period, but rises to 10% during the second period.
If the interest rate is constant in all periods, we only show it in the first period, but if it changes, we show all the relevant rates on the time line.

Single cash flows and annuities

Cash flows forming the basis of time value of money calculations are either single cash flows or annuities.
A single cash flow is a non-repetitive cash inflow or outflow.
An annuity is a series of repetitive cash flows which occurs at fixed intervals for a specified number of periods.
      Ordinary annuity: payments occur at the end of each period
      Annuity due: payments occur at the beginning of each period

Compounding

Refers to the calculation of interest on a principal amount and the addition of that interest to the principal for investment in the subsequent period. The investment thus becomes bigger, or grows, in each subsequent period and interest earned, while calculated at a constant rate, is larger in each succeeding period as a result of the earlier compounding.

Discounting

The reverse of compounding.
It is the process by which a future value resulting from the compounding of interest is used to derive the value of the original investment

Future Value

The amount to which an investment will grow after one or more periods, at a particular compounding rate.
In calculations, future values is abbreviated as FV or S.

Present Value

The current value of future cash flows, determined by application of the discount rate.
In calculations, the present value is abbreviated as PV or P.
The present value of a cash flow due n years in the future is the amount which, if it were on hand today, would grow to equal the future amount.

Perpetuities

A periodic cash flow that has no terminal date, i.e. it will be received or paid periodically, at some interval, forever.

$$\text{PV (Perpetuity)} = \frac{PMT}{i}$$

Calculation of the time value of money

Time value of money calculations, also referred to as "discounted cash flow" or "DCF" analysis, require the application of a compounding or discount rate to one or more cash flows in order to determine a future value or present value of the cash flow(s).

The calculation may be performed by:
1. Use of pre-calculated tables
   The tables contain pre-calculated 'factors' for useful combinations of compounding/discount rate and time periods.

   Four Basic Tables are included - - - - - - - - - - - - - - - - > **Pages 40 – 43 of Study Guide**
   Table A: Present Value of R1 after n years
   Table B: Present Value of R1 per annum received for n years
   Table C: Future Value of R1 after n years
   Table D: Future Value of R1 per annum received for n years

   Perpetuities are not tabled.

   The required future value or present value can be calculated by multiplying the Rand value of the cash flow(s) that will be received/paid by the applicable time value factor.

2. Use of mathematical formulae

Where tables are not available, time value factors can be calculated by means of the following formulae:

Symbol	Description	Formula	Initial Cash Flow	
			Beginning of period	End of period
$(1+i)^n$	Future Value of R1 (Table C)	$(1+i)^n$	X	
v^n	Present Value of R1 (Table A)	$\dfrac{1}{(1+i)^n}$		X
	Future Value of R1 per period (Table D)	$\dfrac{(1+i)^n - 1}{i}$		X
	Present Value of R1 per period (Table B)	$\dfrac{1 - \dfrac{1}{(1+i)^n}}{i}$		X
	Future Value of R1 per period paid in advance	$\left[\dfrac{(1+i)^{n+1} - 1}{i}\right] - 1$	X	
	Present Value of R1 per period paid in advance	$1 + \left[1 - \dfrac{\dfrac{1}{(1+i)^{n-1}}}{i}\right]$	X	

In the formulae:

      n = number of periods

      i = effective interest rate, expressed as a decimal. i.e. 10% is 0,10

3. Use of a pocket calculator

A normal pocket calculator can be used to calculate time value factors as follows:

To Calculate	Required Calculators Steps
Future Value of R1	Key in (1+i) x 1 and press "=" . . . n times
Present Value of R1	Key in 1 / (1+i) and press "=" . . . n times
Future Value of R1 per period	Key in as for future value of R1, then (total – 1) / 1
Present Value of R1 per period	Key in as for present value of R1, then (total – 1) / 1 and thereafter change the sign.

4. Use of a Financial Calculator

A financial calculator may ONLY be used where the question does not stipulate that another approach should be used

5. Other useful relationships

1. Relationship between Tables A and B

$$a_n = \sum_{1}^{n} v^n$$

The above statement means that the sum of individual factors in Table A are equal to the factor stated in Table B.

E.g. According to Table B, the factor of R1 per annum for 4 years at 10% is 3,170.

According to Table A, the factors of R1 after n years at 10 are as follows:

Year 1	0,909
Year 2	0,826
Year 3	0,751
Year 4	0,683
Total	3,710

2. Relationship between Tables C and D

The factor in Table D, for an investment made at the <u>end</u> of the period, can be converted to the factor relating to an investment made at the <u>beginning</u> of the period as follows:

Factor Table D at 10% for 3 years	3,3100
Factor for investment made at the beginning of period (3,310 x 1.10)	3,6410

The factor relating to an investment made at the beginning of the period is equal to the sum of the factors in Table C at 10%:

Year 1	1,1000
Year 2	1,2100
Year 3	1,3310
Total	3,6410

3. Conversion of factor to another year

Where a factor is given in Table B for a certain period, eg years 1-3 at 15% as 2,283, this factor can be converted to the factor relating to years 2-4, by multiplying it by the factor for the first year (namely, 0,870):

Years 1 to 3		= 2,283
Years 2 to 4	(2,283 x 0,870)	= 1,986

Table A:	Year 1	0,756
	Year 2	0,658
	Year 3	0,572
		1,986

Systematic approach to solving problems

The first decision that is required is the method to be applied.

The method will be stated in the questions.

1. Which time value factor must be used?

    Must the future or present value be calculated/
    - If future value, use future value factor
    - If present value, use present value factor

    Must an annuity or a single amount be calculated?

    - If annuity, use a R1 per period factor
    - If single amount, use a R1 factor

2. Which equation can be used?

Future Value (FV)	= present single amount x future value of R1 factor
Present Value (PV)	= future single amount x present value of R1 factor
Future Value (FV)	= annuity x future value of R1 pp factor
Present Value (PV)	= annuity x present value of R1 pp factor

    $$\text{Annuity} = \frac{Future\ Value}{Future\ Value\ of\ R1\ pp\ factor}$$

    $$\text{Annuity} = \frac{Present\ Value}{Present\ Value\ of\ R1\ pp\ factor}$$

3. How will problems be sold?

    Decide on the following:

    - What is required?
    - What is given?
    - What equation must be used?

    Place the required on the left of the equation and the given information on the right of the equation which is to be used, and solve the equation.

4. Reasonability Check

Future Value of R1 factor	always more than 1
Present Value of R1 factor	always less than 1
Future Value of R1 pp factor	always more than number of periods
Present Value of R1 pp factor	always less than number of periods

Study Unit 2: Types of Interest Used in the Time Value of Money

Introduction

1. **Nominal Interest Rate** i_{nom}
   - Nominal or quoted rate
   - This is the rate quoted by banks, brokers, and other financial institutions
   - To be meaningful, the quoted rate must also include the number of compounding periods per year.
   - On loans to consumers is also called the Annual Percentage Rate (APR)
   - Never shown on a time line and never used as an input on a financial calculator, unless compounding occurs only once a year.

2. **Periodic Interest Rate** i_{PER}
   - Rate charged by a lender or paid by a borrower each period
   - Can be per year, six-month period, quarter, month, day, or any other time interval
   - We find the periodic rate as follows:

   $$i_{PER} = \frac{i_{nom}}{m}$$

   This implies that:

   $$i_{nom} = (\text{periodic rate}) (m)$$

   Here m = number of compounding periods per year

   - If interest is only added once a year **(m=1)** then **periodic rate = nominal rate**
   - Periodic rate is the rate generally shown on timelines and used in calculations
   - For compounding more frequently than annually:

   $$FV_n = PV(1+ i_{PER})^{\text{number of periods}} = PV(1+\frac{i_{nom}}{m})^{mn}$$

   See pg 73 of Brigham for example

3. **Effective Interest Rate**
   - Effective (or Equivalent) Annual Rate (EAR)
   - Annual rate that produces the same result as if we had compounded at a given periodic rate m times per year
   - The EAR, also called the EFF% (effective percentage), is found as follows

$$\text{EAR(or EFF \%)} = (1+\frac{i_{nom}}{m}) - 1$$

   - The EAR is not used in calculations.
   - It should be used to compare the effective cost or rate of return on loans or investments when payment periods differ.
   - See pgs 74-75 of Brigham

4. **Flat Rate**
   - Certain financing arrangements make use of a so-called 'flat rate'
   - This is a rate charged throughout the duration of the agreement on the original outstanding amount without taking reductions in the original amount into consideration.

Interest Rate Conversions

1. Conversion from nominal rate to effective rate

$$\text{EAR} = \left(1 + \frac{i_{nom}}{m}\right) - 1$$

2. Conversion from effective rate to nominal rate
   When the effective rate and the number of times per annum that interest must be compounded are known, the nominal rate can be calculated as follows:

$$\text{Nominal Rate} = n\left[(1 + i)^{\frac{1}{n}} - 1\right]$$

   i = effective rate, expressed as a percentage
   n = number of periods per annum

Interpolation and Extrapolation

Interpolation

The process by which a rate that falls <u>within</u> the interval of two known rates is determined.

Using the lower rate = $A + \left(\frac{P-Q}{P-N} x\ (B - A)\right)\%$

Using the higher rate = $B - \left(\frac{Q-N}{P-N} x\ (B - A)\right)\%$

    A is the (Lower) rate of return

    B is the (Higher) rate of return

    P is the amount/NPV calculated using the (lower) rate

    N is the amount/NPV calculated using the (higher) rate

    Q is the amount/NPV calculated using the (unknown) rate

See example on page 15 of Study Guide

Extrapolation

The process by which a rate that falls <u>outside</u> the interval of two known rates is determined.

Using the lower rate = $A - \left(\frac{Q-P}{P-N} x\ (B - A)\right)\%$

Using the higher rate = $B + \left(\frac{N-Q}{P-N} x\ (B - A)\right)\%$

    A is the (Lower) rate of return

    B is the (Higher) rate of return

    P is the amount/NPV calculated using the (lower) rate

    N is the amount/NPV calculated using the (higher) rate

    Q is the amount/NPV calculated using the (unknown) rate

See example on page 16 of Study Guide

Study Unit 3: Illustrative Example