# Tutorial Letter 101/3/2015

## Introductory Financial Mathematics

## DSC1630

## Semesters 1 and 2

### Department of Decision Sciences

**Important Information:**

This tutorial letter contains important information about your module.

Bar code

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# Chapter 1

# General information

## 1.1 Introduction and welcome

It is a pleasure to welcome you as a student to the DSC1630 module. We trust that we will meet each other some time during the semester and that you will find this module in Introductory Financial Mathematics interesting and stimulating.

Introductory Financial Mathematics is a one-semester module. The module may be completed during the first OR second semester of 2015. You have to register for just one of the two semesters. If you complete the module successfully, you do not have to register for it again. If you fail the module, however, you **must** re-register so that you can repeat it.

## 1.2 Purpose and outcomes of the module

**The purpose of the module is:**

This module provides fundamental introductory knowledge, and skills to identify which mathematical formulæ to use in a specific financial problem. Students who complete this module will be able to solve problems involving for interest rates, annuities, amortisation, bond pricing and capital budgeting.

**The outcomes and assessment criteria of this module are as follows**

**Learning outcome 1:**

Learners can apply the basic concepts of various interest rate calculations, simple discount, the concepts of time value of money, the equations of value, the basic concepts and applications of annuities, amortisation and sinking funds, pricing mechanism of bonds and related financial instruments and capital budgeting as well as the various methods that can be used to appraise investments that will yield cash flows at future dates.

**Assessment criteria:**

1. Calculate simple interest and simple interest rates, simple discount and discount rates.
2. Determine and apply the equations and the corresponding time lines relating present and future values of money when simple interest is applicable.
3. Determine and apply the relationship between simple interest and simple discount.

4. Apply the compound interest rate formula to practical problems.
5. Determine and apply the equations for the corresponding time lines relating present and future values of money when compound interest is applicable.
6. Apply the concepts of effective and nominal interest rates to practical problems.
7. Solve practical problems involving odd period calculations.
8. Solve practical problems by applying continuous compounding.
9. Derive formulae relating to annuities.
10. Solve practical problems involving annuities due, ordinary annuities, deferred annuities and perpetuities.
11. Solve practical problems involving the future value and the present value of annuities.
12. Solve practical problems involving increasing annuities.
13. Calculate the repayment of loans by means of both amortisation and sinking funds.
14. Interpret an amortisation schedule as well as a sinking fund schedule.
15. Apply the concept of real cost.
16. Compare two alternative investments with different cash flows using the payback and average rate of return mechanisms.
17. Apply the formulae for the internal rate of return and the net present value to practical problems.
18. Apply the profitability index as well as the modified internal rate of return to practical problems.
19. Apply the formula for pricing a bond on an interest date for a given yield to maturity.
20. Determine the all-in-price of a bond.
21. Apply the concepts of cum and ex interest.
22. Determine the accrued interest of a bond for any settlement date.
23. Determine the clean price of a bond on any settlement date.
24. Determine the settlement date of a bond from given information.
25. Recognise the terms “discount”, “premium” and “par bond”.

**Learning outcome 2:**

Learners can apply the basic concepts and applications of limited statistics and basic linear regression.

**Assessment criteria:**

1. Apply single and double subscripts.
2. Apply single and double summation.

3. Interpret a population and a sample.
4. Determine the arithmetic mean, the weighted mean, the variance and the standard deviation of a set of data.
5. Apply the concept of a linear function, determine the intercepts on the axes, the slope and the equation of a straight line and draw the graph.
6. Apply the concepts of correlation and regression.
7. Draw a scatter diagram.
8. Calculate the Pearson's correlation coefficient and coefficient of determination.
9. Determine the equation of a regression line for given data.
10. Determine how well a regression line fits data by using the correlation coefficient.

## 1.3 Lecturers and contact details

The lecturers for DSC1630 will assist you if you experience any difficulties regarding the study material. Do not hesitate to contact them, but please make an appointment if you want to see them personally.

Information on your lecturers is published in Tutorial Letter 301, 2015 as well as on the Welcome Message of the module on myUnisa. Write down the contact information for module DSC1630 in the box below (or cut it out and paste it here).

## 1.4 Module related resources

(a) Prescribed books
There is no prescribed textbook for this module

(b) E-reserves
There are no E-reserves for this module

(c) Recommended books
There are no recommended books for this module

## 1.5 Student support services for the module

For information on the various student support systems and services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the publication *myStudies @ Unisa*, which you received with your study material.

### 1.5.1 Contact with fellow students

- **Study groups**

  It is advisable to have contact with fellow students. One way to do this is to form study groups. The addresses of students in your area may be obtained from the following department:

  Directorate: Student Administration and Registration
  PO Box 392
  UNISA
  0003

- ***myUnisa***

  If you have access to a computer that is linked to the internet, you can quickly access resources and information at the university. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students (Discussion Forums) and with the administrative departments of Unisa – all through the computer and the internet.

  To go to the *myUnisa* website, start at the main Unisa website, http://www.unisa.ac.za/ and then click on the " *myUnisa*" link. This should take you to the *myUnisa* website. You can also go there directly by typing in https://my.unisa.ac.za/.

  Please consult the publication *myStudies @ Unisa*, on main Unisa's website.

## 1.6 Module specific study plan

### 1.6.1 Study material

The Department of Despatch should supply you with the following study material:

- Tutorial Letter 101, 2015. This contains
  - general information
  - compulsory assignments for each semester
  - self-evaluation exercises with their solutions
  - additional exercises with their solutions
  - notes on the calculators
- other tutorial letters will be **sent out during** the course of the semester
- one study guide: Introductory Financial Mathematics
- a DVD containing instructions for working the recommended calculators

**Please take note that all the study material can be downloaded from the myUnisa.**

**Please consult myUnisa on a regular basis. From time to time the lecturer post additional information on myUnisa or early release of Tutorial letters.**

**Note:** Some of this tutorial matter may not necessarily be available when you register. Tutorial matter that is not available when you register will be mailed to you as soon as possible.

When you register, you will receive an inventory letter containing information about your tutorial matter. See also the booklet entitled *myStudies @ Unisa* (on main Unisa's website).

### 1.6.2 Study time

There are about four months (± 15 weeks) of study time in a semester. In this short period you have to work through the study material and prepare yourself for the examinations. In view of the fact that your study time is limited, you will have to plan carefully. ***As this module is of a mathematical nature you will not be able to master it in a short period of time. You will have to work consistently and according to a timetable if you want to be successful.*** We suggest that you spend at least 45 minutes per day on this module. We have divided your study material into sections. You should be able to complete each section together with the relevant self-evaluation exercises within a week. By the end of each week

1. you should have worked through the designated section of the study material
2. you should have completed the self-evaluation exercises relevant to the study material.

Only then should you continue with the next part of the study material. If you follow this schedule you will have enough time left to revise the study material for the examination.

| **Work covered** | **Study time** | **Self-evaluation exercise** |
|---|---|---|
| Chapter 2 | Week 1 | 1 |
| Chapter 3 up to 3.2 | Week 2 | 2 |
| | **Compulsory assignment 1** | |
| Chapter 3 | Week 3 | 2 |
| Chapters 4 up to 4.2 | Week 4 | 3 |
| | **Compulsory assignment 2** | |
| Chapter 4 | Week 5 | 3 |
| Chapter 5 | Week 6 | 3 |
| | **Compulsory assignment 3** | |
| Chapter 6 | Week 5 | 4 |
| Chapter 7 | Week 6 | 5 |
| Chapter 8 | Week 7 | 6 |
| | **Compulsory assignment 4** | |
| Typical examination questions | Week 8 | 7 and 8 |

### 1.6.3 Calculator

The use of a **financial calculator** **is absolutely essential for this module**.

You may use any make or model that you prefer. However, the Department of Decision Sciences has decided to provide instructions for the following two models:

- SHARP EL-738 or EL738F
- Hewlett Packard HP10BII or HP10BII+

These financial calculators are also recommended by other departments in the College of Economic and Management Sciences.

You need to purchase **ONE** of these financial calculators.

- The SHARP EL-738 calculator may be obtained from most suppliers of SHARP calculators or from a Sharp Electronics branch. Take your Unisa student card with you and ask the sales manager for a special student price. Remember that an ordinary shop will not sell it to you at a student price.
- The Hewlett Packard HP10BII may be purchased from most shops selling electronic appliances.

It is advisable to purchase your calculator as soon as possible as it takes time to master it.

How to operate the SHARP EL-738 and the HEWLETT PACKARD 10BII is explained in

- **notes on the calculators that are included in this tutorial letter** which is the hard copy of
- the **DVD** that you received with your study material.

**PLEASE NOTE:**

The SHARP EL-733A financial calculator or the SHARP EL-5250 and SHARP EL-5120 programmable calculators may still be used. No assistance will however be given to users of these calculators. You may however contact the Department and ask for a manual for the SHARP EL-733A financial calculator, the SHARP EL-5120 programmable calculator and the SHARP EL-5250 programmable calculator.

### 1.6.4 Study methods

We suggest that you follow these steps when you work through your study material:

1. Buy **ONE** of the recommended calculators and master it before you start with your studies.
2. Read through the study material and do all examples that are given.
3. Then do the exercises related to the study material that you have studied. **Use your calculator for the calculations.**
4. Check your answers against the answers that are supplied.
5. Work through the chapter as described above and then do the evaluation exercises at the end of the chapter.
6. Then do the self-evaluation exercises for that chapter that are included in this tutorial letter.

**If you have any problems with the study material at any stage please contact the lecturers immediately so that they can help.**

### 1.6.5 Self-evaluation exercises

To help you in your preparations we provide you with extra exercises and solutions that are known as self-evaluation exercises. Most of these questions originate from old examination papers.

Self-evaluation exercises are important for the following reasons:

1. Self-evaluation exercises assist you in understanding and mastering the study material and its practical applications. They are, therefore, an integral part of the study material.
2. Self-evaluation exercises test your knowledge and understanding of the study material. They provide a way to evaluate your progress.

You are, therefore, strongly advised to do all the self-evaluation exercises.

**Start your studies as soon as possible**. This will enable you to cover the module with enough time left over to study for the examination.

To help you plan your studies, we have divided the work into sections. Each section should be completed within a week, after which the self-evaluation exercises for that section, which are included in this tutorial letter, should also be attempted. Try to do these self-evaluation exercises without looking at our answers. After you have completed your **self-evaluation exercises, compare your answers to our solutions which are included in this tutorial letter. If you experience any problems with the work, contact us immediately so that we can assist you.**

A full set of additional exercises and their solutions follow on page 113 of this tutorial letter.

If you pass this module in one semester, you have completed the module Introductory Financial Mathematics successfully and need not register for it again in the following semester.

## 1.7 Assessments

### 1.7.1 Self-evaluation

You are responsible for evaluating the self-evaluation exercises yourself.

When marking your self-evaluation exercises, compare your answers to the model solutions that are included in this tutorial letter. Our answers are set out step by step so that you can understand the mathematics involved in the calculations. Each calculation and the details of your answer should be checked against the model answer. This will help you to understand each question.

The solutions often contain helpful explanations and comments. This process of self-evaluation will also ensure that you take note of this extra information.

If you have any queries, contact the lecturers as soon as possible.

### 1.7.2 Compulsory assignments

**(See Chapter 2 of this tutorial letter for the first semester's assignments and Chapter 3 for the second semester assignments.)**

There are four compulsory assignments per semester. Submission of every assignment will contribute 25% towards your semester mark. All the assignments MUST be submitted on a mark reading sheet.

Make sure that the compulsory assignments reach the University BEFORE the due dates.

**Remember you have to submit at least one (anyone) of the four assignments to get access to the examination.**

UNDER NO CIRCUMSTANCES WILL EXTENSION BE GRANTED FOR SUBMISSION OF THE ASSIGNMENTS.

| Semester | Assignment number | Study material covered | Unique number | Due date | % contribute towards year mark |
|---|---|---|---|---|---|
| 1 | 1 | Chapter 2 – 3.2 | 549460 | 20 February 2015 | 25% |
| | 2 | Chapter 3 – 4.2 | 505562 | 6 March 2015 | 25% |
| | 3 | Chapter 4, 5 | 570598 | 20 March 2015 | 25% |
| | 4 | Chapter 6, 7, 8 | 570622 | 13 April 2015 | 25% |
| 2 | 1 | Chapter 2 – 3.2 | 589234 | 7 August 2015 | 25% |
| | 2 | Chapter 3 – 4.2 | 589252 | 21 August 2015 | 25% |
| | 3 | Chapter 4, 5 | 570670 | 4 September 2015 | 25% |
| | 4 | Chapter 6, 7, 8 | 570687 | 23 September 2015 | 25% |

### 1.7.3 Submission of assignments

You may submit the COMPULSORY assignments either by mail or internet via myUnisa (https://my.unisa.ac.za). Assignments MAY NOT be submitted by fax or e-mail.

To submit an assignment via myUnisa:

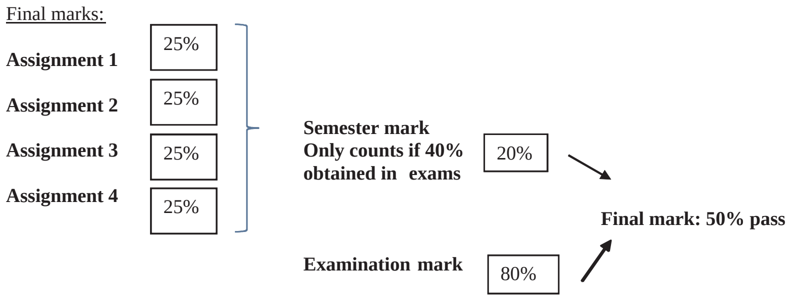
1. Go to myUnisa.
2. Log in with your student number and password.
3. Select the module.
4. Click on assignments in the menu on the left-hand side of the screen.
5. Click on the assignment number you want to submit.
6. Follow the instructions.

**Please check myUnisa to make sure that your assignment has reached Unisa and has been registered after you have submitted your assignment. You are allowed to re-submit an assignment before the due date.**

## 1.8 Examination

**You are automatically admitted to the examinations by submitting assignment 1.**

The composition of your final mark can be summarized as:




For example:

Suppose you obtained 80 marks for Assignment 01, 60 marks for Assignment 02, 20 marks for Assignment 3, 80 marks for Assignment 4 and 40 marks in the examination.

Your semester mark will be calculated as follows:

25% of 80 will give you 20 marks$\left(\frac{25}{100} \times 80 = 20\right)$.

25% of 60 will give you 15 marks$\left(\frac{25}{100} \times 60 = 15\right)$.

25% of 20 will give you 5 marks$\left(\frac{25}{100} \times 20 = 5\right)$.

25% of 80 will give you 20 marks$\left(\frac{25}{100} \times 80 = 20\right)$.

Your total semester mark is thus 60 $(20 + 15 + 5 + 20)$. This 60 will contribute to 20% of your final mark. You will therefore have 12 marks before you start to write the examination.

Your examination mark will be calculated as follows:

80% of 40 will give you 32 marks$\left(\frac{80}{100} \times 40 = 32\right)$.

Your final mark will be 44 $(32 + 12)$.

The examination dates and centres are given in myRegistration @ *unisa*. The duration of the examination for this module is two hours.

Please note the following:

- You may use any financial or programmable calculator in the examination.
- You may only take your calculator and writing material, but no paper, into the examination hall.
- The paper consists of 30 multiple-choice questions that must be answered on a mark-reading sheet.
- You must use an HB pencil to mark the correct option.
- A formulae sheet (similar to the one that you will receive in Tutorial Letter 201) will be supplied.
- The table with the number of each day of a year, will be part of the examination paper.

**The examination will cover all the study material. It is, therefore, important that you do as many examples as possible. Remember practice makes perfect!**

## 1.9 Frequently asked questions

See *myStudies @ unisa* brochure for the most relevant study information.

### Module specific questions

- Must I buy both recommended calculators?

  No! Only one.

- Where can I buy a calculator?

  At any shop that sells electronic devices and see online shopping for example kalahari.com.

- Must I learn the formulæ off by heart?

  No! You will receive a formulæ sheet in the examination that is similar to the one that is in Tutorial letter 201.

- I can't find the $\rceil$ on my calculator.

  It is only a symbol that is used when we want to indicate that the present or future value formulæ of an annuity must be used.

  In Tutorial letter 201 and on myUnisa.

- Rounding of answers.

PLEASE NOTE: You should never round off results in the middle of a problem. For example, consider the calculation of the accumulated value if R1 000 is invested for five years at an interest rate of 16% per year compounded monthly.

If you use the interest rate as 0,16 ÷ 12 = 0,013, your answer will be
$1\ 000\ (1 + 0{,}013)^{60}$ = R2 170,53.

If you use the interest rate as 0,16 ÷ 12 without rounding off, your answer will be R2 213,81, which is the correct answer. The difference is R43,28. Rounding off too early leads to an incorrect answer.

ONLY THE FINAL ANSWER MUST BE APPROXIMATED. Rounding off should only take place in the final step of a calculation. Work for example with five decimals and just round off your answer to the number of decimals equivalent to the answers of the question.

All the above information are either in the study guide or in this tutorial letter.

*In conclusion, we hope we will enjoy a happy and successful semester together. Please remember that we are here to help you, no matter how big or small your problem might be.*

# Chapter 2

# First semester compulsory assignments

**At least one (any one) of the four assignments must be submitted on time to gain access to the examination**

## 2.1 Compulsory Assignment 01

### Due Date: 20 February 2015

### Unique Number: 549460

**Please work through:**

- Guide: Chapter 2 – 3.2
- Tutorial letter 101: Self evaluation exercises week 1 and 2

before answering the assignment questions.

### Question 1

Nola invests R1 500 in an account earning 6,57% simple interest. The balance in the account 16 months later will equal

[1] R1 631,40.

[2] R1 632,82.

[3] R1 636,94.

[4] R1 644,02.

[5] R2 814,00.

## Question 2

A bank's discount rate is 12%. You need to pay the bank R5 000 in six months time. The amount of money that you will *now* receive *from* the bank equals

[1] R4 700,00.
[2] R4 716,98.
[3] R4 724,56.
[4] R5 300,00.
[5] R5 319,15.

## Question 3

Frieda borrows R7 500 from the the bank at an interest rate of 26,00% per year, compounded weekly. The amount that Frieda will have to pay him back after 78 weeks will equal

[1] R10 425,00.
[2] R10 607,60.
[3] R11 031,32.
[4] R11 066,60.
[5] R11 430,24.

## Question 4

An investment of R20 000 accumulated to R45 200. If the applicable simple interest rate is 12%, then the time under consideration is

[1] 3,25 years.
[2] 4,65 years.
[3] 7,19 years.
[4] 10,50 years.
[5] 15,12 years.

## Question 5

Miriam bought a painting for R15 000. For 10 years the value of the painting increased yearly by 20%. Thereafter the value increased yearly by 15%. The amount of money that Miriam can expect to receive if she sells the painting after owning it for 23 years equals

[1] R132 750,00.
[2] R373 371,86.
[3] R571 446,59.
[4] R649 270,78.
[5] R993 710,59.

## Question 6

Two years from now Sipho has to pay Alet R15 000. He decides to pay her back earlier. If a simple interest rate of 12,5% per year is applicable, then the amount that Sipho will have to pay Alet nine months from now equals

[1] R12 656,25.

[2] R12 972,97.

[3] R13 125,00.

[4] R13 664,28.

[5] R13 714,29.

## Question 7

Lulu deposits R900 into a savings account earning $6\frac{1}{2}\%$ interest per year compounded quarterly. After three and a half years she withdraws R1 000 from the account and deposits it into another account paying 11% interest per year compounded semi-annually. How much is the total amount accrued from both accounts two years after making the second deposit?

[1] R1 366,67

[2] R2 138,82

[3] R1 384,27

[4] R2 227,85

[5] None of the above.

## Question 8

If a simple interest rate of 24% is equivalent to a simple discount rate of 20,5% then the time under consideration approximately equals

[1] 260 days.

[2] 304 days.

[3] 312 days.

[4] 711 days.

[5] 854 days.

## Question 9

Richard needs R30 835,42 for an overseas trip. How long will it take him to save towards this amount if he deposits R25 000 now into a savings account earning 10,5% per year compounded weekly?

[1] 2 weeks

[2] 24 weeks

[3] 52 weeks

[4] 104 weeks

[5] None of the above.

## Question 10

If Nkosi earns a nominal interest rate per year of 16,5% per annum compounded at the end of every second month on a savings account, then the effective interest rate equals

[1] 14,527%.

[2] 16,181%.

[3] 16,677%.

[4] 17,677%.

[5] 18%.

## Question 11

Gina invested R25 000 on 5 March in an account earning 8,76% simple interest per year. The accumulated interest on 15 November of the same year will equal

[1] R1 441,76.

[2] R1 524,00.

[3] R1 530,00.

[4] R1 561,29.

[5] none of the above.

## Question 12

It is now the end of May 2015. Piet Hardloop is planning to leave the country at the end of the year. He has three debtors each owing him an amount of R15 000. Sitter must pay his R15 000 back in three months' time from now, Stacy hers in five months' time from now and Skipper his in seven months' time from now. Piet Hardloop tells them that it will be all right if they each pay him back just before he leaves at the end of December 2015. However, he will charge simple interest at 10% from the time that they were each supposed to pay back the amount of R15 000. The amount that Piet will receive at the end of December 2015 will equal

[1] R45 750,00.

[2] R45 757,33.

[3] R46 875,00.

[4] R47 625,00.

[5] none of the above.

## Question 13

Aziza urgently needs R10 000. The bank agrees to give her this amount now, to be paid back seven months from now based on a bank simple discount rate of 16,4%. The amount that she will have to pay back equals

[1] R9 043,33.

[2] R10 956,67.

[3] R10 996,80.

[4] R11 057,87.

[5] R12 000,00.

## Question 14

Mario owes Sweetness R500 due in four months' time and R700 due in nine months' time. He wants to liquidate these obligations with a single payment in 12 months' time. If a simple interest rate of 11% per year is charged on all the amounts, the amount he will pay Sweetness in 12 months' time is equal to

[1] R1 255,92.

[2] R1 276,08.

[3] R1 228,42.

[4] R1 200,00.

[5] none of the above.

## Question 15

The number of years that it will take R6 000 to accumulate to R9 000 at an annual interest rate of 8% compounded every three months is

[1] 5,08 years.

[2] 5,12 years.

[3] 5,27 years.

[4] 6,25 years.

[5] 20,48 years.

## 2.2 Compulsory Assignment 02

## Due Date: 6 March 2015

## Unique Number: 505562

**Please work through:**

- Guide: Chapter 3 – 4.2
- Tutorial letter 101: Self evaluation exercises week 2 and 3

before answering the assignment questions.

This assignment **MUST** be submitted on a **mark-reading sheet**. No written compulsory assignments will be accepted.

## Question 1

An interest rate of 17,5% per year compounded quarterly is equivalent to a continuous compounding rate of

[1] 17,128%.

[2] 17,185%.

[3] 17,500%.

[4] 17,888%.

[5] 19,125%.

## Question 2

An amount borrowed at 29% interest per year compounded continuously has accumulated to R38 279,20 after four years. The initial amount borrowed was

[1] R7 160,73.

[2] R12 000,00.

[3] R12 005,53.

[4] R13 823,05.

[5] R17 721,85.

## Question 3

The effective rate for a continuous compounding rate of 17,5% is

[1] 16,13%.
[2] 17,5%.
[3] 19,12%.
[4] 19,13%.
[5] 21,076%.

***Questions 4 and 5 relate to the following situation:***
*An amount of R10 000 was invested in a special savings account on 15 May at an interest rate of 15% per annum compounded quarterly for seven months. Interest is calculated on 1 January, 1 April, 1 July and 1 October of every year.*

## Question 4

If simple interest is used for the odd periods and compound interest for the rest of the term, the amount of interest received after seven months equals

[1] R665,54.
[2] R896,95.
[3] R901,35.
[4] R1 644,57.
[5] none of the above.

## Question 5

If fractional compounding is used for the full term of seven months, the total amount of interest received will equal

[1] R892,79.
[2] R894,04.
[3] R898,43.
[4] R901,73.
[5] none of the above.

***Questions 6 and 7 relate to the following situation:***
*Three years ago Jake borrowed R7 500 from Martha. The condition was that he would pay her back in seven years' time at an interest rate of 11,21% per year, compounded semi-annually. Six months ago he also borrowed R25 000 from Martha at 9,45% per year, compounded monthly. Jake would like to pay off his debt four years from now.*

## Question 6

The amount of money that Jake will have to pay Martha four years from now equals

[1] R36 607,98.

[2] R45 181,81.

[3] R48 032,20.

[4] R54 278,92.

[5] R55 336,49.

## Question 7

After seeing what he must pay Martha, Jake decides to reschedule his debt as two equal payments: one payment *now* and one three years from *now*. Martha agrees on condition that the new agreement, that will run from *now*, will be subjected to 10,67% interest, compounded quarterly. The amount that Jake will pay Martha three years from *now* equals

[1] R21 171,35.

[2] R22 286,88.

[3] R25 103,93.

[4] R32 500,00.

[5] none of the above.

## Question 8

If R35 000 accumulates to R48 320 at a continuous compounded rate of 8,6%, then the term under consideration is

[1] 2,77 years.

[2] 3,75 years.

[3] 3,91 years.

[4] 4,43 years.

[5] 6,23 years.

## Question 9

Nicolet wants to buy a new state of the art computer for R35 000. She decides to save by depositing an amount of R500 once a month into an account earning 11,32% interest per year, compounded monthly. The approximate time it will take Nicolet to have R35 000 available equals

[1] 40 months.
[2] 54 months.
[3] 70 months.
[4] 115 months.
[5] none of the above.

## Question 10

If money is worth 12% per annum compounded monthly, how long will it take the principal P to double?

[1] 6,12 years
[2] 7,27 years
[3] 8,33 years
[4] 69,66 years
[5] None of the above.

## Question 11

Paul decides to invest R140 000 into an account earning 13,5% interest per year, compounded quarterly. This new account allows him to withdraw an amount of money every quarter for 10 years after which time the account will be exhausted. The amount of money that Paul can withdraw every quarter equals

[1] R1 704,28.
[2] R3 500,00.
[3] R6 429,28.
[4] R8 594,82.
[5] none of the above.

## Question 12

If 15% interest is compounded every two months then the equivalent weekly compounded rate equals

[1] 14,464%.
[2] 14,484%.
[3] 14,816%.
[4] 14,837%.
[5] none of the above.

## Question 13

Dieter owes Paul R3 000 due 10 months from now, and R25 000 due 32 months from now. Dieter asks Paul if he can discharge his obligations by two equal payments: one now and the other one 28 months from now. Paul agrees on condition that a 14,75% interest rate, compounded every two months, is applicable. The amount that Dieter will pay Paul 28 months from now will equal approximately

[1] R11 455.

[2] R11 511.

[3] R11 907.

[4] R14 000.

[5] R20 000.

## Question 14

The accumulated amount after eight years of monthly payments of R1 900 each into an account earning 9,7% interest per year, compounded monthly, equals

[1] R126 532,64.

[2] R182 400,00.

[3] R274 069,25.

[4] R395 077,74.

[5] none of the above.

## Question 15

A saving account pays interest at the rate of 5%, compounded semi-annually. The amount that should be deposited now so that R250 can be withdrawn at the end of every six months for the next 10 years is

[1] R1 930,43.

[2] R3 144,47.

[3] R3 897,29.

[4] R6 386,16.

[5] none of the above.

## 2.3 Compulsory Assignment 03

## Due Date: 20 March 2015

## Unique Number: 570598

**Please work through:**
- Guide: Chapter 4 – 5
- Tutorial letter 101: Self evaluation exercises week 3

before answering the assignment questions.

This assignment **MUST** be submitted on a **mark-reading sheet**. No written compulsory assignments will be accepted.

## Question 1

An amount of R600 is invested every month for eight years. The applicable interest rate is 14,65% per year compounded quarterly. The accumulated amount of these monthly payments equals approximately

[1] R57 600.

[2] R107 500.

[3] R108 400.

[4] R109 300.

[5] R321 200.

## Question 2

At the beginning of each month an amount of $X$ rand is deposited into a savings account earning $k \times 100\%$ interest per year, compounded monthly. The future value of these savings after 24 deposits can be denoted by

[1] $S = Xs_{\overline{24}|k}$.

[2] $S = X(1 + \frac{k}{12})^{24}$.

[3] $S = (1 + k)Xs_{\overline{24}|k}$.

[4] $S = (1 + \frac{k}{12})Xs_{\overline{24}|k \div 12}$.

[5] none of the above.

## Question 3

Bobby borrowed money that must be repaid in nine payments. The first four payments of R2 000 each are paid at the *beginning* of each year. Thereafter five payments of R5 000 each are paid at the end of each year. If money is worth 6,85% per year, then the present value of these payments equals

[1] R22 588,92.

[2] R23 054,54.

[3] R27 381,02.

[4] R27 845,64.

[5] R33 000,00.

## Question 4

In three years' time Lindiwe is going to need R145 000 to pay for a boat cruise on the Queen Mary. She *immediately starts* to make monthly deposits into an account earning 11,05% interest per year, compounded monthly. Lindiwe's monthly deposit equals

[1] R3 384,18.

[2] R3 415,34.

[3] R4 027,78.

[4] R4 707,20.

[5] R4 750,55.

## Question 5

After a golf ball struck Charl on the head he was awarded an amount from the Three Iron Fund as compensation for his injuries. He chose to receive R18 900 per month indefinitely. If money is worth 9,95% per year, compounded monthly, then the amount awarded equals approximately

[1] R189 950.

[2] R2 279 397.

[3] R6 565 554.

[4] R7 252 333.

[5] none of the above.

***Questions 6 and 7 relate to the following situation:***
*Solly will discharge a debt of R500 000 six years from now, using the sinking fund method. The debt's interest is 15,6% per year, paid quarterly. The sinking fund will earn interest at a rate of 8,4% per year, compounded monthly.*

## Question 6

The monthly deposit into the sinking fund will equal

[1] R4 236,10.

[2] R5,364,60.

[3] R10 736,10.

[4] R12 958,53.

[5] R16 235,96.

## Question 7

The total yearly cost to discharge the debt (to the nearest rand) will equal

[1] R42 000.

[2] R78 000.

[3] R93 834.

[4] R128 833.

[5] R142 375.

## Question 8

Monthly deposits of R100 each are made into a bank account earning interest at an interest rate of 18% per annum compounded monthly. The time (in months) that it will take the account to accumulate to R20 000 is given by

[1] $n = \frac{\ln[200(0{,}015)+1]}{\ln(1+0{,}015)}$.

[2] $n = \frac{\ln[200(0{,}015)]}{(0{,}015)}$.

[3] $n = \ln[200(0{,}015) + 1] - \ln(1{,}015)$.

[4] $n = \frac{\ln[200(0{,}015)-1]}{\ln(1+0{,}015)}$.

[5] none of the above.

***Questions 9, 10 and 11 relate to the following situation:***
*The following is an extract from the amortisation schedule of a home loan:*

| Month | Outstanding principal at month beginning | Interest due at month end | Monthly payment | Principal repaid | Outstanding principal at month end |
|---|---|---|---|---|---|
| 147 | R8 155,83 | A | R2 080,54 | R2 014,27 | R6 141,56 |
| 148 | R6 141,56 | R49,90 | R2 080,54 | R2 030,64 | B |
| 149 | B | R33,40 | R2 080,54 | R2 047,14 | R2 063,78 |
| 150 | R2 063,78 | R16,77 | R2 080,54 | R2 063,77 | 0 |

## Question 9

The value of A equals

[1] R41,65.

[2] R49,50.

[3] R66,27.

[4] R166,33.

[5] R167,86.

## Question 10

The value of B equals

[1] R4 061,02.

[2] R4 077,79.

[3] R4 094,21.

[4] R4 110,92.

[5] R4 127,68.

## Question 11

If the interest rate has never changed, the original amount of the home loan was (rounded to the nearest thousand rand)

[1] R21 000,00.

[2] R180 000,00.

[3] R310 000,00.

[4] R312 000,00.

[5] R606 000,00.

***Questions 12 and 13 relate to the following situation:***
*Jay intends to open a small material shop and borrows the money for it from his Uncle Jossop. Jay feels that he will only be able to start repaying his debt after three years. Jay will then pay Uncle Jossop R105 000 per year for five years. Money is worth 19,5% per year.*

## Question 12

The present value of Jay's debt at the time he will start paying Uncle Jossop back will equal

[1] R222 924,04.
[2] R317 500,78.
[3] R408 978,93.
[4] R436 649,07.
[5] R525 000,00.

## Question 13

The amount of money that uncle Jossop originally lent Jay equals

[1] R98 346,23.
[2] R130 288,26.
[3] R130 633,09.
[4] R184 589,43.
[5] R186 054,89.

## Question 14

You started saving to pay for your children's university costs in 20 years' time. Your first payment was R3 600 per year, after which your yearly payments increased by R360 each year. If the expected interest rate per year is 10%, the amount that you expect to receive to the nearest rand on the maturity date will be

[1] R213 030.
[2] R340 380.
[3] R412 380.
[4] R484 380.
[5] none of the above.

## Question 15

Cindy bought a house and managed to secure a home loan for R790 000 with monthly payments of R9 680,70 at a fixed interest rate of 13,75% per year, compounded monthly, over a period of 20 years. If an average yearly inflation rate of 9,2% is expected, then the real cost of the loan (the difference between the total value of the loan and the actual principal borrowed) equals

[1] R87 126.

[2] R201 642.

[3] R270 749.

[4] R588 358.

[5] R1 060 749.

## 2.4 Compulsory Assignment 04

## Due Date: 13 April 2015

## Unique Number: 570622

**Please work through:**
- Guide: Chapter 6, 7 and 8
- Tutorial letter 101: Self evaluation exercises week 4, 5, and 6

before answering the assignment questions.

This assignment **MUST** be submitted on a **mark-reading sheet**. No written compulsory assignments will be accepted.

***Questions 1 and 2 relate to the following situation:***
*Down-To-Earth sells houses. The following table represents the selling price of a house (y) in thousands of rands and the number of houses sold at that price (x).*

| | | | | |
|---|---|---|---|---|
| $x$ | *5* | *15* | *19* | *7* |
| $y$ | *500* | *900* | *1 500* | *2 000* |

## Question 1

The standard deviation for the number of houses sold is

[1] 4.
[2] 5,72.
[3] 6,6.
[4] 11,5.
[5] none of the above.

## Question 2

The correlation coefficient of a linear regression between $x$ and $y$ is approximately

[1] $r = -0{,}16428$.
[2] $r = 0{,}16428$.
[3] $r = 4$.
[4] $r = 5{,}72276$.
[5] none of the above.

***Questions 3 and 4 relate to the following situation:***
*The following table represents the cash inflows for the Two left feet Boutique.*

| ***Year*** | ***Cash inflow*** *(R)* |
|---|---|
| *3* | *45 000* |
| *6* | *90 000* |
| *9* | *115 000* |

*The applicable interest rate is 11,59% per year. The present value of the cash outflows is R95 000.*

## Question 3

The future value of the cash inflows approximately equals

[1] R169 330.

[2] R218 000.

[3] R250 000.

[4] R271 470.

[5] R326 950.

## Question 4

The MIRR equals

[1] 14,72%.

[2] 21,25%.

[3] 31,90%.

[4] 38,06%.

[5] 41,91%.

## Question 5

*Consider Bond F234*

| | |
|---|---|
| *Coupon rate (half yearly)* | *10,5% per year* |
| *Yield to maturity* | *7,955% per year* |
| *Maturity date* | *8 October 2049* |
| *Settlement date* | *29 May 2015* |

The all-in-price equals

[1] R123,49852%.

[2] R126,13814%.

[3] R129,73733%.

[4] R131,24248%.

[5] R134,98733%.

## Question 6

The equation for the present value of Bond AAA on 17/06/2015 is given by

$$107{,}55174 = da_{\overline{n}|z} + 100\left(1 + \frac{0{,}135}{2}\right)^{-29}$$

The yearly coupon rate is

[1] 6,75%.

[2] 7,35%.

[3] 8,55%.

[4] 14,70%.

[5] none of the above.

***Questions 7 and 8 relate to the following situation:***
*Consider Bond ABC*

| | |
|---|---|
| *Coupon rate:* | *9,75% per year* |
| *Yield to maturity:* | *11,4% per year* |
| *Maturity date:* | *15 April 2041* |
| *Settlement date:* | *29 November 2015* |

## Question 7

The accrued interest is

[1] 1,18207.

[2] 1,20205.

[3] 2,34537.

[4] 5,87781.

[5] none of the above.

## Question 8

The clean price equals

[1] R81,69720%.

[2] R85,22964%.

[3] R86,37296%.

[4] R86,39294%.

[5] R88,77706%.

## Question 9

If the NPV of the Calm and Relax Spa is R195 000 and the profitability index is 1,24375, the initial investment in the Spa equals

[1] R86 908.

[2] R156 784.

[3] R195 000.

[4] R800 000.

[5] none of the above.

## Question 10

An estate agent suspects that there is a linear relationship between the number of houses sold and the monthly loan payments. She analyses the following data over the past six months.

| **Number of houses sold** | **Monthly loan payments (in R1 000's)** |
|---|---|
| $x$ | $y$ |
| 160 | 3,7 |
| 250 | 5,6 |
| 800 | 7,5 |
| 450 | 11,3 |
| 120 | 18,9 |
| 50 | 28,4 |

The regression line equation is

[1] $y = -0{,}016x + 17{,}45$.

[2] $y = 17{,}45x - 0{,}016$.

[3] $y = -13{,}99x + 480{,}89$.

[4] $y = 480{,}89x - 13{,}99$.

[5] none of the above.

## Question 11

The next coupon date that follows the settlement date of a bond is 28 October 2015. The half-yearly coupon rate is 7,375%. The accrued interest equals R5,49589%. If this is a cum interest case the settlement date for this bond is

[1] 14 June 2015.

[2] 30 July 2015.

[3] 29 August 2015.

[4] 11 September 2015.

[5] none of the above.

## Question 12

An investment with an initial outlay of R500 000 generates five successive annual cash inflows of R75 000, R190 000, R40 000, R150 000 and R180 000 respectively. The internal rate of return (IRR) equals

[1] 7,78%.

[2] 9,48%.

[3] 21,3%.

[4] 27,0%.

[5] none of the above.

## Question 13

The following figures show the profit of a greengrocer for the past five years: R360 000, R550 000, R200 000, R80 000 and R700 000. The arithmetic mean of the data equals

[1] R225 424.

[2] R252 032.

[3] R378 000.

[4] R1 890 000.

[5] none of the above.

## Question 14

You must choose between two investments, A and B. The profitability index (PI), net present value (NPV) and internal rate of return (IRR) of the two investments are as follows:

| **Criteria** | **Investment A** | **Investment B** |
|---|---|---|
| NPV | 44 000 | −22 000 |
| PI | 1,945 | 0,071 |
| IRR | 16,00% | 8,04% |

What investment/s should you choose, taking all the above criteria into consideration, if the cost of capital is equal to 12% per year?

[1] A

[2] B

[3] Both A and B

[4] Neither A nor B

[5] Too little information to make a decision

## Question 15

The following table represents the annual income (after tax) of an investment:

| Years | After-tax income R |
|---|---|
| 1 | 200 000 |
| 2 | 500 000 |
| 3 | 300 000 |
| 4 | 400 000 |
| 5 | 700 000 |
| 6 | 300 000 |

If the average rate of return is 8,421%, then the original investment (rounded off to the nearest thousand rand) was

[1] R40 000.

[2] R1 497 000.

[3] R2 400 000.

[4] R4 750 000.

[5] none of the above.

# Chapter 3

# Second semester compulsory assignments

**At least one (any one) of the four assignments must be submitted on time to gain access to the examination**

## 3.1 Compulsory Assignment 01

### Due Date: 7 August 2015

### Unique Number: 589234

**Please work through:**

- Guide: Chapter 2 – 3.2
- Tutorial letter 101: Self evaluation exercises week 1 and 2

before answering the assignment questions.

This assignment **MUST** be submitted on a **mark-reading sheet**. No written compulsory assignments will be accepted.

### Question 1

Two years from *now* Sipho has to pay Alet R15 000. He decides to pay her back earlier. If a simple interest rate of 12,5% per year is applicable, then the amount that Sipho will have to pay Alet nine months from *now* equals

[1] R12 656,25.

[2] R12 972,97.

[3] R13 125,00.

[4] R13 664,28.

[5] R13 714,29.

## Question 2

If a simple interest rate of 24% is equivalent to a simple discount rate of 20,5% then the time under consideration approximately equals

[1] 260 days.
[2] 304 days.
[3] 312 days.
[4] 711 days.
[5] 854 days.

## Question 3

If the nominal interest rate per year is 16,5% per annum compounded at the end of every second month, then the effective interest rate equals

[1] 14,527%.
[2] 16,181%.
[3] 16,677%.
[4] 17,677%.
[5] 18%.

## Question 4

Sugar borrows money from the bank at a discount rate of 16,5% per year. She must pay the bank R30 000 in eight months from *now*. The amount of money she receives from the bank now equals

[1] R26 700,00.
[2] R27 027,03.
[3] R33 300,00.
[4] R33 463,26.
[5] R33 707,87.

## Question 5

Moses needs R10 500 in ten months' time to buy herself a new lens for her camera. Two months ago he deposited R9 000 in a savings account at a simple interest rate of 11,5% per year. How much money will Moses still need to buy the lens ten months from now?

[1] R229,50
[2] R408,67
[3] R465,00
[4] R637,50
[5] None of the above.

## Question 6

Chappie borrowed money on 31 August and agreed to pay back the loan on 2 November of the same year. If the discount rate is 18% per year and he received R5 000 on 31 August, what is the value of the loan that Chappie has to pay the bank on 2 November?

[1] R5 000,00
[2] R5 160,32
[3] R4 844,66
[4] R5 155,34
[5] None of the above.

## Question 7

Jacob invests an amount of money in an account earning 13,88% interest per year, compounded weekly. After five years, this amount has accumulated to R50 000. The amount that was invested initially equals

[1] R15 300,00.
[2] R25 001,79.
[3] R26 105,54.
[4] R29 515,94.
[5] R34 700,00.

## Question 8

If R400 accumulates to R460 at a simple interest rate of 8% per year, then the length of time of the investment is given by the expression

[1] $\left(\frac{460}{400} - 1\right) \times \frac{1}{0{,}08}$
[2] $\left(\frac{460}{400} - 1\right) \times 0{,}08$
[3] $\left(\frac{460}{400} + 1\right) \times \frac{1}{0{,}08}$
[4] $\left(\frac{460}{400} + 1\right) \times 0{,}08$
[5] $\left(1 - \frac{460}{400}\right) \times 0{,}08$

## Question 9

An effective rate of 29,61% corresponds to a nominal rate, compounded weekly, of

[1] 26%.
[2] 29,53%.
[3] 29,61%.
[4] 34,35%.
[5] none of the above

## Question 10

Karin won R165 000 and decided to deposit 65% of this amount in an account earning 8,25% interest, compounded every four months. The accumulated amount after five years equals

[1] R151 490,63.
[2] R161 110,84.
[3] R161 332,31.
[4] R247 862,83.
[5] R248 203,55.

## Question 11

An investment of R20 000 accumulated to R45 200. If the applicable simple interest rate is 12%, then the time under consideration is

[1] 3,25 years.
[2] 4,65 years.
[3] 7,19 years.
[4] 10,50 years.
[5] 15,12 years.

## Question 12

Mary invested R40 000 in order to have R56 000 available in 30 months' time. The yearly rate, compounded semi-annually, equals

[1] 7,205%.
[2] 8%.
[3] 13,92%.
[4] 14,41%.
[5] 16%.

## Question 13

An amount of R4 317,26 was borrowed on 5 May at a simple interest rate of 15% per year. The loan will be worth R4 500 on

[1] 12 August.
[2] 16 August.
[3] 21 August.
[4] 9 October.
[5] none of the above.

## Question 14

Dieter owes Paul R3 000 due 10 months from now, and R25 000 due 32 months from now. Dieter asks Paul if he can discharge his obligations by two equal payments: one now and the other one 28 months from now. Paul agrees on condition that a simple interest of 14,75% per year. The amount that Dieter will pay Paul 28 months from now will equal approximately

[1] R11 455.

[2] R11 511.

[3] R11 728.

[4] R14 000.

[5] R20 000.

## Question 15

The accumulated amount that Sipho will receive after 52 months if he deposits R7 300 into an account where money is worth 9,7% interest per year compounded every second month equals

[1] R7 825,36.

[2] R8 388,53.

[3] R10 368,43.

[4] R11 076,73.

[5] none of the above.

## 3.2 Compulsory Assignment 02

## Due Date: 21 August 2015

## Unique Number: 589252

**Please work through:**
- Guide: Chapter 3 – 4.2
- Tutorial letter 101: Self evaluation exercises week 2 and 3

before answering the assignment questions.

This assignment **MUST** be submitted on a mark-reading sheet. No written assignments will be accepted.

## Question 1

Fatima needs R150 000 on 17 November 2015 to upgrade her deli. On 8 January 2015 she deposited an amount into an account earning 13,45% interest compounded monthly and being credited on the 1st of every month. If fractional compounding is used for the full term then the amount that Fatima deposited on 8 January 2015 equalled

[1] R133 662,53.

[2] R133 708,72.

[3] R133 745,47.

[4] R168 230,00.

[5] R168 276,24.

## Question 2

The continuous compounding rate is 13,974%. The equivalent nominal rate, compounded every three months, equals

[1] 13,658%.

[2] 13,735%.

[3] 14,221%.

[4] 14,305%.

[5] 14,997%.

## Question 3

To pay off a loan of R7 000 due now and a loan of R2 000 due in 14 months' time, Lucky agrees to make three payments in two, five and ten months' time respectively. The second payment is to be double the first and the third payment is to be triple the first. What is the size of the payment at month five if interest is calculated at 16% per year, compounded monthly?

[1] R1 582,43

[2] R3 000,00

[3] R3 164,86

[4] R4 500,00

[5] R4 627,26

## Question 4

If R35 000 accumulates to R48 320 at a continuous compounded rate of 8,6%, then the term under consideration is

[1] 2,77 years.

[2] 3,75 years.

[3] 3,91 years.

[4] 4,43 years.

[5] 6,23 years.

## Question 5

Quarterly deposits of R400 each are made into a bank account earning interest at 16% per year compounded quarterly. The approximated time (in number of quarters) that it will take for the account to accumulate to R40 000 is given by

[1] 41 quarters.

[2] 28 quarters.

[3] 2 quarters.

[4] 40 quarters.

[5] 12 quarters.

## Question 6

Six years ago Trevor lent Maria R150 000 on the condition that she would pay him back in nine years time. The applicable interest rate is 15,5% per year, compounded monthly. Maria also owes Trevor another amount of R250 000 that she has to pay back six years from *now* for a loan that earned interest at 16,4% per year, compounded half-yearly. Maria asks Trevor if she can settle both her debts three years from *now*. The total amount that Maria will have to pay Trevor three years from *now* equals

[1] R400 000,00.

[2] R475 017,72.

[3] R488 092,15.

[4] R755 667,10.

[5] R777 202,69.

## Question 7

Mike deposits R1 500 at the end of every month into an account that earns 12,5% interest per year, compounded monthly. After two years, he stops making these monthly contributions because the interest rate changes to 15% per year, compounded every two months. If no withdrawals or deposits are made for four years the balance in the account will equal

[1] R40 660,72.

[2] R62 224,96.

[3] R65 114,13.

[4] R72 517,49.

[5] none of the above.

## Question 8

The Treasure Fund was created for Long John after he lost his leg in a battle with pirates. The fund has undertaken to pay him R1 200 000. Long John prefers to receive three payments: one three years from now; one twice the size of the first payment six years from now, and one four times the size of the first payment ten years from now. The amount of money to the nearest rand that John can expect to receive six years from now if the interest rate changes to 8,6% per year, compounded quarterly, will equal

[1] R325 803.

[2] R333 235.

[3] R651 606.

[4] R666 470.

[5] R1 303 212.

## Question 9

An amount of money accumulates to R45 946 at a continuous compounding rate of 8% after 57 months. The original amount equals

[1] R28 486,52.
[2] R31 420,70.
[3] R31 460,34.
[4] R33 294,20.
[5] R36 756,80.

## Question 10

If $R = \dfrac{x}{s_{\overline{18}|0,15}}$ is simplified, then the equation becomes

[1] $R = x$
[2] $R = 0{,}01319x$
[3] $R = 0{,}16319x$
[4] $R = 6{,}12797x$
[5] $R = 75{,}83638x$

## Question 11

An interest rate of 14,9% per year, compounded quarterly, is equivalent to a weekly compounded interest rate of

[1] 14,65%.
[2] 14,88%.
[3] 15,16%.
[4] 19,02%.
[5] none of the above.

## Question 12

Superman decides that he would like to buy his lovely wife Superwoman a new car when she turns 30 in six years' time. He deposits R6 000 each month into an account earning 8,94% interest per year, compounded monthly. The amount that Superman will have available six years from now will equal

[1] R333 412.
[2] R335 896.
[3] R432 000.
[4] R568 948.
[5] R573 187.

## Question 13

A nominal interest rate of 19,4% per year, compounded monthly, is equivalent to a continuous compounding rate of

[1] 19,4%.

[2] 19,558%.

[3] 21,22%.

[4] 21,41%.

[5] none of the above.

## Question 14

Three years ago Moodley borrowed R10 000 from Linda on the condition that he should pay her back two years from now. He will also pay her R6 000 five years from now. The applicable interest rate for both transactions is 13,75% per year, compounded half yearly. After considering his payback schedule Moodley asks Linda if he can pay her R9 000 now and the rest in four years' time. She agrees on the condition that the new agreement will run from now and that an interest rate of 16,28% per year, compounded monthly, will be applicable from now. The amount that Moodley will have to pay Linda four years from now equals

[1] R8 988,38.

[2] R13 366,24.

[3] R15 245,21.

[4] R17 162,98.

[5] R23 430,38.

## Question 15

A loan will be paid back by means of payments of R25 000 each every second month for six years. An interest rate of 7,5% per year compounded every two months will be applicable. The present value of the loan equals

[1] R238 067,35.

[2] R400 738,72.

[3] R721 181,68.

[4] R900 000,00.

[5] R1 127 887,64.

## 3.3 Compulsory Assignment 03

## Due Date: 4 September 2015

## Unique Number: 570670

**Please work through:**

- Guide: Chapter 4 – 5
- Tutorial letter 101: Self evaluation exercises week 3

before answering the assignment questions.

This assignment **MUST** be submitted on a mark-reading sheet. No written assignments will be accepted.

## Question 1

Monthly payments of R1 200 are made into an account earning 7,75% interest per year, compounded quarterly. The accumulated amount rounded to the nearest hundred rand after 10 years equals

[1] R144 000.

[2] R215 900.

[3] R216 500.

[4] R291 100.

[5] none of the above.

## Question 2

On her 40th birthday Susan decides that she will go for a facelift when she turns 50. She estimates that it will cost her R48 000 when she turns 50. She starts *saving immediately* each month paying an amount into an account earning 8,58% interest per year, compounded monthly. The monthly payment equals

[1] R252,18.

[2] R253,99.

[3] R255,80.

[4] R592,95.

[5] R597,19.

## Question 3

Thulani borrowed an amount of money from his father to open the Generous Pawpaw Shop. The loan will be paid back by means of payments of R25 000 each every second month for six years. An interest rate of 7,5% per year compounded every two months will be applicable. The amount of the loan equals

[1] R238 067,35.
[2] R400 738,72.
[3] R721 181,68.
[4] R900 000,00.
[5] R1 127 887,64.

## Question 4

If $S = (1+i)^n P$ and $P = R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$ then $S$ also equals

[1] $Rs_{\overline{n}|i}$.
[2] $(1+i)Rs_{\overline{n}|i}$.
[3] $(1+i)Ra_{\overline{n}|i}$.
[4] $(1+i)\left(1+\frac{j_m}{m}\right)^{tm}$.
[5] $(1+i)^n$.

***Questions 5 and 6 relate to the following situation:***
*The Gliding Fund must pay Weaver R18 000 every three months indefinitely as compensation. Money is worth 11,4% per year, compounded quarterly*

## Question 5

The opening balance of this fund approximately equals

[1] R157 895.
[2] R474 536.
[3] R631 579.
[4] R1 105 351.
[5] none of the above.

## Question 6

Weaver asks to reschedule the compensation in three payments, the first payment *now*, the second payment twice the size of the first payment four years from *now*, and the third payment three times the size of the first payment nine years from *now*. The Gliding Fund agrees on condition that the interest rate changes to 10,95% per year, compounded monthly. The amount to the nearest hundred rand that Weaver can expect to receive four years from *now* equals

[1] R184 800.

[2] R369 600.

[3] R557 510.

[4] R864 000.

[5] none of the above.

## Question 7

The present value of an annuity is R62 543,42. the time under consideration is 10 years and the applicable interest rate 16% per year, compounded monthly. The future value of this annuity equals

[1] R162 612,89.

[2] R306 521,61.

[3] R581 913,00.

[4] R625 434,20.

[5] R1 333 517,61.

***Questions 8 and 9 relate to the following situation:***
*The last six payments of a loan are reflected in the following amortisation schedule.*

| ***Month*** | ***Outstanding principal at the beginning of the month*** | ***Interest due at the end of the month*** | ***Payment*** | ***Principal repaid*** |
|---|---|---|---|---|
| *175* | *49 694,10* | *422,40* | *8 530,49* | *8 108,09* |
| *176* | *A* | *353,48* | *8 530,49* | *F* |
| *177* | *33 409,01* | *C* | *8 530,49* | *G* |
| *178* | *25 162,50* | *D* | *8 530,49* | *8 316,61* |
| *179* | *B* | *143,19* | *8 530,49* | *H* |
| *180* | *8 458,59* | *E* | *8 530,49* | *I* |

## Question 8

The applicable interest rate per year (compounded monthly) is

[1] 4,95%.
[2] 5,2%.
[3] 8,5%.
[4] 10,2%.
[5] 12,70%.

## Question 9

The value of A equals

[1] R40 540,45.
[2] R40 810,13.
[3] R41 163,61.
[4] R41 586,01.
[5] R42 652,45.

## Question 10

Trivina took out an endowment policy. The first annual payment was $R_x$, whereafter it increased yearly by R1 700. After 20 years the policy paid out R1 005 962. The applicable yearly interest rate is 10%. The value of $x$ equals approximately

[1] R564.
[2] R6 500.
[3] R11 816.
[4] R17 564.
[5] R23 500.

***Questions 11 and 12 relate to the following situation:***
*Carin wants to open the Straight Hair Salon and asks her Aunt Wilma if she will lend her the money.She also indicates that she will only be able to start paying her back after five years, at which time she will then pay R35 000 at the end of every four months, for four years. Aunt Wilma agrees, on the condition that her money must earn 12,2% interest per year, compounded every four months.*

## Question 11

The present value of Carin's debt when she starts paying back Aunt Wilma will equal

[1] R387 335,79.
[2] R420 000,00.
[3] R437 962,78.
[4] R518 312,63.
[5] none of the above.

## Question 12

The amount that Aunt Wilma lends Carin equals

[1] R179 950,37.
[2] R202 809,21.
[3] R213 016,36.
[4] R240 147,06.
[5] R284 205,10.

## Question 13

Ester is getting married 16 months from *now* and estimates that she will need R45 000 for new clothes for her honeymoon. She starts to save *immediately* by depositing R2 200 every month for 16 months into an account earning 15,4% interest per year, compounded monthly. The amount that she still needs when she gets married is denoted by

[1] $X = 45\,000 - 2\,200a_{\overline{16}|0{,}154\div 12}$
[2] $X = 45\,000 - 2\,200s_{\overline{16}|0{,}154\div 12}$
[3] $X = 45\,000 - (1+i)2\,200a_{\overline{16}|0{,}154\div 12}$
[4] $X = 45\,000 - (1+i)2\,200s_{\overline{16}|0{,}154\div 12}$
[5] $X = 45\,000 - 2\,200s_{\overline{17}|0{,}154\div 12}$

***Questions 14 and 15 relate to the following situation:***
*John buys a house and makes a down payment of 16% of the price of the house. For the remaining amount, he manages to secure a loan at an interest rate of 12,05% per year, compounded monthly, for a period of 20 years. His monthly payment is R18 556,84.*

## Question 14

The size of the loan (to the nearest rand) equals

[1] R1 333 820.
[2] R1 680 000.
[3] R2 167 317.
[4] R4 453 642.
[5] none of the above.

## Question 15

The down payment equals

[1] R213 411.
[2] R254 061.
[3] R268 800.
[4] R320 000.
[5] R346 771.

## 3.4 Compulsory Assignment 04

## Due Date: 23 September 2015

## Unique Number: 570687

**Please work through:**

- Guide: Chapter 6, 7 and 8
- Tutorial letter 101: Self evaluation exercises week 4, 5, and 6

before answering the assignment questions.

This assignment **MUST** be submitted on a mark-reading sheet. No written assignments will be accepted.

***Questions 1 and 2 relate to the following situation:***
*The following table represents the cash flows (in Rands) of a company.*

| *Year* | *Cash flows* |
|---|---|
| 3 | 40 000 |
| 5 | −70 000 |
| 7 | −80 000 |
| 9 | 10 000 |
| 11 | 100 000 |

*Money can be borrowed at 14,25% per year and investments can earn 8,27% per year.*

## Question 1

The present value of the cash outflows equals

[1] R67 444,26.
[2] R92 920,21.
[3] R122 557,45.
[4] R132 898,63.
[5] none of the above.

## Question 2

If the future value of the cash inflows is R187 253,00, then the MIRR equals

[1] 8,85%.
[2] 9,73%.
[3] 13,62%.
[4] 22,66%.
[5] none of the above.

***Questions 3, 4 and 5 relate to the following situation:***
*An investment with an initial outlay of R500 000 generates five successive annual cash inflows of R75 000, R190 000, R40 000, R150 000 and R180 000 respectively. The cost of capital K is 10% per annum.*

## Question 3

The internal rate of return (IRR) equals

[1] 7,78%.
[2] 9,48%.
[3] 21,3%.
[4] 27,0%.
[5] none of the above.

## Question 4

The nett present value (NPV) approximately equals

[1] 74 500.
[2] 135 000.
[3] −135 000.
[4] −30 523.
[5] none of the above.

## Question 5

The profitability index (PI) equals

[1] 0,27.
[2] 0,34369.
[3] 0,65631.
[4] 0,93895.
[5] none of the above.

## Question 6

The positive square root of the ........... is called the standard deviation. The missing word is

[1] coefficient.
[2] correlation.
[3] coefficient of determination.
[4] sample.
[5] variance.

***Questions 7, 8 and 9 relate to the following situation:***
*The following table supplies data of the inflation rate and the corresponding prime lending rate during the same time period.*

| ***Inflation rate (%)(x)*** | ***Prime lending rate (%)(y)*** |
|---|---|
| *3,3* | *5,2* |
| *6,2* | *8,0* |
| *11,0* | *10,8* |
| *9,1* | *7,9* |
| *5,8* | *6,8* |
| *6,5* | *6,9* |
| *7,6* | *9,0* |

## Question 7

The linear relationship between the inflation rate and the prime lending rate can be represented by the regression line

[1] $y = 3{,}17477 + 0{,}65407x$.

[2] $y = 0{,}65407 + 3{,}17477x$.

[3] $y = -2{,}76656 + 1{,}26128x$.

[4] $y = 1{,}26128 - 2{,}76656x$.

[5] $y = 2{,}28372 + 0{,}88372x$.

## Question 8

The correlation coefficient equals

[1] −0,908.

[2] +0,495

[3] +0,546.

[4] +0,908.

[5] none of the above.

## Question 9

The coefficient of determination equals

[1] 0,24503.

[2] 0,29812.

[3] 0,82446.

[4] can not be solved.

[5] none of the above.

***Questions 10, 11 and 12 relate to the following situation:***

*Consider BOND ABC*

| | |
|---|---|
| *Coupon rate:* | *9,4% per year* |
| *Yield to maturity:* | *10,6% per year* |
| *Settlement date:* | *16 July 2015* |
| *Maturity date:* | *9 October 2041* |

## Question 10

The all-in price will equal

[1] R82,63215%.

[2] R87,33105%.

[3] R89,45121%.

[4] R91,91965%.

[5] R94,15121%.

## Question 11

The accrued interest equals

[1] R2,18904%.

[2] R2,51694%.

[3] R2,52384%.

[4] R4,36612%.

[5] none of the above

## Question 12

The clean price equals

[1] R84,80721%.

[2] R86,92737%.

[3] R87,26217%.

[4] R89,39581%.

[5] R89,73061%.

## Question 13

The settlement date of Bond E528 is 23 May 2015.
The price on the coupon date that follows the settlement date is denoted by

$$\begin{aligned} P &= \frac{12{,}4}{2} a_{\overline{n}|0{,}108 \div 2} + 10{,}41966. \\ \text{The all-in price} &= 119{,}47116 \left(1 + \frac{0{,}108}{2}\right)^{-20/182}. \end{aligned}$$

The maturity date of Bond E528 is

[1] 12 June 2036
[2] 25 November 2036
[3] 12 December 2036
[4] 23 November 2043
[5] 12 December 2043

## Question 14

The equation for the present value of stock CCC on 17 December 2015 is given by

$$107{,}55174 = da_{\overline{29}|\frac{0{,}135}{2}} + 100\left(1 + \frac{0{,}135}{2}\right)^{-29}$$

The half-yearly coupon rate $d$ is equal to

[1] 6,75%.
[2] 7,35%.
[3] 8,55%.
[4] 14,70%.
[5] none of the above

## Question 15

A .................. is a representative group or subset of the population. It is the portion of the population that is selected for analysis. The missing word is

[1] deviation.
[2] mean.
[3] regression.
[4] sample.
[5] variance.

# Chapter 4

# Self-evaluation exercises and solutions

## 4.1 Self-evaluation exercises

### 4.1.1 Self-evaluation exercise 1: Week 1

**Content: Chapter 2**

1. At what simple interest rate must R7 000 be invested for a period of nine months to accumulate to R7 630?
2. Michael needs R1 200 urgently. Peter is prepared to lend him the money on condition that he pays him R1 295 four months from now. What simple interest rate is Peter earning on this transaction?
3. How large an amount must be invested on 3 March at a simple interest rate of 12% per year, to accumulate to R612 on 2 May of the same year?
4. Mputle Maputle borrows R1 500 on 10 March. How much interest is he paying if he has to pay back the loan on 2 July of the same year and a simple interest rate of 21,5% per year is charged on his loan?
5. How long does it take for an amount of R3 500 to accumulate to R3 755 if simple interest of 18% per year is earned?
6. When will R2 000 invested on 6 March at a simple interest rate of 15% accumulate to R2 240?
7. Sipho borrows money on 31 August and signs an agreement stating that he will pay back the loan on 2 November of the same year. If the discount rate is 18% per year, and he receives R5 000 on 31 August of the same year, what is the face value of the agreement?
8. What is the equivalent simple interest rate of the previous question?
9. John Drake needs to make the following payments against a loan on his lorry:

   R10 000 after six months

   R20 000 after one year

   R40 000 after two years. As a result of drought on his farm, John Drake could not pay the first two payments. After 18 months, John Drake has a record harvest and immediately makes a down payment of R50 000 against his loan. What single size payment should he make two years from now to settle his debt if simple interest of 17% per year is charged on all amounts?

***The solutions to these questions are to be found on p 81 of this tutorial letter.***

**If you need extra exercises you can do the additional exercises on p 113 of this tutorial letter.**

### 4.1.2 Self-evaluation exercise 2: Week 2

**Content: Chapter 3**

1. A deceased had R1 400 in a Swiss bank at the time of his death. Twenty years later the beneficiary of his will learns about the investment and claims it. How much must be paid to the beneficiary if the investment earned 12,5% interest per year compounded half yearly, over the 20 years?

2. How long will it take to save R25 000 for a trip to Europe if you deposit R15 000 into a savings account now, earning interest of 12% per year, compounded monthly? The initial R15 000 will also be used to pay for the trip.

3. An amount of R1 000 has accumulated to R1 500 after two and a half years. Calculate the interest rate per year if interest is compounded monthly.

4. Willem Grobler invests R12 000 at an interest rate of 10,5% per year, compounded monthly. After four years and three months Willem withdraws R15 000 and invests it at 12% interest per year, compounded quarterly. What is the total accumulated amount of both accounts after six years?

5. An investment company invests its funds at an interest rate of 12% per year, compounded monthly. What is the effective interest rate that the company earns?

6. On 3 January Granddad Steyn deposited R2 500 into a savings account for his grandchild who was born on 26 July the previous year. Interest is credited at 18,75% per year on the first day of every month.

   (a) How much money does his grandchild receive on his first birthday if simple interest is used for odd periods and compound interest for the rest of the term?

   (b) How much does he receive if fractional compounding is used for the full period?

7. Joseph would like to buy a lawnmower. He has three options when it comes to borrowing the R3 750 from the bank:

   17,5% per year, compounded semi-annually

   16% per year, compounded quarterly

   16% per year, compounded monthly

   Make use of continuous compounding rates to decide which option Joseph should take.

8. A wholesaler has to pay the following amounts to a manufacturer:

   R200 000 after three months

   R300 000 after one year and three months

   R400 000 after two years. He would like to reschedule his three payments by making only two payments. The first payment will be made at the end of the first year and the second, twice the size of the first, at the end of 21 months. If interest is calculated at 18,75% compounded quarterly, what is the size of each payment? Use month 21 as the comparison date.

***The solutions to these questions are to be found on p 84 of this tutorial letter.***

**Additional exercises are to be found on p 116 of this tutorial letter.**

### 4.1.3 Self-evaluation exercise 3: Week 3

**Content: Chapters 4 and 5**

1. Mr White opened an annuity fund and deposited R3 000 into it. Thereafter he deposited R500 at the end of each month into this fund. Mr Jones, on the other hand, opened his annuity fund by depositing R5 000 into it. He thereafter deposited R300 at the end of each month into this fund. After 15 years of making deposits into the annuity funds, the two friends decided to compare their investments. Calculate the amounts in the two funds after the 15 years if a 12,5% interest rate compounded monthly is applicable.

2. An insurance agent offers services to clients who are concerned about their personal financial planning for retirement. To explain the advantages of an early start to investing, she points out that if the 25-year-old John starts to save R2 000 at the *beginning* of each year for 10 years (and makes no further contributions) John *will earn more* than Jane who waits 10 years and then save R2 000 at the *beginning* of each year until retirement at an age of 65 (a total of 30 contributions). Find the net earnings (compound amount minus total contributions) of John and Jane at age 65. An annual interest rate of 7% is applicable and the deposits are made at the *beginning* of each year.

3. A poor student has to repay his study loan of R80 000 which he received when he enrolled for the first time, in equal monthly payments. The repayment will start after he has finished his education in four years' time. Determine his monthly payments if he wants to repay his debt in five years (after studying for four years) and if interest is calculated at 15% per year, compounded monthly.

4. Determine approximately the accumulated value of R500 payments made every month for a period of eight years if interest is compounded semi-annually at 13,5% per year.

5. Vusi and Vivian want to purchase a new flat and feel that they can afford a mortgage payment of R2 500 a month. They are able to pay R100 000 deposit and obtained a 20-year, 14,75% per annum mortgage bond (compounded monthly).

   (a) How much can they afford to spend on a flat?

   (b) After eight years Vusi receives a huge promotion and decides to buy a much bigger flat. What equity do they have in their present flat after eight years?

6. Maitland Engineering wants to replace machinery after seven years. The company has been investing a sum of R5 000 in a sinking fund every six months for this purpose. The investment has been earning interest at the rate of 16% per year compounded semi-annually. Determine the balance of the fund after seven years.

7. A medical practitioner of Gauteng buys a holiday home on the west coast with a cash deposit of R200 000 plus monthly payments of R10 000 for a period of five years. Interest is 12% per annum compounded monthly. Calculate the cash price of the house.

***The solutions to these questions are to be found on p 89 of this tutorial letter.***

**Additional exercises are to be found on p 119 of this tutorial letter.**

### 4.1.4 Self-evaluation exercise 4: Week 4

**Content: Chapter 6**

1. An investor has to decide between two alternative projects: A and B. The initial investment outlays and the cash inflows of each of the projects are listed in the table below. If the capital cost is 19% per year, use the internal rate of return, the net present value and the profitability index respectively to advise him with regard to the two projects. All funds are in R1 000s.

| **Year** | **Project A INVESTMENT 800** | **Project B INVESTMENT 750** |
|---|---|---|
| | Cash inflows | Cash inflows |
| 1 | 400 | 200 |
| 2 | 300 | 500 |
| 3 | 350 | 450 |

2. Denise and Jan want to start a business. They can choose between two options: a shoe shop and a CD shop. The two shops require the following cash flows (in R'000):

| **Year** | **Shoe shop** | **CD shop** |
|---|---|---|
| 0 | −100 | −400 |
| 1 | 50 | 75 |
| 2 | −50 | 100 |
| 3 | 75 | 400 |

What advice would you give them if you consider the MIRR criterion with an interest rate of 16,5% per year applicable for the cash outflows and an interest rate of 19% per year applicable for the cash inflows?

***The solutions to these questions are to be found on p 92 of this tutorial letter.***

**Additional exercises are to be found on p 121 of this tutorial letter.**

### 4.1.5 Self-evaluation exercise 5: Week 5

**Content: Chapter 7**

1. Consider the following bond

| XYZ: | |
|---|---|
| Coupon rate (half yearly) | 16,5% per annum |
| Redemption date | 1 June 2029 |
| Yield to maturity | 14,2% per annum |
| Settlement date | 14 April 2013 |

   Calculate the all-in price, the accrued interest and the clean price on the settlement date.

2. Calculate the all-in price, the accrued interest and the clean price for the bond in question 1 on the following settlement date: 25 May 2013.

***The solutions to these questions are to be found on p 93 of this tutorial letter.***

**Additional exercises are to be found on p 122 of this tutorial letter.**

### 4.1.6 Self-evaluation exercise 6: Week 6

**Content: Chapter 8**

1. In 2011 the average hourly wage of construction workers was R28,41 . Miners made R27,50 per hour on average and production workers in manufacturing made R26,65. There were 6,2 million production workers in manufacturing, 1 million miners and 4,4 million construction workers in 2011. What was the average hourly wage for workers in all three fields?

2. Assume you are a member of a scholarship committee and are trying to decide between two students who are competing for one award. Your decision must be made on the basis of the grades the students earned in courses taken during the first semester of their third year. The grades are shown below:

| | **Student $A$** | **Student $B$** |
|---|---|---|
| First course | 81 | 83 |
| Second course | 88 | 93 |
| Third course | 83 | 76 |

   (a) If you make the award on the basis of the arithmetic mean, which student would you select?

   (b) If you select the student who is most consistent, which student would you select? Justify your choice.

3. In a regression survey of interest rates and investments made over ten years the following results were observed.

| **Year** | **Yearly investment (Thousands of rands)** | **Average interest (Percent)** |
|---|---|---|
| 1 | 1 060 | 13,8 |
| 2 | 940 | 14,5 |
| 3 | 920 | 13,7 |
| 4 | 1 110 | 14,7 |
| 5 | 1 550 | 14,8 |
| 6 | 1 850 | 15,5 |
| 7 | 2 070 | 16,2 |
| 8 | 2 030 | 15,9 |
| 9 | 1 780 | 14,9 |
| 10 | 1 420 | 15,1 |

   (a) Plot the data on a graph with average interest rate on the horizontal axis and yearly investment on the vertical axis. Comment on the graph.

   (b) Calculate the coefficient of correlation between average interest rate and yearly investment.

   (c) Calculate and interpret the coefficient of determination, $r^2$.

   (d) Develop an effective prediction equation for yearly investment.

   (e) Can we forecast yearly investment if the average interest rate is 16,5?

***The solutions to these questions are to be found on p 95 of this tutorial letter.***

### 4.1.7 Self-evaluation exercise 7: Week 7

**Typical examination questions**

1. On his ninth birthday on 21 February Little John received R420. His parents immediately invested the money in an account that earns 7,5% simple interest. The amount of money that can be withdrawn on 5 June for the same year equals

   [1] R411,21.

   [2] R428,89.

   [3] R428,98.

   [4] R429,07.

   [5] none of the above.

2. An interest rate of 16,4% compounded quarterly is equivalent to a weekly compounded interest of

   [1] 16,073%.

   [2] 16,098%.

   [3] 16,714%.

   [4] 16,741%.

   [5] none of the above.

3. On Dandy Darrell's 21st birthday he notices that he is going bald. He decides that he will go for a hair implant when he turns 30. He estimates that the implant will cost him R12 500. He starts saving immediately by paying an amount monthly into an account earning 9,09% interest compounded monthly. The monthly payment that Dandy Darrell makes into the account equals

   [1] R64,27.

   [2] R74,63.

   [3] R75,20.

   [4] R115,75.

   [5] none of the above.

4. At an interest rate of 14,9% per year compounded quarterly, R1 000 invested monthly for 12 years will accumulate to

   [1] R66 914,38.

   [2] R385 478,48.

   [3] R390 233,94.

   [4] R395 600,34.

   [5] none of the above.

***Questions 5, 6 and 7 refer to the following bond:***

*Consider Bond XYZ.*

| | |
|---|---|
| *Coupon* | 11,59% |
| *Yield to maturity* | 9,46% |
| *Settlement date* | 18 *April* 2013 |
| *Date to maturity* | 15 *November* 2038 |
| *Nominal value* | R750 000 |

5. The all-in price on the settlement date equals

   [1] R119,45625%.

   [2] R119,55642%.

   [3] R119,56986%.

   [4] R125,31160%.

   [5] none of the above.

6. The accrued interest equals

   [1] −R1,72890%.

   [2] −R0,86445%.

   [3] −R0,85734%.

   [4] R4,89003%.

   [5] none of the above.

7. The clean price to the nearest rand equals

   [1] R750 000.

   [2] R896 732.

   [3] R901 008

   [4] R903 162

   [5] none of the above.

8. If the NPV of the Smell Nice Shop is R1 255 and the profitability index is 1,083, then the initial investment in the shop equals

   [1] R1 158,82.

   [2] R1 255,00.

   [3] R10 416,50.

   [4] R15 120,48.

   [5] none of the above.

***Questions 9 and 10 refer to the following situation:***

*Three years ago Malcolm borrowed R7 500 from Sarah on the condition that he would pay her back in five years' time. Interest is calculated at 13,5% per year every three months. Nine months ago he also borrowed R2 500 from her at an interest rate of 15,7% per year compounded monthly payable two years from now.*

9. The total amount that Malcolm owes Sarah two years from now will equal

   [1] R10 000,00.

   [2] R16 141,88.

   [3] R18 353,73.

   [4] R18 406,16.

   [5] none of the above.

10. After seeing what he will owe Sarah two years from now, Malcolm asks Sarah if he can reschedule his debt by paying R9 000 now and the rest four years from now. Sarah agrees on condition that the new agreement will be subject to an interest of 11% per year compounded half-yearly. The amount that Malcolm must pay Sarah in four years' time will equal

    [1] R7 642,92.

    [2] R8 924,87.

    [3] R8 989,83.

    [4] R9 406,16.

    [5] none of the above.

11. In order to settle a debt Trevor agrees to pay Jill R4 500 every six months for six years plus an additional R10 500 at the end of the six years. The present value of Trevor's debt at the beginning of the agreement period if money is worth 9,15% per annum compounded half-yearly equals

    [1] R40 858,13.

    [2] R46 996,52.

    [3] R51 358,13.

    [4] R80 389,66.

    [5] none of the above.

12. If the MIRR for a project lasting eight years is 10,81% and the present value of the outflows equals R291 930,00, then the future value of the cash inflows will approximately equal

    [1] R128 400,00.

    [2] R263 450,00.

    [3] R323 500,00.

    [4] R663 600,00.

    [5] none of the above.

***Questions 13 and 14 refers to the following situation:***

*Just Water Plumbing agreed to establish the Spanner Fund from which they will pay Spanner R2 500 per month indefinitely as compensation for injuries he sustained while working at the No Water Dam. The money is worth 14% per year compounded monthly.*

13. The opening balance of this fund equals

    [1] R161 013,55.

    [2] R214 285,71.

    [3] R250 000,00.

    [4] R448 026,10.

    [5] none of the above.

14. Spanner asks whether Just Water Plumbing could reschedule the compensation into two payments: One payment five years from now when his son will go to university and the other payment exactly the same size as the first one ten years from now when his daughter will attend a beauty school. They agree to this on condition that the interest rate stays the same. The present value of the payments will equal

    [1] R80 506,78.

    [2] R107 142,86.

    [3] R224 013,05.

    [4] R286 783,06.

    [5] none of the above.

***Questions 15 and 16 refer to the following situation:***

*A study was undertaken at eight garages to determine how the resale value of a car is affected by its age. The following data was obtained:*

| **Garage** | **Age of car (in years) ($x$)** | **Re-sale value (R) ($y$)** |
|---|---|---|
| 1 | 1 | 41 250 |
| 2 | 6 | 10 250 |
| 3 | 4 | 24 310 |
| 4 | 2 | 38 720 |
| 5 | 5 | 8 740 |
| 6 | 4 | 26 110 |
| 7 | 1 | 38 650 |
| 8 | 2 | 36 200 |

The garage manager suspects a linear relationship between the two variables. Fit a curve of the form $y = a + bx$ to the data.

15. The equation is equal to

    [1] $y = 7{,}0417 - 0{,}001x$.

    [2] $y = 0{,}001 + 7{,}0417x$.

    [3] $y = 48\,644{,}17 - 6\,596{,}93x$.

    [4] $y = 6\,596{,}93 - 48\,644{,}17x$.

    [5] none of the above.

16. The correlation coefficient equals

    [1] 0,0000.

    [2] −0,9601.

    [3] 0,8450.

    [4] 1,0000.

    [5] none of the above.

***The solutions to these questions are to be found on p 98 of this tutorial letter.***

**Additional exercises are to be found on p 123 of this tutorial letter.**

### 4.1.8 Self-evaluation exercise 8: Week 8

**Typical examination questions**

## Question 1

James borrows R2 000 at a simple interest rate of 8% per annum. The amount that he owes at the beginning of the eighth year equals

[1] R1 120,00

[2] R3 120,00

[3] R3 280,00

[4] R3 427,65

[5] none of the above

## Question 2

After making a down payment of R5 000 on a boat, Mr Clark also had to pay an additional R700 per month for it for three years. Interest was charged at 14,5% per year compounded monthly on the unpaid balance. The original price of the boat equals

[1] R6 611,60

[2] R20 336,44

[3] R25 336,44

[4] R36 337,23

[5] none of the above

## Question 3

If R100 accumulates to R115 at a simple interest rate of 8% per annum, then the length of time (in years) of the investment is given by the expression

[1] $\frac{1}{8}\left(\frac{115}{100} - 1\right)$

[2] $\frac{1}{8}\left(\frac{115}{100} + 1\right)$

[3] $\frac{100}{8}\left(\frac{115}{100} - 1\right)$

[4] $\left(\frac{115}{100} + 1\right) \times \frac{1}{0{,}08}$

[5] none of the above

## Question 4

Jonas needs R14 500 to buy a computer. Compunet is prepared to lend him the money on condition that he pays the money back in ten months' time. The amount that he must pay back if a discount rate of 28% is applicable will equal

[1] R11 116,67
[2] R11 756,76
[3] R17 883,33
[4] R18 913,04
[5] none of the above

## Question 5

If the continuous compounding rate for a nominal rate compounded every three months is 11,832%, then the nominal rate equals

[1] 11,66%
[2] 11,832%
[3] 12,01%
[4] 12,07%
[5] 12,56%

## Question 6

If R25 000 accumulates to R32 850 after 39 months, then the continuous compounding rate equals

[1] 7,5%
[2] 7,6%
[3] 8,4%
[4] 8,8%
[5] 9,7%

## Question 7

Nene is making monthly payments towards a loan of R250 000 which she borrowed for six years. An interest rate of 11,8% per year, compounded monthly, is applicable. After 33 months the interest rate changes to 15,6% per year, compounded quarterly. The amount that Nene has paid off when the interest rate changes equals

[1] R93 151,85
[2] R102 009,77
[3] R147 990,23
[4] R156 848,15
[5] R160 432,47

## Question 8

The effective rate for a continuous compounding rate of 17,5% is

[1] 16,13%
[2] 17,5%
[3] 19,12%
[4] 19,13%
[5] none of the above

***Questions 9 and 10 relate to the following situation:***
*Tracy deposited R25 000 into an account earning 9,75% interest per year, compounded quarterly. After five years the interest rate changed to 10% per year, compounded weekly. She then decided to deposit R500 every week into this account.*

## Question 9

The balance in this account after five years equals

[1] R34 750,00
[2] R37 187,50
[3] R41 198,25
[4] R48 780,49
[5] none of the above

## Question 10

After owning this account for nine years Tracy decides to close it. The amount of money that Tracy can expect to withdraw then equals

[1] R127 725,46
[2] R129 000,00
[3] R167 519,80
[4] R168 194,18
[5] R188 074,51

## Question 11

Jonathan bought a 107 cm plasma screen television set. He agrees to *immediately* start to pay R1 403 per month. The term of the agreement is 24 months and the applicable interest rate is 20,124% per year, compounding monthly. The original price of the television set equals

[1] R27 079,22

[2] R27 533,34

[3] R27 995,08

[4] R30 385,36

[5] R33 672,00

## Question 12

A simple interest rate of 9,68% is equivalent to a simple discount rate of 7,5%. The time under consideration is

[1] 2,2 years

[2] 2,4 years

[3] 2,8 years

[4] 3 years

[5] 6 years

## Question 13

The net present value (NPV) of the Beautiful People Shop is R14 983 and the profitability index (PI) is 1,034. The initial investment in the shop approximately equals

[1] R7 366

[2] R14 490

[3] R14 983

[4] R15 492

[5] none of the above

***Questions 14 and 15 relate to the following situation:***
*Charlene intends to open a hairdressing salon and borrows the money from Aunt Amor. Charlene feels that she will only be able to start repaying her debt after five years. Charlene will then pay Aunt Amor R35 000 every six months for four years. Money is worth 17,9% per year compounded semi-annually.*

## Question 14

The present value of Charlene's debt at the time she starts paying back will equal

[1] R69 484,18
[2] R194 079,19
[3] R225 113,21
[4] R280 000,00
[5] R385 298,07

## Question 15

The amount of money that Aunt Amor lends Charlene equals

[1] R82 358,16
[2] R95 527,55
[3] R118 818,94
[4] R163 502,53
[5] R163 741,31

## Question 16

The equation for the present value of Bond ABC on 01/07/2012 is given by

$$P(01/07/2012) = \frac{14{,}7}{2} a_{\overline{29}|0{,}135 \div 2} + 100\left(1 + \frac{0{,}135}{2}\right)^{-29}.$$

The fraction of the half year to be discounted back is

$$f = \frac{74}{181}.$$

The accrued interest equals R4,30932%. The clean price for Bond ABC equal

[1] R100,40824%
[2] R104,71756%
[3] R107,56456%
[4] R111,87388%
[5] R114,90174%

## Question 17

Consider Bond 567

| | |
|---|---|
| Coupon rate: | 12,4% per year |
| Yield to maturity: | 10,8% per year |
| Settlement date: | 23 May 2013 |
| Maturity date: | 12 December 2034 |

The all-in-price equals

[1] R112,61841%

[2] R112,62197%

[3] R113,27116%

[4] R118,78268%

[5] R119,47116%

***Questions 18 and 19 relate to the following situation:***
*Three years ago Daniel borrowed R10 000 from Sarah on condition that he would pay her back in six years' time. Interest is calculated at 14,75% per year, compounded quarterly. Six months ago he also borrowed R17 500 from her at an interest rate of 10,5% per year, compounded monthly. This loan will be paid back three years from now.*

## Question 18

The total amount that Daniel will owe Sarah three years from now is

[1] R33 881,55

[2] R45 656,47

[3] R46 362,95

[4] R47 778,84

[5] R49 078,73

## Question 19

Daniel asks Sarah if he can reschedule his debt, paying R18 000 now and the rest two years from now. Sarah agrees to this on condition that the interest rate for the new agreement starting *now* changes to 13,4% per year compounded half-yearly. The amount that Daniel must pay Sarah two years from now equals

[1] R12 313,49

[2] R20 584,99

[3] R23 330,83

[4] R33 881,55

[5] R43 915,82

## Question 20

The accumulated amount (rounded to the nearest thousand rand) of semi-annual payments of R5 500 for ten years into an account earning 8,9% interest per year compounded monthly, equals

[1] R72 000,00

[2] R83 000,00

[3] R110 000,00

[4] R172 000,00

[5] R173 000,00

## Question 21

The following figures show the profit of a greengrocer for the past five years: R360 000, R550 000, R200 000, R80 000 and R700 000.
The standard deviation of the data equals

[1] R225 424

[2] R252 032

[3] R378 000

[4] R1 890 000

[5] none of the above

## Question 22

Fawzia took out an endowment policy that matures in 20 years. The expected interest rate per year is 10%. Her first payment is R3 600 per year, after which the yearly payments will increase by R360 each year. The amount that she can expect to receive on the maturity date will be

[1] R213 030

[2] R340 380

[3] R412 380

[4] R484 380

[5] none of the above

## Question 23

The following table shows the number of loans approved for different amounts during the second half of 2011.

| Amount of loan in R100 000 ($x$) | Number of loans ($y$) |
|---|---|
| 2 | 45 |
| 3 | 250 |
| 4 | 250 |
| 5 | 175 |
| 6 | 125 |

The regression line equation is

[1] $y = 0{,}00279x + 3{,}528$

[2] $y = 3{,}528x + 0{,}00279$

[3] $y = 8{,}5x + 135$

[4] $y = 135x + 8{,}5$

[5] $y =$ none of the above

***The solutions to these questions are to be found on p 105 of this tutorial letter.***

**Additional exercises are to be found on p 128 of this tutorial letter.**

## 4.2 Solutions: self-evaluation exercises

### 4.2.1 Solution to self-evaluation exercise 1

1. Simple interest $I = Prt$

$$\begin{aligned} I &= \text{interest earned} = \text{R7 630} - \text{R7 000} = \text{R630} \\ P &= \text{present value} = \text{R7 000} \\ r &= \text{simple interest rate} = ? \\ t &= \text{term} = \text{nine months} = \tfrac{9}{12} = \tfrac{3}{4} \text{ year} \end{aligned}$$

$$\begin{aligned} I &= Prt \\ 630 &= 7\,000 \times r \times \tfrac{3}{4} \\ r &= \tfrac{630 \times 4}{7\,000 \times 3} \\ r &= 0{,}12 \\ &= 12\% \end{aligned}$$

The simple interest rate is 12,0%.

2. Simple interest: $I = Prt$

$$\begin{aligned} I &= \text{interest earned} = \text{R1 295} - \text{R1 200} = \text{R95} \\ P &= \text{present value} = \text{R1 200} \\ r &= \text{simple interest rate} = ? \\ t &= \text{term} = \text{four months} = \tfrac{4}{12} \text{ year} \end{aligned}$$

$$\begin{aligned} I &= \text{Pr}\,t \\ 95 &= 1\,200 \times r \times \tfrac{4}{12} \\ r &= \tfrac{95 \times 12}{4 \times 1\,200} \\ r &= 0{,}2375 \\ &= 23{,}75\% \end{aligned}$$

The simple interest rate is 23,75%.

3. Simple interest: $S = P(1 + rt)$

$$\begin{aligned} S &= \text{Accumulated amount} = \text{R612} \\ P &= \text{present value} = ? \\ r &= \text{simple interest rate} = 0{,}12 \\ t &= \text{term of loan} = \text{from 3 March until 2 May.} \end{aligned}$$

| **Period** | Number of days |
|---|---|
| 3–31 March | 29 (3rd included) |
| 1–30 April | 30 |
| 1–2 May | 1 (2nd excluded) |
| | 60 days |

**OR**

Use the number of each day of the year table. Day number 122 (2 May) minus day number 62 (3 March) equals 60.

$$\begin{aligned} S &= P(1+rt) \\ P &= \frac{S}{1+rt} \\ &= \frac{612}{\left(1+0{,}12\times\frac{60}{365}\right)} \\ &= 600{,}16 \end{aligned}$$

R600,16 must be invested on 3 March.

4. Simple interest: $I = Prt$

$I$ = interest earned = ?
$P$ = present value = R1 500
$r$ = simple interest rate = 0,215
$t$ = term of loan = from 10 March until 2 July

Day number 183 (2 July) minus day number 69 (10 March) equals 114.

$$\begin{aligned} I &= Prt \\ &= 1\,500 \times 0{,}215 \times \frac{114}{365} \\ &= 100{,}73 \end{aligned}$$

He has to pay R100,73 interest on the loan of R1 500.

5. Simple interest: $S = P(1+rt)$

$S$ = accumulated amount = R3 755
$P$ = present value = R3 500
$r$ = simple interest rate = 0,18
$t$ = term of investment = ?

$$\begin{aligned} S &= P(1+rt) \\ \frac{S}{P} &= 1+rt \\ \frac{S}{P}-1 &= rt \\ t &= \left(\frac{S}{P}-1\right)\div r \\ &= \left(\frac{3\,755}{3\,500}-1\right)\div 0{,}18 \\ &= 0{,}4048 \text{ years} \\ &= 0{,}4048\times 365 \text{ days} \\ &= 147{,}7 \approx 148 \text{ days.} \end{aligned}$$

R3 500 must be invested for 148 days for an interest rate of 18% per year to accumulate to R3 755.

6. Simple interest: $I = Prt$

$I$ = interest earned = R2 240 − R2 000 = R240
$P$ = present value = R2 000
$r$ = simple interest rate = 0,15
$t$ = term of investment = ?

$$\begin{aligned} I &= Prt \\ t &= \frac{I}{P\times r} \\ t &= \frac{240}{2\,000\times 0{,}15} \\ &= 0{,}8 \text{ years} \\ &= 0{,}8\times 365 \text{ days} \\ &= 292 \text{ days.} \end{aligned}$$

Day number 65 (6 March) plus 292 equals day number 357 that is 23 December.

If R2 000 is invested on 6 March at an interest rate of 15% per year, it will accumulate to R2 240 on 23 December of the same year.

7. Discount: $P = S(1 - dt)$

| | | |
|---|---|---|
| $P$ | $=$ | present value $=$ amount that he receives $=$ R5 000 |
| $S$ | $=$ | face value or future value $=$ ? |
| $d$ | $=$ | discount rate $=$ 0,18 |
| $t$ | $=$ | term of loan $=$ 31 August until 2 November of the same year. |

Day number 306 (2 November) minus day number 243 (31 August) equals 63.

$$\begin{aligned} P &= S(1 - dt) \\ S &= \frac{P}{(1 - dt)} \\ &= \frac{5\,000}{\left(1 - 0{,}18 \times \frac{63}{365}\right)} \\ &= 5\,160{,}32 \end{aligned}$$

The face value is R5 160,32.

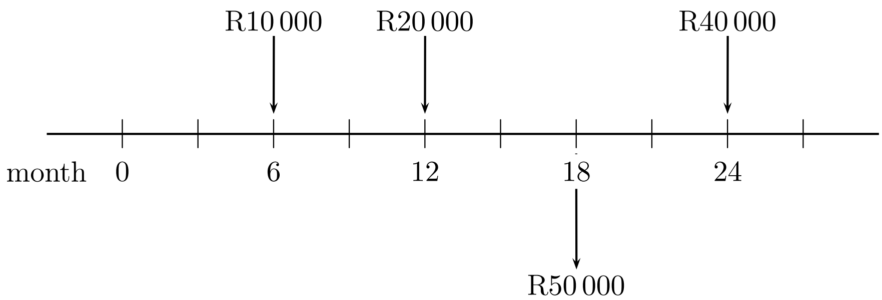
8. Interest paid in the previous question is R160,32(5 160,32 − 5 000):

The simple interest rate equivalent to the above interest can be calculated as $I = Prt$.

$$\begin{aligned} I &= Prt \\ 160{,}32 &= 5\,000 \times r \times \frac{63}{365} \\ \frac{160{,}32}{5\,000} \times \frac{365}{63} &= r \\ r &= 0{,}18577 \\ &= 18{,}58\%. \end{aligned}$$

The equivalent simple interest is 18,58%.

9.




He has to pay his debt. We must calculate the value of all his payments and obligations at the same time, namely at month 24.

**Obligations:**

R10 000 must be moved 18 months forward:

$$10\,000\left(1 + 0{,}17 \times \frac{18}{12}\right) = 12\,550{,}00$$

R20 000 must be moved 12 months forward:

$$20\ 000\left(1+0{,}17\times\frac{12}{12}\right)=23\,400{,}00$$

There is no need to move the R40 000.

**Payments:**

R50 000 must be moved six months forward:

$$50\ 000\left(1+0{,}17\times\frac{6}{12}\right)=54\,250{,}00$$

The amount that he has to pay at month 24 is:

$$\begin{aligned}\text{Obligations} - \text{payments} &= (12\,550{,}00+23\,400{,}00+40\,000{,}00)-54\,250{,}00\\ &= 21\,700{,}00\end{aligned}$$

The amount to be paid is R21 700,00.

### 4.2.2 Solution to self-evaluation exercise 2

1. Compound interest: $S=P\left(1+\frac{j_m}{m}\right)^{tm}$

$$\begin{aligned}S &= \text{future value} = ?\\ P &= \text{present value} = \text{R}1\,400\\ m &= \text{number of compounded periods per year} = 2\\ t &= \text{the number of years for which the investment is made} = 20 \text{ years.}\\ j_m &= \text{interest per year} = 12{,}5\%\end{aligned}$$

$$\begin{aligned}S &= P\left(1+\frac{j_m}{m}\right)^{tm}\\ &= 1\,400\left(1+\frac{0{,}125}{2}\right)^{20\times 2}\\ &= 15\,822{,}88\end{aligned}$$

After 20 years R15 822,88 must be paid to the inheritor.

2. Compound interest: $S=P\left(1+\frac{j_m}{m}\right)^{tm}$

$$\begin{aligned}S &= \text{R}25\,000\\ P &= \text{R}15\,000\\ m &= 12\\ t &= ?\\ j_m &= 0{,}12\end{aligned}$$

$$
\begin{aligned}
S &= P\left(1 + \tfrac{j_m}{m}\right)^{tm} \\
25\ 000 &= 15\ 000\left(1 + \tfrac{0{,}12}{12}\right)^{12t} \\
\tfrac{25\ 000}{15\ 000} &= \left(1 + \tfrac{0{,}12}{12}\right)^{12t} \\
\ln\left(\tfrac{25\ 000}{15\ 000}\right) &= 12\,t\ln\left(1 + \tfrac{0{,}12}{12}\right) \\
\frac{\ln\left(\frac{25\ 000}{15\ 000}\right)}{\ln\left(1+\frac{0{,}12}{12}\right)} &= 12t \\
12t &= 51{,}34 \\
t &\approx 4\tfrac{1}{4}
\end{aligned}
$$

It will take $4\frac{1}{2}$ years.

3. Compound interest: $S = P\left(1 + \frac{j_m}{m}\right)^{tm}$.

$$
\begin{aligned}
S &= \text{R}1\,500 \\
P &= \text{R}1\,000 \\
j_m &= ? \\
t &= 2\tfrac{1}{2} \\
m &= 12.
\end{aligned}
$$

$$
\begin{aligned}
S &= P\left(1 + \tfrac{j_m}{m}\right)^{tm} \\
1\ 500 &= 1\ 000\left(1 + \tfrac{j_m}{12}\right)^{2{,}5\times 12} \\
\left(\tfrac{1\ 500}{1\ 000}\right)^{\frac{1}{30}} &= 1 + \tfrac{j_m}{12} \\
\left(\tfrac{1\ 500}{1\ 000}\right)^{\frac{1}{30}} - 1 &= \tfrac{j_m}{12} \\
12\left[\left(\tfrac{1\ 500}{1\ 000}\right)^{\frac{1}{30}} - 1\right] &= j_m \\
j_m &= 0{,}1633 \\
&= 16{,}33\%.
\end{aligned}
$$

The interest rate per year is 16,33%.

4. We firstly calculate the value of R12 000 after four years and three months with:

$$
\begin{aligned}
P &= 12\,000 \\
j_m &= 0{,}105 \\
t &= 4\tfrac{3}{12}\text{ years} = 4{,}25\text{ years} \\
m &= 12 \\
S &= ?
\end{aligned}
$$

$$
\begin{aligned}
S &= P\left(1 + \tfrac{j_m}{m}\right)^{tm} \\
S &= 12\ 000\left(1 + \tfrac{0{,}105}{12}\right)^{4{,}25\times 12} \\
&= 18\,712{,}95
\end{aligned}
$$

Then R15 000 of the R18 712,95 is withdrawn and invested for the remaining one year and nine months at an interest rate of 12% per year compounded quarterly, with
$j_m = 0{,}12$ $m = 4$ and $t = 1\frac{9}{12}$.

After one year and nine months the R15 000 will accumulate to:

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 15\ 000\left(1+\frac{0{,}12}{4}\right)^{1{,}75\ \times\ 4} \\ &= 18\,448{,}11 \end{aligned}$$

From the R18 712,95 there is R3 712,95 (18 712,95 − 15 000,00) left that earns 10,5% interest, compounded monthly, therefore $j_m = 0{,}105$, $m = 12$ and $t = 1{,}75$ for the remaining one year and nine months (21 months).

After one year and nine months (21 months) the R3 712,95 will accumulate to:

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 3\,712{,}95\left(1+\frac{0{,}105}{12}\right)^{1{,}75 \times 12} \\ &= 4\,458{,}34 \end{aligned}$$

The total accumulated amount for both accounts at the end of the six year period is

R22 906,45 (18 448,11 + 4 458,34).

5. Effective interest: $j_{eff} = 100\left[\left(1+\frac{j_m}{m}\right)^m - 1\right]$

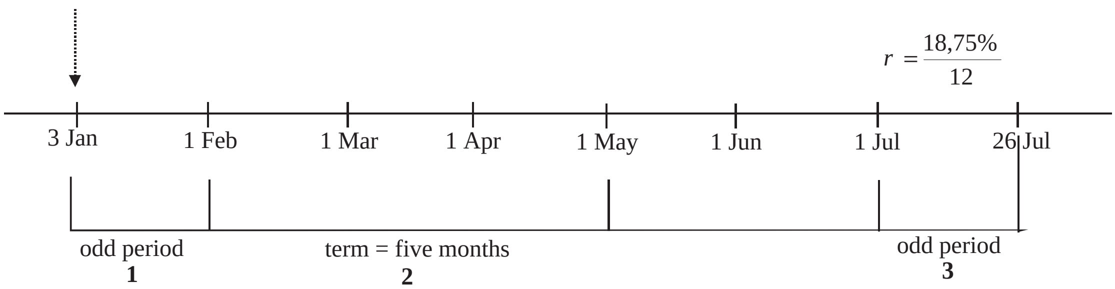
$j_m$ = nominal rate = 0,12
$m$ = number of times per year that the interest are calculated = 12

$$\begin{aligned} j_{eff} &= 100\left[\left(1+\frac{0{,}12}{12}\right)^{12} - 1\right] \\ &= 12{,}68\%. \end{aligned}$$

The effective interest rate is 12,68%.

6.

(a)



Period 1: odd period of 29 days (Day number 32 - 3 = 29.)
Period 2: term = five months
Period 3: odd period of 25 days

Value of R2 500 on 1 February:

$$\begin{aligned} S_1 &= P(1 + rt) \\ &= 2\,500\left(1 + \tfrac{29}{365} \times 0{,}1875\right) \\ &= 2\,537{,}24 \end{aligned}$$

Value of R2 537,24 on 1 July: with $m = 12$ and $t = \frac{5}{12}$.

$$\begin{aligned} S_2 &= 2\,537{,}24\left(1 + \tfrac{j_m}{m}\right)^{tm} \\ &= (2\,537{,}24) \times \left(1 + \tfrac{0{,}1875}{12}\right)^{\left(\frac{5}{12} \times \frac{12}{1}\right)} \\ &= 2\,741{,}75 \end{aligned}$$

Value of R2 741,75 on 26 July:

$$\begin{aligned} S_3 &= 2\,741{,}75(1 + rt) \\ &= (2\,741{,}75) \times \left(1 + \tfrac{25}{365} \times 0{,}1875\right) \\ &= 2\,776{,}96 \end{aligned}$$

Thus the value on 26 July is

$$\begin{aligned} &2\,500\left(1 + \tfrac{29}{365} \times 0{,}1875\right)\left(1 + \tfrac{0{,}1875}{12}\right)^{\left(\frac{5}{12} \times \frac{12}{1}\right)}\left(1 + \tfrac{25}{365} \times 0{,}1875\right) \\ &= 2\,776{,}96. \end{aligned}$$

The grandchild will receive R2 776,96 on his first birthday.

(b) $S = P\left(1 + \frac{j_m}{m}\right)^{tm}$

$j_m$ = interest rate per year = 0,1875
$m$ = number of compounding periods per year = 12
$t$ = term of investment = five compounding periods = $\frac{5}{12}$ years plus the number of odd days as a fraction of a year $\left(\frac{5}{12} + \frac{29+25}{365}\right)$.

$$\begin{aligned} S &= 2\,500\left(1 + \tfrac{0{,}1875}{12}\right)^{\left(\frac{5}{12} + \frac{29+25}{365}\right) \times 12} \\ &= 2\,776{,}90 \end{aligned}$$

He will receive R2 776,90 if fractional compounding is used.

7. Continuous compounding rate: $c = m \ln\left(1 + \frac{j_m}{m}\right)$

**Option A:**

$$\begin{aligned} c &= 2\ln\left(1 + \tfrac{0{,}175}{2}\right) \\ &= 16{,}78\% \end{aligned}$$

**Option B:**

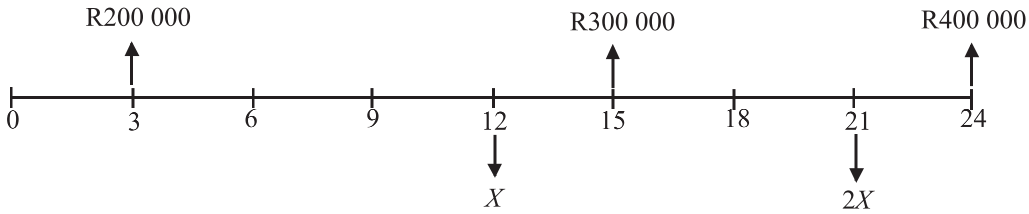
$$\begin{aligned} c &= 4\ln\left(1 + \tfrac{0{,}16}{4}\right) \\ &= 15{,}69\% \end{aligned}$$

**Option C:**

$$\begin{aligned} c &= 12\ln\left(1+\tfrac{0{,}16}{12}\right) \\ &= 15{,}89\% \end{aligned}$$

The best option for Joseph is the one with the lowest interest rate, thus option B.

8.




Due to the time value of money, we must move all the moneys to the same date namely the comparison date, month 21.

The value of the debt after 21 months:

For R200 000:

$$\begin{aligned} t &= 18 \div 12 \text{ (month 21 minus month 3)} = 1{,}5 \text{ years} \\ m &= 4 \\ j_m &= 0{,}1875. \end{aligned}$$

$$\begin{aligned} S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\ &= 200\ 000 \times \left(1+\tfrac{0{,}1875}{4}\right)^{(1{,}5\times 4)} \end{aligned}$$

For R300 000: $300\ 000 \times \left(1+\tfrac{0{,}1875}{4}\right)^{(0{,}5\times 4)}$. $t$ = six months = 0,5 years.

For R400 000: $400\ 000\left(1+\tfrac{0{,}1875}{4}\right)^{-(0{,}25\times 4)}$. $t$ = three months = 0,25 years.

The total debt after 21 months is thus:

$$200\,000\left(1+\tfrac{0{,}1875}{4}\right)^{(1{,}5\times4)} + 300\ 000\left(1+\tfrac{0{,}1875}{4}\right)^{(0{,}5\times4)} + 400\ 000\left(1+\tfrac{0{,}1875}{4}\right)^{-(0{,}25\times4)}$$

$$= 263\,268{,}54 + 328\,784{,}18 + 382\,089{,}55$$

$$= 974\,142{,}27$$

The value of the payments at the end of the 21 months:

First payment: $X\left(1+\tfrac{0{,}1875}{4}\right)^{(0{,}75\times4)}$. $t$ = nine months = 0,75 years.

Second payment: $2X$. (no interest is applicable)

$$\begin{aligned}
\text{Payment} &= \text{Obligations}\\
X\left(1+\tfrac{0,1875}{4}\right)^{(0,75\times4)}+2X &= 974\ 142{,}27.\\
X\left[\left(1+\tfrac{0,1875}{4}\right)^{(0,75\times4)}+2\right] &= 974\,142{,}27 \text{ (Take the common factor } X\text{, out.)}\\
X &= \frac{974\,142{,}27}{\left[\left(1+\frac{0,1875}{4}\right)^{(0,75\times4)}+2\right]}\\
X &= 309\,514{,}87
\end{aligned}$$

The size of the first payment at the end of the first year is thus R309 514,87.

The payment after 21 months is thus:

$$\begin{aligned}
&2\times 309\,514{,}87\\
&= 619\,029{,}74
\end{aligned}$$

The payment is R619 029,74.

### 4.2.3 Solution to self-evaluation exercise 3

1. **Mr White**

   The initial payment accumulated to:

$$\begin{aligned}
S &= P\left(1+\tfrac{j_m}{m}\right)^{tm}\\
&= 3\,000\left(1+\tfrac{0,125}{12}\right)^{15\times12}\\
&= 19\,373{,}65
\end{aligned}$$

   The monthly payments from an ordinary annuity and accumulated to:

$$\begin{aligned}
S &= Rs_{\overline{n}|i}\\
&= 500s_{\overline{15\times12}|0,125\div12}\\
&= 261\,978{,}42
\end{aligned}$$

   Mr White's fund accumulated to R281 352,07 (261 978,42 + 19 373,65):

   **Mr Jones**

   The initial deposit accumulated to:

$$\begin{aligned}
S &= P\left(1+\tfrac{j_m}{m}\right)^{tm}\\
&= 5\,000\left(1+\tfrac{0,125}{12}\right)^{15\times12}\\
&= 32\,289{,}42
\end{aligned}$$

   The monthly payments accumulated to:

$$\begin{aligned}
S &= Rs_{\overline{n}|i}\\
&= 300s_{\overline{15\times12}|0,125\div12}\\
&= 157\,187{,}05
\end{aligned}$$

Mr Jones's fund accumulated to R189 476,47 (157 187,05 + 32 289,42)

Mr White has R91 875,60 (281 352,07 − 189476,47) more than Mr Jones in his fund.

2. John invests R2 000 for 10 years:

| | |
|---|---|
| Time: | 10 years |
| Payments: | R2 000 |
| Interest: | 7% per year |

$$\begin{aligned} S &= (1+i)Rs_{\overline{n}|i} \\ &= (1+0{,}07)2\ 000 s_{\overline{10}|0{,}07} \\ &= 29\,567{,}20 \end{aligned}$$

This amount now accumulates compound interest for 30 years:

$$\text{where}\quad j_m = 0{,}07,\quad m = 1,\quad t = 30.$$

$$\begin{aligned} S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\ &= 29\,567{,}20\,(1+0{,}07)^{30} \\ &= 225\,073{,}07 \end{aligned}$$

Net earnings of John:

$$\begin{aligned} S &= 225\,073{,}07 - (2\,000 \times 10) \\ &= 225\,073{,}07 - 20\,000 \\ &= 205\,073{,}07 \end{aligned}$$

John's earnings are R205 073,07.

Jane investing R2 000 for 30 years:

$$\begin{aligned} S &= (1+i)Rs_{\overline{n}|i} \\ &= (1+0{,}07)2\,000 s_{\overline{30}|0{,}07} \\ &= 202\,146{,}08 \end{aligned}$$

Net earnings of Jane:

$$\begin{aligned} S &= 202\,146{,}08 - (2\,000 \times 30) \\ &= 142\,146{,}08 \end{aligned}$$

Jane's earnings are R142 146,08.

3. The R80 000 accumulates interest in the four years' time:

$$\begin{aligned} S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\ &= 80\,000\left(1+\tfrac{0{,}15}{12}\right)^{4\times 12} \\ &= 145\,228{,}39 \end{aligned}$$

This R145 228,39 is the amount money that he has to repay in equal monthly payments:

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 145\,228{,}39 &= Ra_{\overline{5\times 12}|0{,}15\div 12} \\ R &= 3\,454{,}97 \end{aligned}$$

If he wants to repay the loan in five years' time he must pay R3 454,97 per month.

4. As the payments made and the interest dates don't correspond we must first convert the semi-annually compounding interest to monthly compounding.

$$\begin{aligned} i &= n\left[\left(1+\tfrac{j_m}{m}\right)^{m\div n}-1\right] \\ \text{with} \quad n &= 12 \\ j_m &= 0{,}135 \\ m &= 2 \end{aligned}$$

$$\begin{aligned} i &= 12\left[\left(1+\tfrac{0{,}135}{2}\right)^{(2\div 12)}-1\right] \\ &= 0{,}13135... \\ &= 13{,}135...\% \end{aligned}$$

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ \text{with} \quad R &= 500 \\ n &= 8\times 12 \\ i &= 0{,}13135... \\ S &= 500s_{\overline{8\times 12}|0{,}13135...\div 12} \\ &= 84\,218{,}28 \end{aligned}$$

5. (a) Value of the flat:

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= 2\,500a_{\overline{20\times 12}|0{,}1475\div 12} \\ &= 192\,551{,}30 \end{aligned}$$

They can afford a flat for R192 551,30 plus R100 000 deposit that is R292 551,30.

(b) After eight years they have made $8 \times 12 = 96$ payments. The present value of the loan at that stage is:

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= 2\,500a_{\overline{(20-8)\times 12}|0{,}1475\div 12} \\ &= \text{R}168\,370{,}01 \end{aligned}$$

Their equity in the flat is R124 181,29 $(192\,551{,}30 - 168\,370{,}01 + 100\,000)$

6. Sinking fund:

| | |
|---|---|
| Semi-annually payments: | R5 000 |
| Interest: | 16% per year |
| Time: | 7 years |

Accumulated value will be:

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= 5\,000s_{\overline{7\times 2}|0{,}16\div 2} \\ &= 121\,074{,}60 \end{aligned}$$

The balance will be R121 074,60.

7.
$$\begin{aligned}\text{Price} &= \text{Deposit} + Ra_{\overline{n}|i}\\ &= 200\,000 + 10\ 000a_{\overline{5\times 12}|0{,}12\div 12}\\ &= 200\,000 + 449\ 550{,}38\\ &= 649\,550{,}38\end{aligned}$$

The cash price of the house is R649 550,38.

### 4.2.4 Solution to self-evaluation exercise 4

1. **Internal rate of return:**

A: $f(I) = \frac{400}{1+I} + \frac{300}{(1+I)^2} + \frac{350}{(1+I)^3} - 800 = 0.$

The IRR = 15,37%.

B: $f(I) = \frac{200}{1+I} + \frac{500}{(1+I)^2} + \frac{450}{(1+I)^3} - 750 = 0.$

The IRR = 21,82%.

As the internal rate of return $> K$ (cost of capital) for project $B$, invest in $B$.

**Net Present Value:**

A: $N = \frac{400}{(1+0{,}19)} + \frac{300}{(1+0{,}19)^2} + \frac{350}{(1+0{,}19)^3}, - 800.$

The NPV $= -44$.

B: $N = \frac{200}{(1+0{,}19)} + \frac{500}{(1+0{,}19)^2} + \frac{450}{(1+0{,}19)^3} - 750.$

The $NPV = 38$.

Advise $B$ over $A$ since it has a greater $NPV$.

**Profitability index:**

A: $PI_A = \dfrac{\text{NPV} + \text{Outlay}}{\text{Outlay}} = \frac{-44\,+\,800}{800} = 0{,}945.$

B: $PI_B = \dfrac{\text{NPV} + \text{Outlay}}{\text{Outlay}} = \frac{38\,+\,750}{750} = 1{,}051.$

$PI_B > 1$ therefore select $B$.

2. (a) Calculate the present value of the cash outlays:

**Shoe:**

$$\begin{aligned} I &= 100 + \frac{50}{(1+0{,}165)^2}\\ &= 100 + 36{,}84\\ &= 136{,}84\end{aligned}$$

The present value is R136,84.

**CD:**

$$I = 400$$

(b) Calculate the future value of the cash inflows at the end of the project.

**Shoe:**

$$\begin{aligned} C &= 50\,(1+0{,}19)^2 + 75\\ &= 70{,}81 + 75\\ &= 145{,}81\end{aligned}$$

The future value is R145,81.

**CD:**

$$\begin{aligned} C &= 75\,(1+0{,}19)^2 + 100\,(1+0{,}19) \\ &= 106{,}21 + 119{,}00 + 400 \\ &= 625{,}21 \end{aligned}$$

The future value is R625,21.

(c) Calculate the MIRR

$$\text{MIRR} = \left[\left(\frac{C}{PV_{out}}\right)^{\frac{1}{n}} - 1\right]$$

**Shoe:**

$$\begin{aligned} \text{MIRR} &= \left(\tfrac{145{,}81}{136{,}84}\right)^{\left(\frac{1}{3}\right)} - 1 \\ &= 2{,}13\% < 19\%. \end{aligned}$$

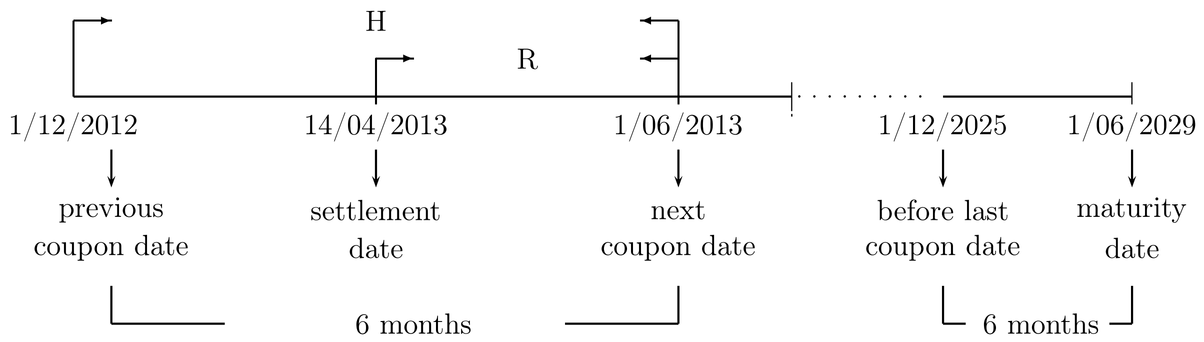
**CD:**

$$\begin{aligned} \text{MIRR} &= \left(\tfrac{625{,}21}{400}\right)^{\left(\frac{1}{3}\right)} - 1 \\ &= 16{,}05\% < 19\%. \end{aligned}$$

Since both options MIRR values are smaller than 19%, not one of the two options is advisable because he can earn more interest if he invests his money at 19%.

### 4.2.5 Solution to self-evaluation exercise 5

1.




The number of half yearly coupon periods is

$$\begin{aligned} \text{Years} &= 1/06/2013 \text{ to } 1/06/2029 \\ &= (2029 - 2013) \\ &= 16 \end{aligned}$$

We multiply by 2 to get the number of half yearly coupons – thus 32 ($16 \times 2$).

The number of days from the settlement date until the next coupon (interest) date is $R$:

The day number 152 (1 June) minus day number 104 (14 April) equals 48. Thus $R = 48$.

The number of days in the half year in which the settlement date falls (1/12/2012 to 1/06/2013) is $H$.

Day number 365 (31 December) minus 335 (1 December) plus 152 (1 June) equals 182. Thus $H = 182$.

The present value of the bond on 1/06/2013 is:

$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1 + z)^{-n} \\ &= \tfrac{16,5}{2} a_{\overline{32}|0,142 \div 2} + 100 \left(1 + \tfrac{0,142}{2}\right)^{-32} \\ &= 114{,}39343. \end{aligned}$$

Since the settlement date is more than ten days from the next coupon (interest) date it is a cum interest case and we must add the coupon.

$$\begin{aligned} P(1/06/2013) &= 114{,}39343 + 8{,}25 \\ &= 122{,}64343\% \end{aligned}$$

We must now discount this present value of the bond back to the settlement date to obtain the all-in-price.

$$\begin{aligned} \text{All-in-price} &= 122{,}64343 \times \left(1 + \tfrac{0,142}{2}\right)^{-\left(\frac{48}{182}\right)} \\ &= 120{,}44471\% \end{aligned}$$

The all-in price is R120,44471%.

The accrued interest:

$$\begin{aligned} &= \tfrac{H-R}{365} \times c \\ &= \tfrac{182-48}{365} \times 16{,}5 \\ &= 6{,}05753 \end{aligned}$$

The accrued interest is R6,05753%.

$$\begin{aligned} \text{Clean price} &= \text{All-in price} - \text{accrued interest} \\ &= 120{,}44471 - 6{,}05753 \\ &= 114{,}38718 \end{aligned}$$

The clean price is 114,38718%.

2. The settlement date is 25 May 2013.

   The price on the next interest date (1 June 2013):

   $P$(1 June 2013) = R114,39343% - see solution to question 1.

   Since this is an ex interest case no coupon must be added.

   The remaining number of days from 25 May 2013 to 1 June 2013 (152 − 145):

   $$R = 7$$

   The number of days in the half year 1 December 2012 to 1 June 2013:

   $$H = 182$$

   Thus the fraction of the half year for discounting:

   $$f = \frac{7}{182}$$

The all-in price is:

$$\begin{aligned} P &= 114{,}39343 \times \left(1 + \tfrac{0{,}142}{2}\right)^{-\left(\frac{7}{182}\right)} \\ &= 114{,}09204 \end{aligned}$$

The all-in price is R114,09204%.

The accrued interest:

$$\begin{aligned} &= \tfrac{-R}{365} \times c \\ &= \tfrac{-7}{365} \times 16{,}5 \\ &= -0{,}31644 \end{aligned}$$

The accrued interest is −R0,31644%

$$\begin{aligned} \text{Clean price} &= \text{All-in price} - \text{Accrued interest} \\ &= 114{,}09204 - (-0{,}31644) \\ &= 114{,}40848 \end{aligned}$$

The clean price is R114,40848%.

### 4.2.6 Solution to self-evaluation exercise 6

1. This is an example of a weighted mean calculation where the wages are the data values and the number of workers in each field is the weights. The weighted mean is

$$\begin{aligned} \bar{x}_w &= \frac{\sum_{i=1}^{3} x_i w_i}{\sum_{i=1}^{3} w_i} \\ &= \frac{x_1 w_1 + x_2 w_2 + x_3 w_3}{w_1 + w_2 + w_3} \\ &= \frac{(28{,}41 \times 4{,}4) + (27{,}50 \times 1) + (26{,}65 \times 6{,}2)}{4{,}4 + 1 + 6{,}2} \\ &= 27{,}39. \end{aligned}$$

2. (a) The arithmetic mean for student $A$ is

$$\begin{aligned}\bar{x} &= \frac{\sum_{i=1}^{3} x_i}{3} \\ &= \frac{x_1 + x_2 + x_3}{3} \\ &= \frac{81 + 88 + 83}{3} \\ &= \frac{252}{3} \\ &= 84.\end{aligned}$$

The arithmetic mean for student $B$ is

$$\begin{aligned}\bar{x} &= \tfrac{252}{3} \\ &= 84.\end{aligned}$$

There is no choice between student $A$ and $B$ because they have the same arithmetic mean.

(b) The standard deviation for student $A$ is

$$\begin{aligned}S &= \sqrt{\frac{\sum_{i=1}^{3} (x_i - \bar{x})^2}{n-1}} \\ &= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2}{3-1}} \\ &= \sqrt{\frac{(81-84)^2 + (88-84)^2 + (83-84)^2}{2}} \\ &= \sqrt{\frac{9+16+1}{2}} \\ &= \sqrt{13} \\ &= 3{,}61.\end{aligned}$$

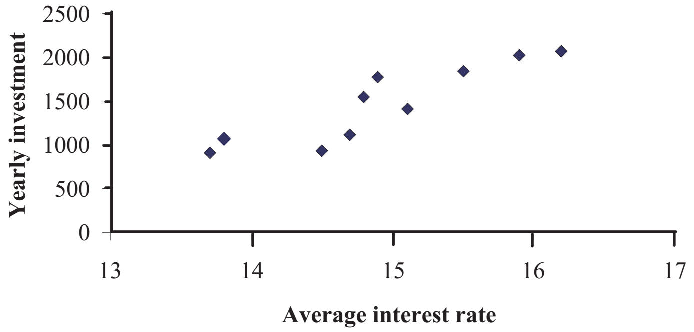
The standard deviation for student $B$ is

$$\begin{aligned}S &= \sqrt{\tfrac{1+81+64}{2}} \\ &= \sqrt{73} \\ &= 8{,}54.\end{aligned}$$

**These calculations can be done directly on your calculator. See Notes on the calculator for the key operations.**

Student $A$ will be selected because he has a smaller standard deviation than student $B$. His performance is more stable than that of student $B$.

3. (a) It looks as if there exists a positive linear correlation between average interest rate and yearly investment. This means that if the average interest rate increases, then yearly investment will also increase.




(b) **You must do these calculations on your calculator using the statistical functions directly. You do not need to do the following in-between steps for the calculations.**

| Year $i$ | Average interest $x_i$ | Yearly investment $y_i$ | $x_i^2$ | $x_iy_i$ | $y_i^2$ |
|---|---|---|---|---|---|
| 1 | 13,8 | 1 060 | 190,44 | 14 628 | 1 123 600 |
| 2 | 14,5 | 940 | 210,25 | 13 630 | 883 600 |
| 3 | 13,7 | 920 | 187,69 | 12 604 | 846 400 |
| 4 | 14,7 | 1 110 | 216,09 | 16 317 | 1 232 100 |
| 5 | 14,8 | 1 550 | 219,04 | 22 940 | 2 402 500 |
| 6 | 15,5 | 1 850 | 240,25 | 28 675 | 3 422 500 |
| 7 | 16,2 | 2 070 | 262,44 | 33 534 | 4 284 900 |
| 8 | 15,9 | 2 030 | 252,81 | 32 277 | 4 120 900 |
| 9 | 14,9 | 1 780 | 222,01 | 26 522 | 3 168 400 |
| 10 | 15,1 | 1 420 | 228,01 | 21 442 | 2 016 400 |
| **$n$ =10** | **149,1** | **14 730** | **2 229,03** | **222 569** | **23 501 300** |

The coefficient of correlation is

$$\begin{aligned} r &= \frac{n\sum_{i=1}^{n} x_iy_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}\sqrt{n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2}} \\ &= \frac{10\,(222\,569) - (149{,}1)\,(14\,730)}{\sqrt{10\,(2\,229{,}03) - (149{,}1)^2}\sqrt{10\,(23\,501\,300) - (14\,730)^2}} \\ &= \frac{29\,447}{32\,759{,}8161} \\ &= 0{,}8989. \end{aligned}$$

(c) The coefficient of determination is $r^2 = 0{,}8989^2 = 0{,}8080$. This means that almost 81% of the variation in yearly investments can be declared by the average interest rate.

(d) The equation of the straight line is $y = a + bx$ where

$$
\begin{aligned}
b &= \frac{\sum_{i=1}^{10} x_i y_i - \sum_{i=1}^{10} x_i \sum_{i=1}^{10} y_i}{n\sum_{i=1}^{10} x_i^2 - (\sum_{i=1}^{10} x_i)^2} \\
&= \frac{10(222\,569) - (149{,}1)(14\,730)}{10(2\,229{,}03) - (149{,}1)^2} \\
&= \frac{29\,447}{59{,}49} \\
&= 494{,}99
\end{aligned}
$$

and

$$
\begin{aligned}
a &= \frac{\sum_{i=1}^{10} y_i}{n} - \frac{b\sum_{i=1}^{10} x_i}{n} \\
&= \frac{14\ 730}{10} - \frac{(494{,}99)(149{,}1)}{10} \\
&= -5\ 907{,}30.
\end{aligned}
$$

Thus $y = -5\ 907{,}30 + 494{,}99x$.

**NOTE: Use the statistical functions on your calculator for these calculations. It is much easier.**

(e) Although an interest rate of 16,5% is not in the span of $x$-values, it is not too far from the rest of the $x$-values, and because the coefficient of correlation is large enough, we can forecast the corresponding yearly investment (extrapolate). The yearly investment for an interest rate of 16,5% is

$$
\begin{aligned}
y &= -5\,907{,}30 + 494{,}99 \times 16{,}5 \\
&= 2\,260{,}04.
\end{aligned}
$$

### 4.2.7 Solution to self-evaluation exercise 7

**TYPICAL EXAM QUESTIONS**

1.

$$
\begin{aligned}
S &= P(1 + rt) \\
\text{with } P &= 420 \\
r &= 0{,}075 \\
t &= (156 - 52) \\
&= \frac{104}{365}
\end{aligned}
$$

$$\begin{aligned} S &= 420\left(1+0{,}075\times\frac{104}{365}\right) \\ &= 428{,}98 \end{aligned}$$

Little John can withdraw R428,98.

2.

$$\begin{aligned} i &= n\left(\left(1+\frac{j_m}{m}\right)^{\frac{m}{n}}-1\right) \\ \text{with } j_m &= 0{,}164 \\ m &= 4 \\ n &= 52. \end{aligned}$$

$$\begin{aligned} i &= 52\left(\left(1+\frac{0{,}164}{4}\right)^{\frac{4}{52}}-1\right) \\ &= 0{,}16098 \\ &= 16{,}098\%. \end{aligned}$$

The equivalent interest rate is 16,098%.

3. This is an annuity due problem due to the fact that the word immediately is in the sentence.

$$\begin{aligned} S &= (1+i)\,Rs_{\overline{n}|i} \\ \text{with } I &= 0{,}0909\div 12 \\ N &= 9\times 12 \\ S &= 12\,500\div\left(1+\frac{0{,}0909}{12}\right) \\ 12\,500 &= \left(1+\frac{0{,}0909}{12}\right)Rs_{\overline{9\times 12}|0{,}0909\div 12} \\ R &= 74{,}63 \end{aligned}$$

The monthly payments are R74,63.

4. The quarterly interest rate must first be converted to monthly compounded.

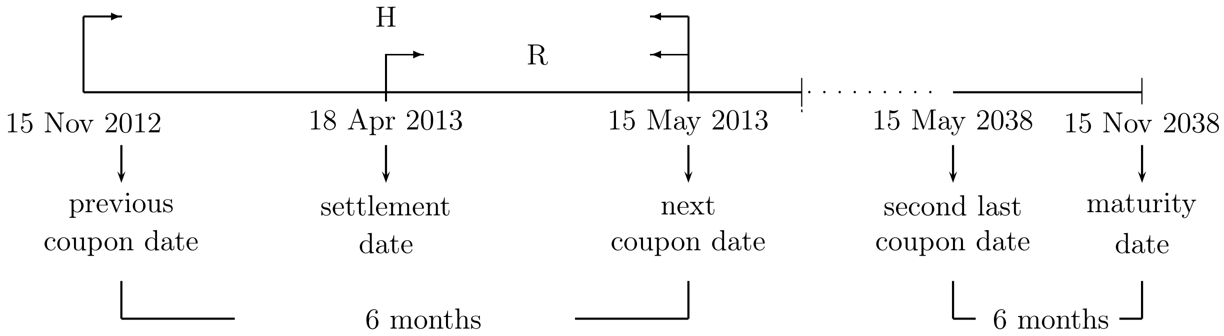
$$\begin{aligned} i &= n\left(\left(1+\frac{j_m}{m}\right)^{\frac{m}{n}}-1\right) \\ \text{with } j_m &= 0{,}149 \\ m &= 4 \\ n &= 12 \end{aligned}$$

$$\begin{aligned} i &= 12\left(\left(1+\frac{0{,}149}{4}\right)^{\frac{4}{12}}-1\right) \\ &= 0{,}14718... \end{aligned}$$

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ \text{with } I &= 0{,}14718 \div 12 \\ N &= 12 \times 12 \\ R &= 1\,000 \\ S &= 1\,000s_{\overline{12\times 12}|0{,}14718\div 12} \\ &= 390\,225{,}94 \end{aligned}$$

The accumulated amount is R390 225,94.

5.




In order to calculate the number of coupons still outstanding we first determine the number of years – from the next coupon date to the maturity date – and then multiply it by two to get the number of half years. As the months May and November differ and we want to calculate the number of years we move the next coupon date six months on to 15/11/2013.

$$\begin{aligned} \text{Years} &= 15/11/2038 - 15/11/2012 \\ &= 25. \end{aligned}$$

This 25 is the number of years in which the coupon payments will be made. We must multiply this now by (2).

Thus

$$\begin{aligned} n &= 25 \times 2 \\ &= 50. \end{aligned}$$

Our calculations were done from 15/11/2012 to 15/11/2038 but the next coupon date that follows the settlement date is 15/05/2013. We must therefore add one (1) to the $n$.

$$\begin{aligned} n &= 50+1 \\ &= 51. \end{aligned}$$

The number of days from the settlement date 18/04/2013 to the next coupon date 15/05/2013 is R: Day number 135 (15 May) minus day number 108 (18 April) equals 27, thus $R = 27$.

The number of days in the half year in which the settlement date falls, (15/11/2012 to 15/05/2013) is $H$. Day number 365 (31 December) minus day number 319 (15 November) plus day number 135(15 May) equals 181, thus $H = 181$.

The present value of the bond on 15/05/2013 is:

$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ &= \frac{11{,}59}{2} a_{\overline{51}|0{,}0946 \div 2} + 100\left(1+\frac{0{,}0946}{2}\right)^{-51} \\ &= 120{,}38349 \end{aligned}$$

As the settlement date is more than ten days from the next coupon date, we add a coupon − cum interest case.

$$\begin{aligned} P &= 120{,}38349 + 5{,}795 \\ &= 126{,}17849 \end{aligned}$$

We must now discount the present value of the bond back to the settlement date to obtain the all-in price.

$$\begin{aligned} \text{All-in price} &= 126{,}17849\left(1+\frac{0{,}0946}{2}\right)^{-\frac{27}{181}} \\ &= 125{,}31160 \end{aligned}$$

The all-in price is R125,31160%.

6.

$$\begin{aligned} \text{The accrued interest} &= \frac{H-R}{365} \times c \\ &= \frac{181-27}{365} \times 11{,}59 \\ &= 4{,}89003 \end{aligned}$$

The accrued interest is R4,89003%.

7.

$$\begin{aligned} \text{Clean price} &= \text{All-in price} - \text{accrued interest} \\ &= 125{,}31160 - 4{,}89003 \\ &= 120{,}42157 \end{aligned}$$

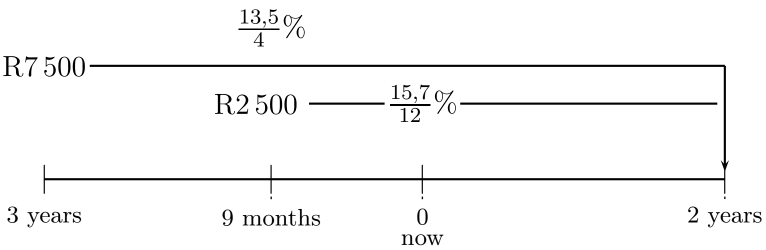
The clean price for one bond is R120,42157%. The given nominal value is R750 000 therefore 7 500 bonds were bought. The clean price is R903 162 (7 500 × 120,42157).

8.

$$
\begin{aligned}
\text{PI} &= \frac{\text{NPV} + \text{initial investment}}{\text{initial investment}} \\
1{,}083 &= \frac{1\,255 + \text{initial investment } (x)}{\text{initial investment } (x)} \\
1{,}083x &= 1\,255 + x \\
1{,}083x - x &= 1\,255 \\
0{,}083x &= 1\,255 \\
x &= \frac{1\,255}{0{,}083} \\
&= 15\,120{,}48
\end{aligned}
$$

The initial investment is R15 120,48.

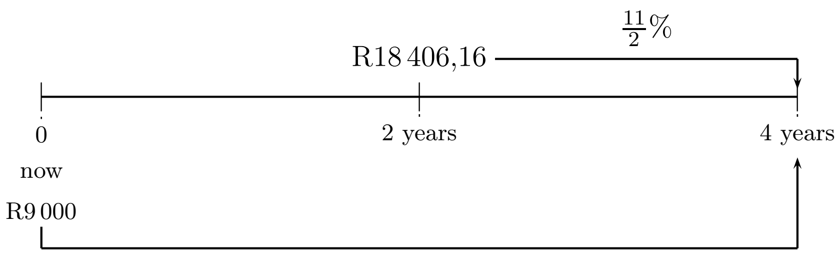
9.




$$
\begin{aligned}
\text{The amount due} &= 7\,500\left(1+\frac{0{,}135}{4}\right)^{5\times 4} + 2\,500\left(1+\frac{0{,}157}{12}\right)^{2{,}75\times 12} \\
&= 14\,567{,}05 + 3\,839{,}11 \\
&= 18\,406{,}16
\end{aligned}
$$

The amount due is R18 406,16.

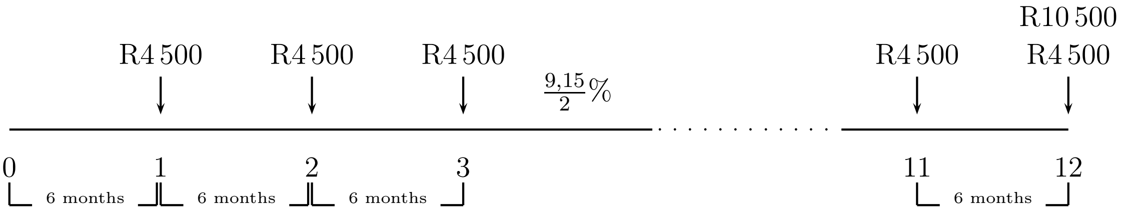
10.



$$
\begin{aligned}
\text{Payments} &= \text{Obligations}\\
9\,000\left(1+\frac{0{,}11}{2}\right)^{4\times 2}+X &= 18\,406{,}16\left(1+\frac{0{,}11}{2}\right)^{2\times 2}\\
X &= 22\,802{,}01-13\,812{,}18\\
&= 8\,989{,}83.
\end{aligned}
$$

The amount is R8 989,83.

11.




$$
\begin{aligned}
\text{PV} &= Ra_{\overline{n}|i}\\
&= 4\,500a_{\overline{6\times 2}|0{,}0915\div 2}\\
&= 40\,858{,}13
\end{aligned}
$$

We must discount the R10 000 back to now.

$$
\begin{aligned}
S &= P\left(1+\frac{j_m}{m}\right)^{tm}\\
10\,500 &= P\left(1+\frac{0{,}0915}{2}\right)^{6\times 2}\\
P &= 6\,138{,}39
\end{aligned}
$$

$$
\begin{aligned}
\text{Thus total amount} &= PV+P\\
&= 40\,858{,}13+6\,138{,}39\\
&= 46\,996{,}52
\end{aligned}
$$

The total amount is R46 996,52.

12.

$$
\begin{aligned}
\text{MIRR} &= \left(\frac{C}{PV_{\text{out}}}\right)^{\frac{1}{n}}-1\\
\text{with}\quad M &= 10{,}81\%\\
P &= 291\,930\\
N &= 8
\end{aligned}
$$

$$\begin{aligned}
0{,}1081 &= \left(\frac{C}{291\,930}\right)^{\frac{1}{8}} - 1 \\
1{,}1081 &= \left(\frac{C}{291\,930}\right)^{\frac{1}{8}} \\
C &= (1{,}1081)^{8} \times 291\,930 \\
&= 663\,606{,}09 \\
&\approx 663\,600{,}00
\end{aligned}$$

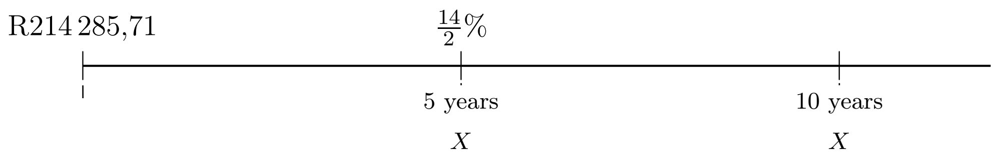
The future value of the cash inflows is R663 600,00.

13.

$$\begin{aligned}
P &= \frac{R}{i} \\
\text{with } R &= 2\,500 \\
i &= 0{,}14 \div 12 \\
P &= \frac{2\,500}{0{,}14 \div 12} \\
&= 214\,285{,}71
\end{aligned}$$

The opening balance is R214 285,71.

14.




$$\begin{aligned}
\text{Payments} &= \text{Obligations} \\
214\,285{,}71 &= X\left(1+\frac{0{,}14}{12}\right)^{-5\times 12} + X\left(1+\frac{0{,}14}{12}\right)^{-10\times 12} \\
&= X\left[\left(1+\frac{0{,}14}{12}\right)^{-60} + \left(1+\frac{0{,}14}{12}\right)^{-120}\right] \\
X &= \frac{214\,285{,}71}{\left[\left(1+\frac{0{,}14}{12}\right)^{-60} + \left(1+\frac{0{,}14}{12}\right)^{-120}\right]} \\
&= 286\,783{,}06.
\end{aligned}$$

The present value is R286 738,06.

15. Using your calculator directly the equation for the regression line is

$$y = 48\,644{,}17 - 6\,596{,}93x$$

16. The correlation coefficient is $r = -0{,}9601$

### 4.2.8 Solution to self-evaluation exercise 8

**TYPICAL EXAM QUESTIONS**

1.

$$\begin{aligned} S &= P(1+rt) \\ P &= 2\,000 \\ r &= 8\% \\ t &= 7 \end{aligned}$$

$$\begin{aligned} S &= 2\,000(1+0{,}08\times 7) \\ P &= 3\,120{,}00 \end{aligned}$$

James owes R3 120,00.

2.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ R &= 700 \\ n &= 3\times 12 \\ i &= 14{,}5\% \div 12 \end{aligned}$$

$$\begin{aligned} P &= 700a_{\overline{3\times 12}|0{,}145\div 12} \\ &= 20\,336{,}44 \end{aligned}$$

The original price was R25 336,44 (20 336,44 + 5 000).

3.

$$\begin{aligned} S &= P(1+rt) \\ 115 &= 100(1+0{,}08\times t) \\ \frac{115}{100} &= 1+\frac{8}{100}t \\ \frac{8}{100}t &= \frac{115}{100}-1 \\ t &= \frac{100}{8}\left(\frac{115}{100}-1\right) \end{aligned}$$

4.

$$\begin{aligned} P &= S(1-dt) \\ P &= 14\,500 \\ d &= 28\% \\ t &= \frac{10}{12} \\ 14\,500 &= S\left(1-0{,}28\times\frac{10}{12}\right) \\ S &= \frac{14\,500}{1-0{,}28\times\frac{10}{12}} \\ &= 18\,913{,}04 \end{aligned}$$

Jonas must pay R18 913,04.

5.

$$\begin{aligned} c &= m\ln\left(1+\frac{j_m}{m}\right) \\ 0{,}11832 &= 4\ln\left(1+\frac{j_m}{4}\right) \\ \frac{0{,}11832}{4} &= \ln\left(1+\frac{j_m}{4}\right) \\ e^{\frac{0{,}11832}{4}} &= 1+\frac{j_m}{4} \\ j_m &= 4\left[\left(e^{\frac{0{,}11832}{4}}\right)-1\right] \\ &= 12{,}01\% \end{aligned}$$

The nominal rate is 12,01%.

6.

$$\begin{aligned} S &= Pe^{ct} \\ 32\,850 &= 25\,000e^{c\times\frac{39}{12}} \\ e^{c\times\frac{39}{12}} &= \frac{32\,850}{25\,000} \\ \frac{39}{12}c\ln e &= \ln\left(\frac{32\,850}{25\,000}\right) \\ c &= \ln\left(\frac{32\,850}{25\,000}\right)\times\frac{12}{39} \\ &= 8{,}4\% \end{aligned}$$

7.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 250\,000 &= Ra_{\overline{6\times 12}|0{,}118\div 12} \\ R &= 4\,861{,}59 \end{aligned}$$

Amount outstanding:

$$\begin{aligned} P &= 4\,861{,}59 a_{\overline{72-33}|0{,}118\div 12} \\ &= 156\,848{,}15 \end{aligned}$$

The amount paid off is R93 151,85 (250 000 − 156 848,15).

Please note: If you enter the value for PMT as 4 861,59 your answer will be R156 848,15. If you however continue with the calculations without re-entering the value for the payment your answer will be R156 848,01.

8.

$$\begin{aligned} J_\alpha &= 100(e^c - 1) \\ &= 100(e^{0{,}175} - 1) \\ &= 19{,}12\% \end{aligned}$$

9.

$$\begin{aligned} S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\ &= 25\,000\left(1 + \frac{0{,}0975}{4}\right)^{5\times 4} \\ &= 40\,468{,}72 \end{aligned}$$

The balance in the account is R40 468,72.

10.

$$\begin{aligned} S &= P\left(1 + \frac{j_m}{m}\right)^{tm} + Rs_{\overline{n}|i} \\ &= 40\,468{,}72\left(1 + \frac{0{,}10}{52}\right)^{4\times 52} + 500 s_{\overline{4\times 52}|0{,}10\div 52} \\ &= 60\,349{,}05 + 127\,725{,}46 \\ &= 188\,074{,}51 \end{aligned}$$

Tracy can expect to withdraw R188 074,51.

11.

$$\begin{aligned} P &= (1+i)Ra_{\overline{n}|i} \\ &= (1+i)1\,403 a_{\overline{24}|0{,}20124\div 12} \\ &= (1+i)27\,533{,}34 \\ &= 27\,995{,}08. \end{aligned}$$

The original price of the television set was R27 995,08.

12.

$$
\begin{aligned}
S &= P(1+rt) \\
P &= S(1-dt) \\
S &= S(1-dt)(1+rt) \\
\frac{S}{S} &= (1-dt)(1+rt) \\
1+rt &= \frac{1}{(1-dt)} \\
rt &= \frac{1}{1-dt} - 1 \\
&= \frac{1-1(1-dt)}{(1-dt)} \\
&= \frac{1-1+dt}{1-dt} \\
rt &= \frac{dt}{1-dt} \\
r &= \frac{dt}{1-dt} \times \frac{1}{t} \\
r &= \frac{d}{1-dt} \\
1-dt &= \frac{d}{r} \\
dt &= 1-\frac{d}{r} \\
t &= \left(1-\frac{d}{r}\right)/d \\
t &= \left(1-\frac{0{,}075}{0{,}0968}\right) \div 0{,}075 \\
t &= 3
\end{aligned}
$$

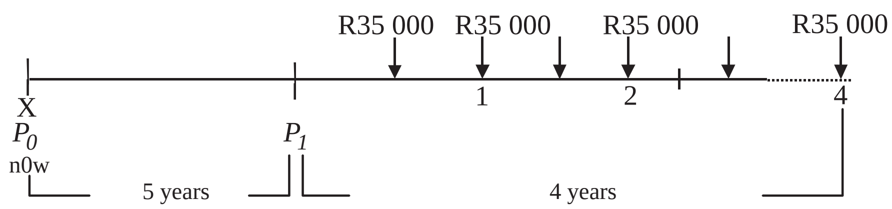
The time under consideration is 3 years.

13.

$$
\begin{aligned}
PI &= \frac{NPV + \text{original outlay}}{\text{original outlay}} \\
1{,}034 &= \frac{14\,983 + \text{outlay}}{\text{outlay}} \\
1{,}034 \text{ outlay} &= 14\,983 + \text{outlay} \\
1{,}034 \text{ outlay} - \text{outlay} &= 14\,983 \\
0{,}034 \text{ outlay} &= 14\,983 \\
\text{outlay} &= \frac{14\,983}{0{,}034} \\
&= 440\,676{,}47
\end{aligned}
$$

The original outlay was R440 676,47.

14.




$$\begin{aligned} P_1 &= Ra_{\overline{n}|i} \\ i &= 0{,}179 \div 2 \\ n &= 4 \times 2 \\ R &= 35\,000 \\ P_1 &= 35\,000a_{\overline{4\times2}|0{,}179\div2} \\ &= 194\,079{,}19 \end{aligned}$$

Charlene owes Aunt Amor R194 079,19.

15. This amount must be discounted back to the time that aunt Amor gave Charlene the money.

$$\begin{aligned} S &= P(1+\frac{j_m}{m})^{tm} \\ j_m &= 0{,}179 \\ m &= 2 \\ S &= 194\,079{,}19 \\ t &= 5 \\ P_0 &= 194\,079{,}19\left(1+\frac{0{,}179}{2}\right)^{-5\times2} \\ &= 82\,358{,}16 \end{aligned}$$

Aunt Amor lent Charlene R82 358,16.

16.

$$\begin{aligned} P &= \frac{14{,}7}{2}a_{\overline{29}|0{,}135\div2} + 100\left(1+\frac{0{,}135}{2}\right)^{-29} \\ &= 107{,}55174 \end{aligned}$$

This is a cum-interest case due to the fact that $R = 74$.

$$\begin{aligned} \text{Price}(01/07/2012) &= 107{,}55174 + 7{,}35 \\ &= 114{,}90170 \end{aligned}$$

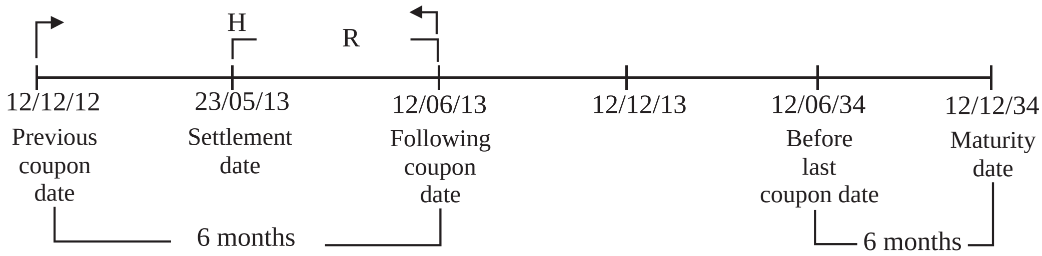
This amount must be discounted back to the settlement date.

$$\begin{aligned} P &= 114{,}90170\left(1+\frac{0{,}135}{2}\right)^{-74/181} \\ &= 111{,}87388 \end{aligned}$$

$$\begin{aligned} \text{Clean price} &= 111{,}87388 - 4{,}30932 \\ &= 107{,}56456 \end{aligned}$$

The clean price is R107,56456%.

17.




$$\begin{aligned} \text{Years} &= 12/12/34 - 12/12/13 \\ &= 21. \end{aligned}$$

We must multiply by two to get the number of half yearly coupons and add one to get the $n$.

$R$ is the number of days from the settlement date until the next coupon date.

$$R = 163(12\text{ June}) - 143(23\text{ May}) = 20$$

$H$ is the number of days in the half year in which the settlement date falls. (The number of days from the coupon date just before the settlement date until the coupon date that follows the settlement date.)

$$H = 365 - 346 + 163 = 182$$

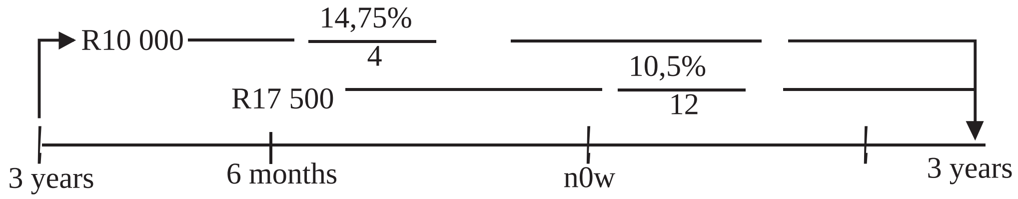
$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ &= \frac{12{,}4}{2}a_{\overline{43}|0{,}108\div 2} + 100\left(1+\frac{0{,}108}{2}\right)^{-43} \\ &= 113{,}27116 \end{aligned}$$

The present value on 12 June 2013 is R113,27116% and it is cum-interest. Add the coupon $-113{,}27116+6{,}2 = 119{,}47116$. This amount must be discounted back to the settlement date 23 May 2013.

$$\begin{aligned} P(23/05/13) &= 119{,}47116\left(1+\frac{0{,}108}{2}\right)^{-\frac{20}{182}} \\ &= 118{,}78268 \end{aligned}$$

The present value is R118,78268%.

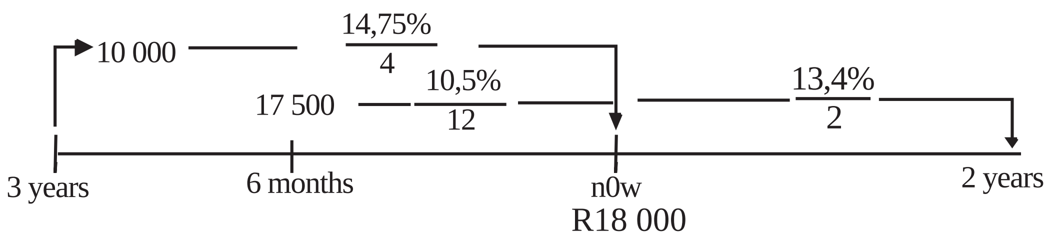
18.




$$
\begin{aligned}
S &= P_1\left(1+\frac{j_m}{m}\right)^{tm} + P_2\left(1+\frac{j_m}{m}\right)^{tm}\\
&= 10\,000\left(1+\frac{0{,}1475}{4}\right)^{6\times 4} + 17\,500\left(1+\frac{0{,}105}{12}\right)^{3{,}5\times 12}\\
&= 23\,847{,}00 + 25\,231{,}73\\
&= 49\,078{,}73
\end{aligned}
$$

Daniel owes Sarah R49 078,73 three years' from now.

19.




$$
\begin{aligned}
S &= 10\,000\left(1+\frac{0{,}1475}{4}\right)^{3\times 4} + 17\,500\left(1+\frac{0{,}105}{12}\right)^{0{,}5\times 12}\\
&= 15\,442{,}47 + 18\,439{,}08\\
&= 33\,881{,}55
\end{aligned}
$$

$$
\begin{aligned}
\text{Amount due} &= 33\,881{,}55 - 18\,000\\
&= 15\,881{,}55
\end{aligned}
$$

$$
\begin{aligned}
S &= P\left(1+\frac{j_m}{m}\right)^{tm}\\
&= 15\,881{,}55\left(1+\frac{0{,}134}{2}\right)^{2\times 2}\\
&= 20\,584{,}99
\end{aligned}
$$

Amount payable two years from now is R20 584,99.

20. We must first convert the monthly interest rate to semi-annually.

$$\begin{aligned} i &= n\left(\left(1+\frac{j_m}{m}\right)^{m\div n}-1\right) \\ &= 2\left(\left(1+\frac{0{,}089}{12}\right)^{12\div 2}-1\right) \\ &= 0{,}09067... \end{aligned}$$

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= 5\,500s_{\overline{10\times 2}|0{,}09067\div 2} \\ &= 173\,149{,}47 \end{aligned}$$

The accumulated amount is R173 149,47.

21.

$$S=\sqrt{\frac{\sum_{i=1}^{n}(x_1-\bar{x})^2}{n}}$$

We use our calculator to do this question.

The standard deviation is R252 032.

22.

$$\begin{aligned} S &= \left(R+\frac{Q}{i}\right)s_{\overline{n}|i}-\frac{nQ}{i} \\ R &= 3\,600 \\ Q &= 360 \\ i &= 0{,}10 \\ n &= 20 \end{aligned}$$

$$\begin{aligned} S &= \left(3\,600+\frac{360}{0{,}10}\right)s_{\overline{20}|0{,}10}-\frac{20\times 360}{0{,}10} \\ &= 412\,380-72\,000 \\ &= 340\,380 \end{aligned}$$

Fawzia can expect to receive R340 380.

23. The regression line equation is

$$y=8{,}5x+135$$

We enter the data directly into our calculator.

# Chapter 5

# Additional exercises and solutions

## 5.1 Additional exercises

### 5.1.1 Additional exercise 1

**Content: Chapter 2**

1. At what simple interest rate must R2 000 be invested to accumulate to R4 640 at the end of 10 years?

   [1] 23,2%.

   [2] 10,0%.

   [3] 13,2%.

   [4] 5,69%.

   [5] None of the alternatives listed above.

2. What was the present value of a loan on 5 May if it accumulates to R4 500 on 16 August of the same year at a simple interest rate of 15% per year?

   [1] R4 315,55.

   [2] R4 318,96.

   [3] R4 317,26.

   [4] R4 688,63.

   [5] None of the alternatives listed above.

3. Lulu wants to buy a television set that costs R3 500. She approaches a bank, which agrees to lend her the money, if she repays the loan in 10 months' time. What size loan should she apply for if the bank charges 13% per year bank discount on short-term loans?

   [1] R3 879,17.

   [2] R3 120,83.

   [3] R3 500,00.

   [4] R3 925,23.

   [5] None of the alternatives listed above.

4. What is the equivalent simple interest rate $r$ in question 3?

   [1] 14,58%.

   [2] 13,00%.

   [3] 12,99%.

   [4] 11,73%.

   [5] None of the alternatives listed above.

5. Joseph owes R500 due in four months' time and R700 due in nine months' time. What single payment in 12 months' time will liquidate these obligations if a simple interest rate of 11% per year is charged on all the amounts?

   [1] R1 255,92.

   [2] R1 276,08.

   [3] R1 228,42.

   [4] R1 200,00.

   [5] None of the alternatives listed above.

6. Joseph (mentioned in question 5), has a debt of R500 payable in four months' time and R700 in nine months' time from now, will receive R1 000 from his grandmother two months from now, with which he will immediately pay off against his debts. The single payment in 12 months' time from now that will liquidate his obligations, if a simple interest rate of 11% per year is charged on all the amounts, is

   [1] R200,00.

   [2] R190,44.

   [3] R164,25.

   [4] R228,42.

   [5] none of the alternatives listed above.

7. Mario has been given the option of either paying his R2 500 bill now or settling it for R2 730 after four months. If he chooses to pay after four months, the simple interest rate which he would be charged is

   [1] 0,276%.

   [2] 2,3%.

   [3] 25,27%.

   [4] 27,60%.

   [5] none of the alternatives listed above.

8. The amount of capital needed to yield R300 simple interest in 18 months' time if the interest rate is 9,5% per year, equals

   [1] R2 105,26.

   [2] R2 850,00.

   [3] R4 275,00.

   [4] R4 736,84.

   [5] none of the alternatives listed above.

9. The simple interest rate that is equal to a discount rate of 12% per year for a period of 18 months is

   [1] 0,12%.

   [2] 10,34%.

   [3] 12,00%.

   [4] 14,63%.

   [5] none of the alternatives listed above.

***The solutions to these questions are to be found on p 133 of this tutorial letter.***

### 5.1.2 Additional exercise 2

**Content: Chapter 3**

1. Mrs Naidoo deposits R15 000 into a new savings account. How much money will she have in the bank after three years if interest is compounded monthly at 8% per year?

   [1] R15 302,00

   [2] R19 053,56

   [3] R18 895,68

   [4] R18 204,44

   [5] None of the alternatives listed above.

2. How long will it take R25 000 to accumulate to R30 835,42 if interest is compounded weekly at 10,5% per year?

   [1] 2 weeks

   [2] 24 weeks

   [3] 52 weeks

   [4] 104 weeks

   [5] None of the alternatives listed above.

3. At what yearly rate, compounded semi-annually, should Mary invest R20 000 in order to have R28 000 available after 30 months?

   [1] 13,92%

   [2] 2,26%

   [3] 13,5%

   [4] 6,96%

   [5] None of the alternatives listed above.

4. John deposits R900 into a savings account paying $6\frac{1}{2}$% interest per year compounded quarterly. After three and a half years he withdraws R1 000 from the account and deposits it into another account paying 11% interest per year compounded semi-annually. How much is the total amount accrued from both accounts two years after making the second deposit?

   [1] R1 366,67

   [2] R2 138,82

   [3] R1 384,27

   [4] R2 227,85

   [5] None of the alternatives listed above.

5. What is the effective interest rate of a nominal rate of 18,75% per year compounded every three months?
   [1] 19,95%
   [2] 20,11%
   [3] 26,52%
   [4] 18,75%
   [5] None of the alternatives listed above.

6. Ben deposits R8 000 into a savings account on 18 April. Ben's money earns 16,5% interest per year compounded half-yearly credited on 1 June and 1 December every year. How much money will he have in the bank at the end of nine months, if simple interest is used for odd periods and compound interest for the full terms?
   [1] R9 023,90
   [2] R9 020,16
   [3] R8 832,25
   [4] R9 015,91
   [5] None of the alternatives listed above.

7. How much money will Ben (see question 6) have in the bank after nine months, if fractional compounding is used for the full term?
   [1] R9 013,08
   [2] R8 746,95
   [3] R9 005,26
   [4] R9 685,76
   [5] None of the alternatives listed above.

8. Fatima wants to buy a hi-fi set. She has three options when borrowing the R2 500 from the bank:
   (A) 16 % per year compounded monthly
   (B) 19% per year compounded semi-annually
   (C) 17% per year compounded annually

   Use continuous rates to decide which is the better option for Fatima.
   [1] A
   [2] B
   [3] C
   [4] B and C
   [5] None of the alternatives listed above.

9. Nkosi owes James R500 due in two months, R1 000 due in five months, R1 500 due in eight months and R200 due in 18 months. He wants to discharge his obligations by three equal payments, one due in three months, one due in six months and one due in 10 months. Calculate the payments if interest is compounded monthly at 15% per year and the end of 10 months is taken as the comparison date.

   Which answer is correct?

   [1] R1 111,71

   [2] R1 061,54

   [3] R1 074,21

   [4] R1 172,46

   [5] None of the alternatives listed above.

***The solutions to these questions are to be found on p 137 of this tutorial letter.***

### 5.1.3 Additional exercise 3

**Content: Chapters 4 and 5**

1. On his 35th birthday Ken decides to put away an amount of R400 at the end of every month towards a retirement annuity that earns interest at 10% per year, compounded monthly. The balance of Ken's account on his 50th birthday is

   [1] R165 788,14.

   [2] R6 929 756,31.

   [3] R37 222,98.

   [4] none of the alternatives listed above.

2. What is the present value of the annuity in question 1?

   [1] R165 788,14.

   [2] R6 929 756,31.

   [3] R37 222,98.

   [4] None of the alternatives listed above.

3. Susan decides to rent a television set for 20 weeks because her "soapies" and her husband's cricket viewing coincide. She has enough money in a savings account to cover the cost of renting a television set. She has two options: firstly to pay an amount of R500 upfront or secondly to pay R25 a week for 20 weeks, paid in advance. She will pay either of the two options out of her savings account, which earns interest at 14% per year compounded weekly. The amount of money that she can save if she chooses option 2 is

   [1] R476,45.

   [2] R12,55.

   [3] R500.

   [4] none of the alternatives listed above.

4. A businessman buys R100 000 of equipment on the following terms:

   Interest will be charged at a rate of 12% per year compounded semi-annually, but no payment will be made until two years after purchase. Thereafter equal semi-annual payments will be made for five years. The semi-annual payment is

   [1] R29 970,75.

   [2] R13 586,80.

   [3] R22 343,84.

   [4] R17 153,02.

   [5] none of the alternatives listed above.

***Questions 5, 6 and 7 relate to the following situation:***

*Magopi wants to buy a townhouse that costs R200 000. After making a down payment of 20% of the price of the townhouse, he manages to secure a loan for 20 years from the Nest Tree Bank at an interest rate of 15% compounded monthly.*

5. His monthly payment equals

   [1] R1 952,29.

   [2] R2 106,86.

   [3] R2 130,15.

   [4] R2 633,58.

   [5] none of the alternatives listed above.

6. Mogapi's equity in the townhouse after twelve years equals

   [1] R53 244,87.

   [2] R59 624,72.

   [3] R64 530,56

   [4] R82 595,95.

   [5] none of the alternatives listed above.

7. After twelve years the bank decides to adjust the interest rate to 12% per year, compounded monthly. Mogapi's new monthly payment equals

   [1] R1 908,15.

   [2] R2 281,50.

   [3] R2 385,19.

   [4] R2 201,76.

   [5] none of the alternatives listed above.

***The solutions to these questions are to be found on p 141 of this tutorial letter.***

### 5.1.4 Additional exercise 4

**Content: Chapter 6**

1. An investor must choose between three alternative proposals A, B and C. The initial investment outlay and the cash inflows for each are set out in the table below. His cost of capital is K = 18%. Use the internal rate of return, the net present value and the profitability index respectively to advise him with regard to the three proposals. All funds are in R1 000s.

| **Year** | **Proposal *A*** | **Proposal *B*** | **Proposal *C*** |
|---|---|---|---|
| **0** | **Investment: 500** | **Investment: 500** | **Investment: 550** |
| | Cash inflow | Cash inflow | Cash inflow |
| 1 | 200 | 200 | 280 |
| 2 | 280 | 250 | 280 |
| 3 | 300 | 280 | 280 |
| 4 | 200 | 300 | 280 |
| 5 | 180 | 280 | 280 |

2. John Modise has an opportunity to invest in a construction company. The company foresees the following cash flows during the next four years.

| **Years** | **Cash Flow** |
|---|---|
| 0 | −200 |
| 1 | 100 |
| 2 | −200 |
| 3 | 400 |
| 4 | 400 |

Money can be borrowed at 15% per year while an investment, can earn 18% interest per year in a high risk development. Considering the MIRR criterion, what advice will you give him in connection with his possible investment.

***The solutions to these questions are to be found on p 143 of this tutorial letter.***

### 5.1.5 Additional exercise 5

**Content: Chapter 7**

1. Consider the following bond: Bond XXX:

| | |
|---|---|
| Coupon rate (half yearly) | 14,7% per year |
| Redemption date | 1 January 2028 |
| Yield to maturity | 13,5% per year |
| Settlement date | 18 April 2013 |

   Calculate the all-in price, the accrued interest and the clean price.

2. Calculate the all-in price, the accrued interest and the clean price for the bond in question one on the settlement date 24 June 2013.

3. Consider the following bond:

| Bond AAA | |
|---|---|
| Coupon | 11% |
| Redemption date | 1 March 2026 |
| Yield to maturity | 13,75% |
| Settlement date | 28 September 2011 |

   The bond is sold at

   [1] discount.

   [2] premium.

   [3] par.

   [4] none of the alternatives listed above.

4. Consider the following bond: BBB

| | |
|---|---|
| Coupon rate | 7,5% |
| Redemption date | 1 April 2028 |
| Yield to maturity | 14% |
| Settlement date | 28 September 2015 |

   The clean price is given by

   [1] R38,56812%

   [2] R61,03717%

   [3] R71,91128%

   [4] none of the alternatives listed above.

***The solutions to these questions are to be found on p 145 of this tutorial letter.***

### 5.1.6 Additional exercise 6

**Content: Typical exam questions**

## Question 1

Fifteen months from now Jenny has to pay Jonas R5 000. She decides to pay him back earlier. If a simple interest rate of 13% per annum is applicable, then the amount that Jenny will have to pay Jonas seven months from now equals

[1] R4 566,67

[2] R4 601,23

[3] R4 627,24

[4] R4 655,94

[5] none of the above

## Question 2

If R17 500 accumulates to R22 000 after 53 months, the continuous compounding interest rate equals

[1] 5,181%

[2] 5,193%

[3] 5,318%

[4] 5,822%

[5] none of the above

***Questions 3 and 4 relate to the following situation:***
*Dube invested an amount of money at j% interest per annum, compounded half yearly. After four years the accumulated amount of R2 844,20 was reinvested at 1,5% per annum more than previously while the compounding period became quarterly.*

## Question 3

If, after a further three years, the accumulated amount was R3 881,49, then the original interest rate equals

[1] 9%

[2] 10,5%

[3] 11,37%

[4] 12%

[5] none of the above

## Question 4

The original amount invested equals

[1] R1 784,49

[2] R1 827,48

[3] R1 888,79

[4] R2 000,00

[5] none of the above

## Question 5

The settlement date of Bond E534 with a coupon rate of 14,7% per annum and a yield to maturity of 13,5% per annum is 18 April 2013. If the equation of the price on the next coupon date is

$$P = da_{\overline{n}|z} + 15{,}04289$$

and the all-in price equation is

$$P(18/04/2013) = 107{,}55174(1{,}0675)^{-74/182},$$

then the maturity date will be

[1] 18 April 2027

[2] 1 July 2027

[3] 18 October 2027

[4] 1 January 2028

[5] none of the above

## Question 6

Francois's friend James has decided to open a rugby jersey shop next to Francois's rugby ball stall. Starting on 1 January he invested a monthly amount into an account earning 9,4% interest per annum compounded monthly. If he had R250 000 in this account at the end of September of the same year, his monthly deposit equalled approximately

[1] R23 123,46

[2] R24 131,36

[3] R26 709,50

[4] R26 918,73

[5] none of the above

## Question 7

The equation for the present value of Bond ABC on 8/12/2015 is

$$P(8/12/2015) = \tfrac{5}{2}a_{\overline{6}|0,12\div 2} + 100(1 + \tfrac{0,12}{2})^{-6}$$

with

$$f = \frac{33}{184}$$

and the accrued interest equal to R2,06849%.
The clean price of Bond ABC equals

[1] R79,86029%

[2] R82,33420%

[3] R84,40269%

[4] R86,41118%

[5] none of the above

***Questions 8 and 9 relate to the following situation:***
*Clever Chris took out an endowment policy with an annual payment of R6 500 that increases each year by R1 700.*

## Question 8

If money is worth 10% per annum, then after 20 years the policy is worth

[1] R200 068,74

[2] R459 257,94

[3] R874 574,99

[4] R1 005 962,49

[5] none of the above

## Question 9

The amount of interest earned by the policy equals approximately

[1] R162 300

[2] R453 000

[3] R390 000

[4] R552 960

[5] none of the above

## Question 10

Adriana's investment with an initial outlay of R225 000 returns a constant cash flow of R36 000 per annum for 15 years. The internal rate of return on the investment equals

[1] 6,01%
[2] 6,25%
[3] 13,65%
[4] 14,46%
[5] none of the above

***Questions 11 and 12 relate to the following situation:***
*The following table supplies data of the average interest rates and corresponding number of policies sold.*

| *Average interest rate* ($x$) | *Number of policies sold* ($y$) |
|---|---|
| *13,00%* | *200* |
| *13,25%* | *250* |
| *14,00%* | *300* |
| *14,50%* | *450* |
| *15,00%* | *450* |
| *16,00%* | *300* |
| *17,00%* | *200* |
| *17,50%* | *150* |
| *18,00%* | *120* |

## Question 11

The arithmetic mean of the average interest rate equals

[1] 13,825%
[2] 15,00%
[3] 15,36%
[4] 15,5%
[5] none of the above

## Question 12

The relationship between the average interest rates and number of policies sold can be represented by the regression line

[1] $y = -5\,000 + 400x$

[2] $y = -3{,}67 + 15{,}67x$

[3] $y = 17{,}26 - 0{,}00705x$

[4] $y = 718{,}35 - 29{,}26x$

[5] none of the above

***Questions 13 and 14 relate to the following situation:***
*When Caitlyn was born her grandma decided that she would like to give Caitlyn R100 000 on her 21st birthday. For 10 years grandma could manage, every six months, to pay an amount into an account earning 9,25% per annum, compounded half yearly. After 10 years when the interest rate changed to 8% per annum, compounded monthly, grandma stopped paying money into the account but left the money there.*

## Question 13

The balance in the account when the interest rate changed to 8% per annum, compounded monthly, equals

[1] R36 984,49

[2] R41 599,60

[3] R42 888,29

[4] R53 191,49

[5] none of the above

## Question 14

The half-yearly payment into the account equals

[1] R1 242,00

[2] R1 308,77

[3] R1 349,31

[4] R1 673,46

[5] none of the above

***The solutions to these questions are to be found on p 149 of this tutorial letter.***

### 5.1.7 Additional exercise 7

**Content: Typical exam questions**

## Question 1

Ipi invested R16 000 in an account on 21 February. It will accumulate to R16 570,08 on 7 December of the same year. The simple interest rate applicable is

[1] 3,44%
[2] 3,56%
[3] 4,3%
[4] 4,4%
[5] 4,5%

## Question 2

The continuous compounding rate for an effective rate of 16,13% is

[1] 16,13%
[2] 17,5%
[3] 19,12%
[4] 21,076%
[5] none of the above

## Question 3

Three years ago Jack borrowed R7 000 from Jill at 11% per year compounded quarterly. Eighteen months ago he borrowed another R9 000 at 9% per year compounded monthly. The amount that Jack will owe Jill three years from now equals

[1] R26 654,00
[2] R26 896,73
[3] R26 936,17
[4] R30 682,02
[5] none of the above

***Questions 4 and 5 relate to the following situation:***
*Ian wants to open a bicycle shop at the Bi-a-Ride complex. He estimates that he will have R250 000 available on 16 June from an investment made on 7 September of the previous year, earning 7,65% interest per year compounded on the 1st day of each month.*

## Question 4

If simple interest is used for odd periods and compound interest for the full terms, then the amount that Ian deposited on 7 September equals

[1] R235 677,98

[2] R235 727,23

[3] R264 776,03

[4] R265 192,37

[5] none of the above

## Question 5

If fractional compounding is used for the whole period, then the amount that Ian deposited on 7 September equals

[1] R235 635,69

[2] R235 679,96

[3] R236 048,56

[4] R264 776,03

[5] R265 190,14

***Questions 6 and 7 relate to the following situation:***
*Paul took out an endowment policy. Its initial annual payment of R6 000 will increase annually by R1 500. The policy matures in 30 years. Its expected annual interest rate is 9,70%.*

## Question 6

The amount that Paul can expect to receive after 30 years equals

[1] R932 583,78

[2] R1 165 729,72

[3] R2 872 232,69

[4] R3 336 150,21

[5] none of the above

## Question 7

The total amount paid for the policy equals

[1] R120 575,78

[2] R223 500,00

[3] R225 000,00

[4] R832 500,00

[5] none of the above

## Question 8

An investment with an initial outlay of R500 000 generates five successive annual cash inflows of R75 000, R190 000, R40 000, R150 000 and R180 000 respectively. The internal rate of return (IRR) equals

[1] 7,78%

[2] 9,48%

[3] 21,3%

[4] 27,0%

[5] none of the above

***Questions 9, 10 and 11 relate to the following situation:***
*The following is an extract from the amortisation schedule of the home loan of John Pele:*

| Month | Outstanding principal at month beginning | Interest due at month end | Monthly payment | Principal repaid | Principal outstanding at month end |
|---|---|---|---|---|---|
| 147 | R8 155,83 | A | R2 080,54 | R2 014,27 | R6 141,56 |
| 148 | R6 141,56 | R49,90 | R2 080,54 | R2 030,64 | B |
| 149 | B | R33,40 | R2 080,54 | R2 047,14 | R2 063,78 |
| 150 | R2 063,78 | R16,77 | R2 080,54 | R2 063,77 | R0 |

## Question 9

The value of A equals

[1] R41,65

[2] R49,50

[3] R66,27

[4] R166,33

[5] R167,86

## Question 10

The value of B equals

[1] R4 061,02

[2] R4 077,79

[3] R4 094,21

[4] R4 110,92

[5] R4 127,68

## Question 11

If the interest rate has never changed, the original amount of John Pele's home loan was (rounded to the nearest thousand rand)

[1] R21 000,00

[2] R180 000,00

[3] R310 000,00

[4] R312 000,00

[5] R606 000,00

***Questions 12 and 13 relate to the following situation:***
*Consider Bond ABC.*

| | |
|---|---|
| *Coupon rate:* | *9,91% per year* |
| *Yield to maturity:* | *7,47% per year* |
| *Settlement date:* | *17 June 2013* |
| *Maturity date:* | *11 January 2042* |

## Question 12

The all-in-price equals

[1] R127,85047%

[2] R128,00061%

[3] R128,47362%

[4] R128,62450%

[5] R132,93158%

## Question 13

The clean price of Bond ABC equals

[1] R127,34899%

[2] R127,97288%

[3] R128,50209%

[4] R128,66892%

[5] R129,27612%

***Questions 14 and 15 relate to the following situation:***
*The following table supplies data of the inflation rate and the corresponding prime lending rate during the same time period.*

| *Inflation rate (%)(x)* | *Prime lending rate (%)(y)* |
|---:|---:|
| *3,3* | *5,2* |
| *6,2* | *8,0* |
| *11,0* | *10,8* |
| *9,1* | *7,9* |
| *5,8* | *6,8* |
| *6,5* | *6,9* |
| *7,6* | *9,0* |

## Question 14

The linear relationship between the inflation rate and the prime lending rate can be represented by the regression line

[1] $y = 3{,}17477 + 0{,}65407x$

[2] $y = 0{,}65407 + 3{,}17477x$

[3] $y = -2{,}76656 + 1{,}26128x$

[4] $y = 1{,}26128 - 2{,}76656x$

[5] $y = 2{,}28372 + 0{,}88372x$

## Question 15

The correlation coefficient equals

[1] −0,908

[2] +0,495

[3] +0,546

[4] +0,908

[5] none of the above

***The solutions to these questions are to be found on p 153 of this tutorial letter.***

## 5.2 Solutions: additional exercises

### 5.2.1 Solution: additional exercise 1

1. Simple interest: $I = Prt$

$$\begin{aligned} I &= \text{interest earned} = \text{R4 640} - \text{R2 000} = \text{R2 640} \\ P &= \text{principal amount} = \text{R2 000} \\ r &= \text{simple interest rate} = ? \\ t &= \text{period of investment} = 10 \text{ years} \end{aligned}$$

$$\begin{aligned} I &= Prt \\ 2\ 640 &= 2\ 000 \times r \times 10 \\ \tfrac{2640}{2000\times 10} &= r \\ r &= 0{,}132 \\ r &= 13{,}2\%. \end{aligned}$$

**OR**

$$\begin{aligned} S &= P\,(1+rt) \\ 4\,640 &= 2\,000\,(1 + r \times 10) \\ \tfrac{4\,640}{2\,000} &= 1 + 10r \\ 10r &= \tfrac{4\,640}{2\,000} - 1 \\ 10r &= 1{,}32 \\ r &= 0{,}132 \\ r &= 13{,}2\%. \end{aligned}$$

R2 000 must be invested at an interest rate of 13,2% to accumulate to R4 640 at the end of ten years.

2. Simple interest: $S = P(1 + rt)$

$$\begin{aligned} S &= \text{accumulated amount} = \text{R4 500} \\ P &= \text{principal or present value} = ? \\ r &= \text{simple interest rate} = 0{,}15 \\ t &= \text{period of loan} = \text{from 5 May tot 16 August} \end{aligned}$$

| **Period** | **Number of days** |
|---|---|
| 5 - 31 May | 27 (including 5th) |
| 1 - 30 June | 30 |
| 1 - 31 July | 31 |
| 1 - 16 August | 15 (excluding 16th) |
| | 103 days |

**OR**

Use the number of each day of the year table. Day number 228 (16 August) minus 125 (5 May) equals 103.

$$\begin{aligned} S &= P(1+rt) \\ P &= \tfrac{S}{1+rt} \\ &= \frac{4500}{1+0{,}15\times\frac{103}{365}} \\ &= 4\,317{,}26. \end{aligned}$$

The present value is R4 317,26.

3. Discount: $P = S(1 - dt)$.

$$\begin{aligned} P &= \text{present value} = \text{money she will receive} = \text{R3 500} \\ S &= \text{future value} = \text{money she applies for} = ? \\ d &= \text{discount rate} = 0{,}13 \\ t &= \text{period of loan} = 10 \text{ months} = \tfrac{10}{12}\text{years.} \end{aligned}$$

$$\begin{aligned} P &= S(1 - dt) \\ S &= \tfrac{P}{1-dt} \\ S &= \tfrac{3\ 500}{(1-0{,}13\times\frac{10}{12})} \\ &= 3\,925{,}23 \end{aligned}$$

She has to apply for a loan of R3 925,23.

4.

$$\begin{aligned} I &= 3\,925{,}23 - 3\,500 \\ &= 425{,}23 \end{aligned}$$

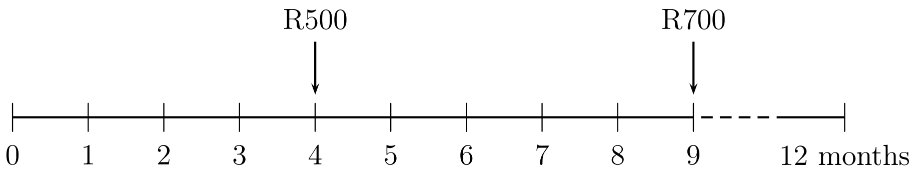
The interest paid in question 3 is R425,23.

Thus the interest rate equivalent to the above interest can be calculated using $I = Prt$.

$$\begin{aligned} I &= Prt \\ 425{,}23 &= 3\,500 \times r \times \tfrac{10}{12} \\ \tfrac{425{,}23}{3500} \times \tfrac{12}{10} &= r \\ r &= 0{,}1458 \end{aligned} \qquad \text{OR} \qquad \begin{aligned} r &= \tfrac{d}{1-dt} \\ &= \tfrac{0{,}13}{1-0{,}13} \times \tfrac{10}{12} \\ &= 0{,}1458. \end{aligned}$$

The equivalent interest rate is 14,58%.

5.




r = 11% per year

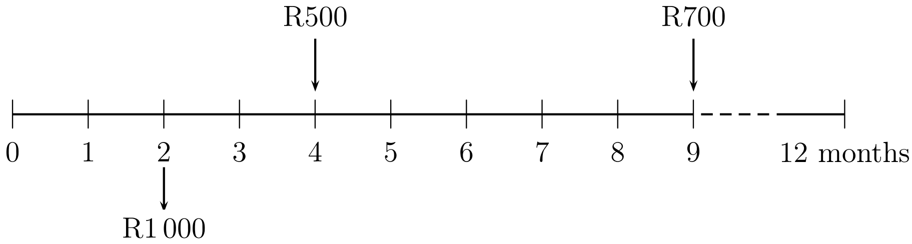
What he owes he must pay back. But because of the time value of money we have to calculate the value of all his obligations and payments on the same date namely month 12. The R500 must move eight months forward and the R700, three months forward.

R500's value at month 12: $500(1 + 0{,}11 \times\frac{8}{12}) = 536{,}67$.

R700's value at month 12: $700(1 + 0{,}11 \times\frac{3}{12}) = 719{,}25$.

Thus he owes R1 255,92 ($536{,}67 + 719{,}25$) at month 12. He has to make a single payment of R1 255,92 in 12 months' time.

6.




What he owes he must pay back. But because of the time value of money we have to calculate the value of all his obligations and payments on the same date, that is at month 12.

R500 must be moved eight months forward:

$$500\left(1+0{,}11\times\frac{8}{12}\right)=536{,}67$$

R700 must be moved three months forward:

$$700\left(1+0{,}11\times\frac{3}{12}\right)=719{,}25$$

R1 000 must be moved 10 months forward:

$$1\ 000\left(1+0{,}11\times\frac{10}{12}\right)=1\ 091{,}67$$

Thus the amount that he still has to pay at month 12:

Obligations − payments made is R164,25 (536,67 + 719,25 − 1 091,67)

7.

$$\begin{aligned}
I &= Prt\\
\text{Use } I &= 2\,730\ -\ 2\,500\ =\ 230,\\
P &= 2\,500\quad\text{and}\\
t &= \tfrac{4}{12}.\\
\text{From } I &= Prt\quad\text{it follows that}\\
230 &= 2\,500\ \times\ \tfrac{4}{12}\ \times\ r\\
r &= \tfrac{230\times 12}{2\,500\times 4}\\
&= 27{,}6\%.
\end{aligned}$$

Mario will pay 27,60% simple interest.

8.

$$\begin{aligned} I &= \mathrm{Pr}\,t \\ \text{with} \quad I &= 300, \\ r &= 0{,}095 \quad \text{and} \\ t &= \tfrac{18}{12}. \end{aligned}$$

$$\begin{aligned} \text{Now} \quad 300 &= P \times 0{,}095 \times \tfrac{18}{12} \\ P &= \frac{300 \times 12}{0{,}095 \times 18} \\ &= 2\,105{,}26. \end{aligned}$$

The amount needed is R2 105,26.

9.

$$\begin{aligned} D &= Sdt \\ \text{with} \quad S &= 100, \\ d &= 0{,}12 \quad \text{and} \\ \mathrm{t} &= \frac{18}{12}. \end{aligned}$$

$$\begin{aligned} \text{Now} \quad D &= 100 \times 0{,}12 \times \frac{18}{12} \\ &= 18{,}00 \end{aligned}$$

In order to determine the applicable interest rate we use

$$\begin{aligned} I &= Prt \\ \text{with} \quad I &= 18, \\ P &= (100 - 18) \quad \text{and} \\ \mathrm{t} &= \frac{18}{12}. \end{aligned}$$

$$\begin{aligned} \text{Therefore} \quad r &= \frac{18 \times 12}{(100 - 18) \times 18} \\ &= 14{,}63. \end{aligned}$$

OR

$$\begin{aligned} r &= \frac{d}{1 - dt} \\ &= \frac{0{,}12}{1 - 0{,}12} \times \frac{18}{12} \\ &= 0{,}1463. \end{aligned}$$

The applicable interest rate is 14,63%.

### 5.2.2 Solution: additional exercise 2

1. Compound interest: $S = P\left(1+\frac{j_m}{m}\right)^{tm}$

$$\begin{array}{rcl} S &=& \text{Future value} = ? \\ P &=& \text{Present value} = \text{R15 000} \\ j_m &=& \text{interest rate per year} = 0{,}08 \\ m &=& \text{number of compounding periods per year} = 12 \\ \text{t} &=& \text{term of investment} = 3 \text{ years.} \end{array}$$

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 15\,000\left(1+\frac{0{,}08}{12}\right)^{3\times 12} \\ &= 19\,053{,}56 \end{aligned}$$

She will have R19 053,56 in the bank after three years.

2. Compound interest:

$$\begin{array}{rcl} S &=& \text{R30 835,42} \\ P &=& \text{R25 000,00} \\ j_m &=& 0{,}105. \\ m &=& 52 \\ t &=& ? \end{array}$$

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ 30\,835{,}42 &= 25\,000\left(1+\frac{0{,}105}{52}\right)^{52t} \\ \frac{30\,835{,}42}{25\,000} &= \left(1+\frac{0{,}105}{52}\right)^{52t} \\ \ln\left(\frac{30\,835{,}42}{25\,000}\right) &= 52t\,\ln\left(1+\frac{0{,}105}{52}\right) \\ \frac{\ln\left(\frac{30\,835{,}42}{25\,000}\right)}{\ln\left(1+\frac{0{,}105}{52}\right)} &= 52t \\ 52t &= 104 \\ t &= 2 \end{aligned}$$

It will take $2 \times 52 = 104$ weeks for R25 000 to accumulate to R30 835,42.

3. Compound interest: $S = P\left(1+\frac{j_m}{m}\right)^{tm}$.

$$\begin{array}{rcl} S &=& \text{R28 000} \\ P &=& \text{R20 000} \\ j_m &=& \text{interest rate per year} = ? \\ m &=& \text{number of compounding periods per year} = 2 \\ t &=& \text{term of investment} = 30 \text{ months} = \text{two and a half years} \end{array}$$

$$
\begin{aligned}
S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
28\,000 &= 20\,000\left(1+\tfrac{j_m}{2}\right)^{2,5\times 2} \\
\tfrac{28\,000}{20\,000} &= \left(1+\tfrac{j_m}{2}\right)^{5} \\
\left(\tfrac{28\,000}{20\,000}\right)^{\left(\frac{1}{5}\right)} &= 1+\tfrac{j_m}{2} \\
\left(\tfrac{28\,000}{20\,000}\right)^{\left(\frac{1}{5}\right)} - 1 &= \tfrac{j_m}{2} \\
j_m &= \left(\left(\tfrac{28\,000}{20\,000}\right)^{\left(\frac{1}{5}\right)} - 1\right)\times 2 \\
&= 13{,}92\%
\end{aligned}
$$

Mary should invest the R20 000 at a yearly rate of 13,92% compounded semi-annually.

4. First we calculate the value of R900 after three and a half years.

   With $P = 900$, $j_m = 0{,}065$, $m = 4$ and $t = 3{,}5$.

$$900\left(1+\frac{0{,}065}{4}\right)^{4\times 3{,}5} = \text{R}1\,127{,}85$$

   Of the R1 127,85, R1 000 must be invested for two years at 11% compounded semi-annually and R127,85 (1 127,85 − 1 000) must be invested for two years at $6\frac{1}{2}\%$ compounded quarterly.

   After two years R1 000 will accumulate to:

$$
\begin{aligned}
S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\
&= 1\,000\left(1+\tfrac{0{,}11}{2}\right)^{2\times 2} \\
&= 1\,000\left(1+\tfrac{0{,}11}{2}\right)^{4} \\
&= 1\,238{,}82
\end{aligned}
$$

   After two years R127,85 will accumulate to:

$$
\begin{aligned}
S &= 127{,}85\left(1+\tfrac{0{,}065}{4}\right)^{4\times 2} \\
&= 127{,}85\left(1+\tfrac{0{,}065}{4}\right)^{8} \\
&= 145{,}45
\end{aligned}
$$

   His total accrued amount after two years of making the second deposit is R1 384,27 (1 238,82 + 145,45).

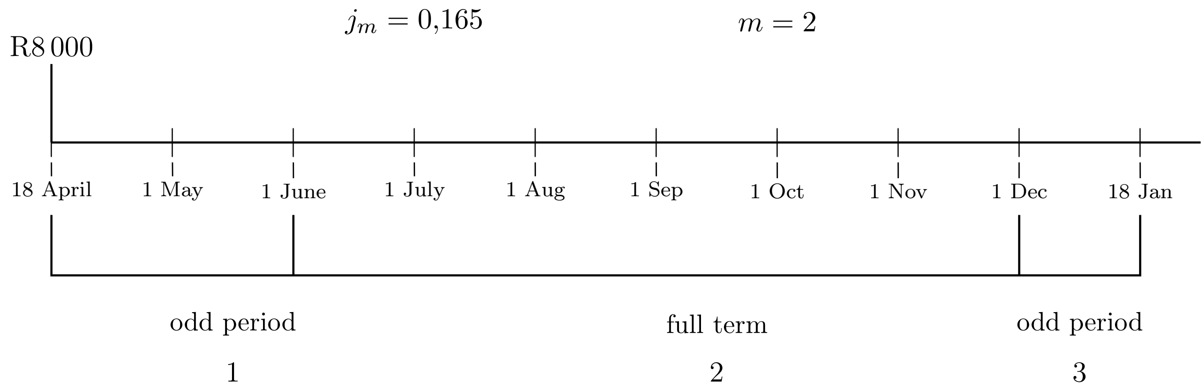
5. Effective interest rate: $J_{eff} = 100\left[\left(1+\frac{j_m}{m}\right)^{m} - 1\right]$

   $j_m$ = nominal rate = 0,1875

   $m$ = number of times the interest is calculated per year $= \frac{12}{3} = 4$

$$
\begin{aligned}
J_{eff} &= 100[(1+\tfrac{0{,}1875}{4})^{4} - 1] \\
&= 20{,}11\%
\end{aligned}
$$

6.




Period 1: odd period of 13 + 31 = 44 days (Day number 152 (1 June) minus 108 (18 April) = 44)

Period 2: one full term = one half year (1 June - 1 Dec)

Period 3: odd period of 31 + 17 = 48 days

Value of R8 000 on 1 June:

$$\begin{aligned} S_1 &= P(1+rt) \\ &= 8\,000\left(1+\tfrac{44}{365}\times 0{,}165\right) \end{aligned}$$

Value of R8 000 on 1 December: ($t = \frac{6}{12}$ and $m = 2$)

$$\begin{aligned} S_2 &= S_1\left(1+\tfrac{j_m}{m}\right)^{tm} \\ &= (\text{value on 1 June}) \times \left(1+\tfrac{0{,}165}{2}\right)^{\left(\frac{6}{12}\times 2\right)} \end{aligned}$$

Value of R8 000 on 18 January:

$$\begin{aligned} S_3 &= S_2(1+rt) \\ &= (\text{value on 1 December}) \times \left(1+\tfrac{48}{365}\times 0{,}165\right) \end{aligned}$$

Thus value of R8 000 on 18 January:

$$\begin{aligned} S &= 8\,000\left(1+\tfrac{44}{365}\times 0{,}165\right)\left(1+\tfrac{0{,}165}{2}\right)\left(1+\tfrac{48}{365}\times 0{,}165\right) \\ &= 9\,023{,}90 \end{aligned}$$

He will have R9 023,90 in the bank at the end of nine months.

7. $S = P\left(1+\frac{j_m}{m}\right)^{tm}$

$$\begin{aligned} j_m &= \text{interest rate} = 0{,}165 \\ m &= \text{number of compounding periods per year} = 2 \\ t &= \text{term of investment} \\ &= \text{one compounding period of six months} \\ &\quad\ \text{plus the number of odd days express as a fraction of a year} \\ &= \left(\tfrac{1}{2}+\tfrac{44+48}{365}\right) \end{aligned}$$

$$\begin{aligned} S &= 8000\left(1+\frac{0{,}165}{2}\right)^{\left(\frac{6}{12}+\frac{44+48}{365}\right)\times\frac{2}{1}} \\ &= \text{R}9\,013{,}08 \end{aligned}$$

He will have R9 013,08 in the bank after nine months if fractional compounding is used.

8. Continuous rate: $c = mln\left(1+\frac{j_m}{m}\right)$.

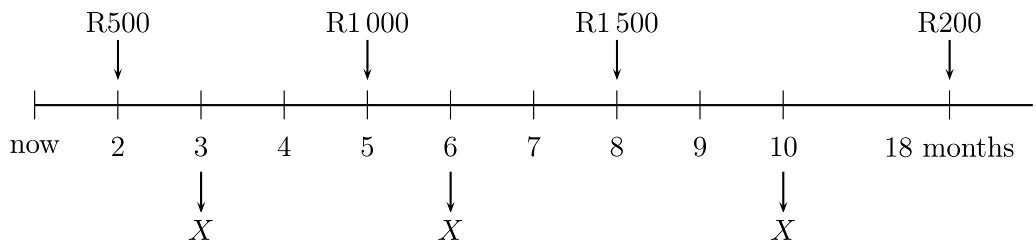
Option A: $$\begin{aligned} c &= 12\ \ln\left(1+\frac{0{,}16}{12}\right) \\ &= 15{,}89\%. \end{aligned}$$

Option B: $$\begin{aligned} c &= 2\ \ln\left(1+\frac{0{,}19}{2}\right) \\ &= 18{,}15\%. \end{aligned}$$

Option C: $$\begin{aligned} c &= 1\ \ln\left(1+\frac{0{,}17}{1}\right) \\ &= 15{,}70\%. \end{aligned}$$

The best option for Fatima is the one with the lowest interest rate, thus option C.

9.




Because of the time value of money all the moneys must be taken to the same date namely the comparison date, month 10.

The values of the obligations at the end of month 10:

For R500: $500 \times \left[1+\frac{0{,}15}{12}\right]^{8}$ $\quad t = \frac{8}{12}$ and $m = 12$.

For R1 000: $1\,000 \times \left[1+\frac{0{,}15}{12}\right]^{5}$ $\quad t = \frac{5}{12}$ and $m = 12$.

For R1 500: $1\,500 \times \left[1+\frac{0{,}15}{12}\right]^{2}$ $\quad t = \frac{2}{12}$ and $m = 12$.

For R200: $200 \times \left[1+\frac{0{,}15}{12}\right]^{-8}$ $\quad t = \frac{-8}{12}$ and $m = 12$.

Thus the total obligation at the end of month 10 is:

$$\begin{aligned} \text{Obligations} &= 500\left[1+\frac{0{,}15}{12}\right]^{8} + 1\,000\left[1+\frac{0{,}15}{12}\right]^{5} + 1\,500\left[1+\frac{0{,}15}{12}\right]^{2} + 200\left[1+\frac{0{,}15}{12}\right]^{-8} \\ &= 3\,335{,}14 \end{aligned}$$

The values of the payments at the end of month 10 are:

First payment: $X\left[1+\frac{0{,}15}{12}\right]^7$.

Second payment: $X\left[1+\frac{0{,}15}{12}\right]^4$.

Third payment: X.

Take out the common factor $X$ in all the terms:

$$
\begin{aligned}
\text{Payments} &= \text{Obligations}\\
X\left[1+\tfrac{0{,}15}{12}\right]^7 + X\left[1+\tfrac{0{,}15}{12}\right]^4 + X &= 3\,335{,}14 \quad \text{Take out the common factor } X\text{:}\\
X\left[\left(1+\tfrac{0{,}15}{12}\right)^7 + \left(1+\tfrac{0{,}15}{12}\right)^4 + 1\right] &= 3\,335{,}14\\
X &= \frac{3\,335{,}14}{\left(1+\frac{0{,}15}{12}\right)^7 + \left(1+\frac{0{,}15}{12}\right)^4 + 1}\\
&= \frac{3\,335{,}14}{3{,}14180}\\
X &= 1\,061{,}54
\end{aligned}
$$

Nkosi has to make three payments of R1 061,54 each.

### 5.2.3 Solution: additional exercise 3

1. Future value of an annuity: $S = Rs_{\overline{n}|i}$

   $R$: payments made = R400

   $n$: term of payments = 15 years = 15 × 12 months = 180 months

   $i$: interest rate = 0,10 ÷ 12 (NB Don't round this off.)

   $$
   \begin{aligned}
   S &= 400s_{\overline{15\times 12}|0{,}10\div 12}\\
   &= 165\,788{,}14
   \end{aligned}
   $$

   Peter's account balance on his 50th birthday is R165 788,14.

2. Present value of an annuity : $P = Ra_{\overline{n}|i}$

   $R$: payments made = R400

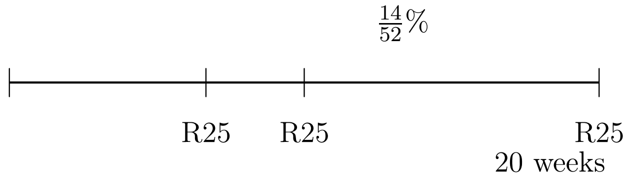
   $n$: term of payments = 15 years = 15 × 12 months = 180 months

   $i$: interest rate = 0,10 ÷ 12

   $$
   \begin{aligned}
   P &= 400a_{\overline{15\times 12}|0{,}10\div 12}\\
   &= 37\ 222{,}98
   \end{aligned}
   \qquad \text{OR} \qquad
   \begin{aligned}
   S &= P\left(1+\frac{j_m}{m}\right)^{tm}\\
   165\,788{,}14 &= P\left(1+\frac{0{,}10}{12}\right)^{15\times 12}\\
   P &= 37\,222{,}98
   \end{aligned}
   $$

   The present value of the annuity is R37 222,98.

3.




PV of first option: R500

PV of second option:

$$\begin{aligned} P &= (1+i)\,Ra_{\overline{n}|i} \\ &= \left(1+\tfrac{0{,}14}{52}\right) 25a_{\overline{20}|0{,}14\div 52} \\ &= 487{,}45 \end{aligned}$$

PV of first option − PV of second option

$$\begin{aligned} &= 500 - 487{,}45 \\ &= 12{,}55 \end{aligned}$$

She can save R12,55 if she chooses option 2.

4. Value of his obligation after two years, thus four half-year periods:

$$\begin{aligned} S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\ &= 100\,000\left(1+\tfrac{0{,}12}{2}\right)^{2\times 2} \\ &= 126\,247{,}70 \end{aligned}$$

The new value of the loan is now R126 247,70.

Value of semi-annual payments for the next five years, thus $5 \times 2 = 10$ half year periods:

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 126\ 247{,}70 &= Ra_{\overline{5\times 2}|0{,}12\div 2} \\ &= 17\,153{,}02 \end{aligned}$$

The semi-annual payment is R17 153,02.

5. First work out the amount of the loan for which Mogapi must apply.

20% of R200 000 = R40 000. The amount of the loan is R160 000.

OR

80% of R200 000 equals R160 000.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ \text{with}\quad P &= 160\,000 \\ n &= 20 \times 12 \\ i &= 0{,}15 \div 12 \\ R &= ? \end{aligned}$$

$$\begin{aligned} 160\,000 &= Ra_{\overline{20\times 12}|0{,}15\div 12} \\ R &= 2\,106{,}86 \end{aligned}$$

The monthly payments are R2 106,86.

6.

$$\begin{array}{rrcl} & P & = & Ra_{\overline{n}\rceil i} \\ \text{with} & R & = & 2\ 102{,}86 \\ & n & = & (20-12)\times 12 \\ & i & = & 0{,}15 \div 12 \\ & P & = & ? \\ \\ & P & = & 2\ 106{,}86 a_{\overline{8\times 12}\rceil 0{,}15\div 12} \\ & & = & 117\,404{,}05 \end{array}$$

Mogapi's equity in his townhouse after twelve years equals R82 595,95 $(160\,000 - 117404{,}05 + 40000)$

7.

$$\begin{array}{rrcl} & P & = & Ra_{\overline{n}\rceil i} \\ \text{with} & P & = & 117\ 404{,}05 \\ & n & = & 8\times 12 \\ & i & = & 0{,}12 \div 12 \\ & R & = & ? \\ \\ & 117\,404{,}05 & = & Ra_{\overline{8\times 12}\rceil 0{,}12\div 12} \\ & & = & 1\,908{,}15 \end{array}$$

The payments are R1 908,15.

### 5.2.4 Solution: additional exercise 4

1. **Internal rate of return:**

$$\text{A: } f(I) = \tfrac{200}{1+I} + \tfrac{280}{(1+I)^2} + \tfrac{300}{(1+I)^3} + \tfrac{200}{(1+I)^4} + \tfrac{180}{(1+I)^5} - 500$$

IRR $=$ 37,7%

$$\text{B: } f(I) = \tfrac{200}{1+I} + \tfrac{250}{(1+I)^2} + \tfrac{280}{(1+I)^3} + \tfrac{300}{(1+I)^4} + \tfrac{280}{(1+I)^5} - 500$$

IRR=40,3%

$$\text{C: } f(I) = \tfrac{280}{1+I} + \tfrac{280}{(1+I)^2} + \tfrac{280}{(1+I)^3} + \tfrac{280}{(1+I)^4} + \tfrac{280}{(1+I)^5} - 550$$

Use IRR $=$ 42,1%

Because $I > K = 18\%$ all three proposals are acceptable.

As C has the highest internal rate of return, we advise to choose C.

**Net present value**

A: $N = \frac{200}{1{,}18} + \frac{280}{1,18^2} + \frac{300}{1,18^3} + \frac{200}{1,18^4} + \frac{180}{1,18^5} - 500 = 235$

B: $N = \frac{200}{1{,}18} + \frac{250}{1,18^2} + \frac{280}{1,18^3} + \frac{300}{1,18^4} + \frac{280}{1,18^5} - 500 = 297$

C: $P = 280a_{\overline{5}|0{,}18} = 876$

$NPV = 876 - 550 = 326$

Choose C because it has the highest $NPV$.

**OR**

$$\begin{aligned} N &= \frac{280}{1{,}18} + \frac{280}{1{,}18^2} + \frac{280}{1{,}18^3} + \frac{280}{1{,}18^4} + \frac{280}{1{,}18^5} - 550 \\ &= 326 \end{aligned}$$

**Profitability index**

A: $PI = \frac{235+500}{500} = \frac{735}{500} = 1{,}47.$

B: $PI = \frac{\text{NPV+Outlay}}{\text{Outlay}} = \frac{297+500}{500} = 1{,}594$

C: $PI = \frac{\text{NPV+Outlay}}{\text{Outlay}} = \frac{326+550}{550} = 1{,}593$

Recommendation: Choose B.

2. MIRR $= (C/PV_{out})^{1/n} - 1$

$I$: Present value of the cash outflows

$$\begin{aligned} PV_{out} &= 200 + 200(1 + 0{,}15)^{-2} \\ &= 351{,}23 \end{aligned}$$

$C$: Future value of cash inflows

$$\begin{aligned} C &= 100(1 + 0{,}18)^3 + 400(1 + 0{,}18)^1 + 400 \\ &= 1036{,}30 \end{aligned}$$

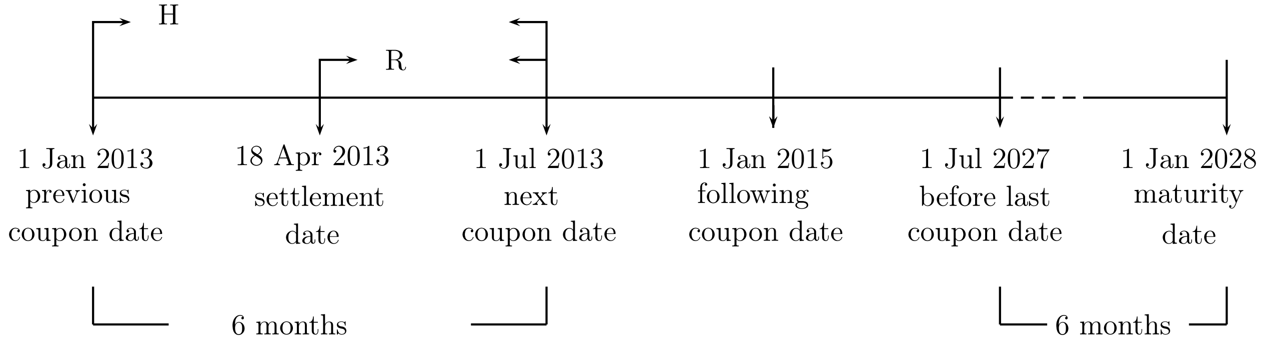
$n$: number of years $= 4$

$$\begin{aligned} \text{MIRR} &= \left(\frac{C}{PV_{out}}\right)^{\frac{1}{n}} - 1 \\ \text{MIRR} &= \left(\frac{1036{,}30}{351{,}23}\right)^{\left(\frac{1}{4}\right)} - 1 \\ &= 0{,}3106 \\ &= 31{,}06\% \end{aligned}$$

Because 31,06% is larger than the compound interest rate of 18% he is going to earn more by investing in the Construction Company than leaving the money in the bank.

### 5.2.5 Solution: additional exercise 5

1.




The number of years = 1/1/2028 − 1/7/2013.

As the months (January and July) aren't the same we must move the next coupon date to the following coupon date. Thus 1/7/2013 becomes 1/1/2015.

$$\begin{aligned}\text{Years} &= (1/1/2028 - 1/1/2015)\\ &= 14\end{aligned}$$

We must now multiply by two to get the half yearly coupons.

$$\begin{aligned}n &= 14 \times 2\\ &= 28\end{aligned}$$

We must now add the period 1/7/2013 to 1/1/2015 that is one coupon.

$$\begin{aligned}n &= 28 + 1\\ &= 29\end{aligned}$$

The number of days from the settlement date 18 April, until the next coupon date 1 July, is $R$: The day number 182 (1 July) minus 108 (18 April) equals 74 thus R = 74.

The number of days in the half year in which the settlement date falls (1/1/2013 to 1/7/2013) is $H$: The day number 182 (1 July) minus 1 (1 January) equals 181, thus $H = 181$.

The present value of the bond on 1/7/2013 is:

$$\begin{aligned}P &= da_{\overline{n}|z} + 100(1+z)^{-n}\\ &= \frac{14{,}7}{2}a_{\overline{29}|0{,}135\div 2} + 100\left(1+\frac{0{,}135}{2}\right)^{-29}\\ &= 107{,}55174\end{aligned}$$

As the settlement date is more than ten days from the next coupon date we must add the coupon.

$$\begin{aligned}P(1/7/2013) &= 107{,}55174 + 7{,}35\\ &= 114{,}90174\end{aligned}$$

We must now discount the present value of the bond back to the settlement date to obtain the all-in-price.

$$\begin{aligned}\text{All-in price} &= 114{,}90174\left(1+\tfrac{0{,}135}{2}\right)^{-\left(\frac{74}{181}\right)}\\ &= 111{,}87388\end{aligned}$$

The All-in price is R111,87388%.

$$\begin{aligned}\text{Accrued interest} &= \tfrac{H-R}{365}\times c\\ &= \tfrac{181-74}{365}\times 14{,}7\\ &= 4{,}30932\end{aligned}$$

$$\begin{aligned}\text{Clean price} &= \text{All-in price} - \text{accrued interest}\\ &= 111{,}87388 - 4{,}30932\\ &= 107{,}56456\end{aligned}$$

The clean price is R107,56456%.

2. The present value of 1 July 2013 is

   $P=$ R107,55174%. (see question one)

   As the settlement date 24 June 2013 is less than ten days from the next coupon date we don't add the coupon. It is an ex-interest case. We must discount the present value of the bond on 1 July 2013 back to the settlement date of 24 June 2013.

   The number of days ($R$) from the settlement date to the next coupon date is 182 (1 July) minus 174 (24 June) equals 7.

$$\begin{aligned}\text{All-in price} &= 107{,}55174\left(1+\tfrac{0{,}135}{2}\right)^{-\left(\frac{7}{181}\right)}\\ &= 107{,}28039\end{aligned}$$

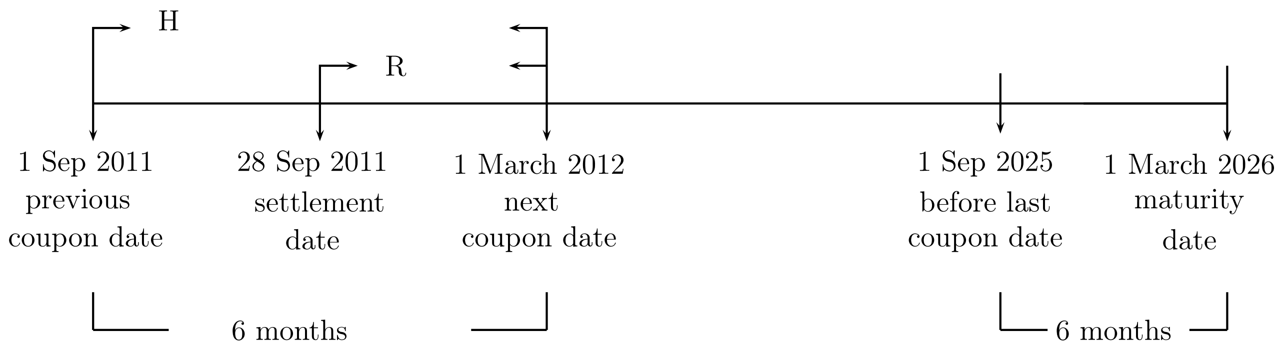
The all-in price is R107,28039%.

$$\begin{aligned}\text{Accrued interest} &= \tfrac{-R}{365}\times c\\ &= \tfrac{-7}{365}\times 14{,}7\\ &= -0{,}28192\end{aligned}$$

$$\begin{aligned}\text{Clean price} &= \text{All-in-price} - \text{accrued interest}\\ &= 107{,}28039-(-0{,}28192)\\ &= 107{,}56231\end{aligned}$$

The clean price is R107,56231%.

3.




$$\begin{aligned} n &= 01/03/2026 - 01/03/2012 \\ &= 14 \times 2 \\ &= 28 \end{aligned}$$

$$\begin{aligned} P\,(01/03/12) &= d a_{\overline{n}|z} + 100\,(1+z)^{-n} \\ &= \tfrac{11}{2} a_{\overline{28}|0{,}1375\div 2} + 100\left(1+\tfrac{0{,}1375}{2}\right)^{-28} \\ &= 83{,}10812 \end{aligned}$$

Cum interest thus we add the coupon:

$$\begin{aligned} \text{Price} &= 83{,}10812 + 5{,}5 \\ &= 88{,}60812 \end{aligned}$$

The remaining days from 28/9/2011 to 1/3/2012 is:

Day number 365 (31 December) minus 271 (28 September) plus 60 (1 March).

$$R = 365 - 271 + 60 = 154.$$

The number of days in the half-year in which the settlement date falls is between 1/9/2011 and 1/3/2012. Day number 365 (31 December) minus 244 (1 September) plus 60 (1 March).

$$H = 365 - 244 + 60 = 181.$$

The fraction to be discounted back is thus

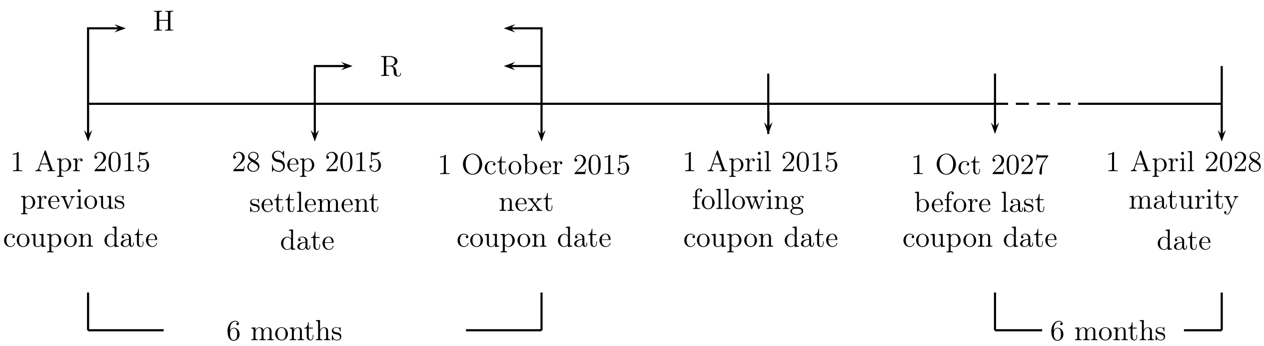
$$\begin{aligned} f &= \tfrac{R}{H} \\ &= \tfrac{154}{181} \end{aligned}$$

The all-in price is:

$$\begin{aligned} P\,(28/09/2011) &= 88{,}60812\left[1+\tfrac{0{,}1375}{2}\right]^{-\left(\frac{154}{181}\right)} \\ &= 83{,}73459 \end{aligned}$$

The price of R83,73459% is less than R100%, thus the bond is sold at a discount.

4. Bond BBB:




$$\begin{aligned} \text{Years} &= 1/4/28 - 1/4/11 \\ &= 13 \end{aligned}$$

Multiply by two and add one to get $n$.

$$\begin{aligned} &= 13 \times 2 \\ &= 26 + 1 \\ &= 27 \end{aligned}$$

$$\begin{aligned} P\,(1/10/2015) &= da_{\overline{n}|z} + 100\,(1+z)^{-n} \\ &= \tfrac{7{,}5}{2} \times a_{\overline{27}|0{,}14 \div 2} + 100\,\left(1 + \tfrac{0{,}14}{2}\right)^{-27} \\ &= 61{,}04320 \end{aligned}$$

The bond is sold ex-interest.

The remaining days from 28/9/2015 to 1/10/2015 is $R = 3$.

The number of days in the half year 1/4/2015 to 1/10/2015 is:

$$H = 274 \text{ (1 October)} - 91 \text{ (1 April)} = 183.$$

The all-in price is:

$$\begin{aligned} P(28/09/2015) &= 61{,}04320\left(1 + \tfrac{0{,}14}{2}\right)^{-\left(\frac{3}{183}\right)} \\ &= 60{,}97553 \end{aligned}$$

The all-in price is R60,97553%.

The accrued interest is

$$\begin{aligned} &= \tfrac{-R}{365} \times C \\ &= \tfrac{-3}{365} \times 7{,}5 \\ &= -0{,}06164 \end{aligned}$$

$$\begin{aligned} \text{The clean price} &= \text{All-in-price} - \text{accrued interest} \\ &= 60{,}97553 + 0{,}06164 \\ &= 61{,}03717 \end{aligned}$$

The clean price is R61,03717%.

### 5.2.6 Solution: additional exercise 6

1.


$$\begin{aligned} S &= P(1+rt) \\ S &= 5\,000 \\ r &= 13\% \\ t &= \frac{8}{12} \end{aligned}$$

$$\begin{aligned} 5\,000 &= P\left(1 + 0{,}13 \times \frac{8}{12}\right) \\ P &= 4\,601{,}23 \end{aligned}$$

Jenny will pay Jonas R4601,23.

2.

$$\begin{aligned} S &= Pe^{ct} \\ S &= 22\,000 \\ P &= 17\,500 \\ t &= \frac{53}{12} \end{aligned}$$

$$\begin{aligned} 22\,000 &= 17\,500e^{\left(\frac{53}{12}c\right)} \\ e^{\left(\frac{53}{12}\cdot c\right)} &= \frac{22\,000}{17\,500} \\ \frac{53}{12}c\ln e &= \ln\left(\frac{22\,000}{17\,500}\right) \\ c &= \ln\left(\frac{22\,000}{17\,500}\right) \div \frac{53}{12} \\ &= 5{,}181\% \end{aligned}$$

The continuous rate is 5,181%.

3.

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tn} \\ P &= 2\,844{,}20 \\ S &= 3\,881{,}49 \\ t &= 3 \\ m &= 4 \end{aligned}$$

$$\begin{aligned} 3\,881{,}49 &= 2\,844{,}20\left(1+\frac{j_m}{4}\right)^{3\times 4} \\ j_m &= 10{,}50\% \end{aligned}$$

The original interest rate was 9% (10,5−1,5).

4.

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ S &= 2\,844{,}20 \\ j_m &= 9\% \\ m &= 2 \\ t &= 4 \end{aligned}$$

$$\begin{aligned} 2\,844{,}20 &= P\left(1+\frac{0{,}09}{2}\right)^{4\times 2} \\ P &= 2\,000{,}00 \end{aligned}$$

The original amount invested was R2 000.

5.

$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ 15{,}04289 &= 100(1+0{,}0675)^{-n} \\ n &= 29 \end{aligned}$$

The number of years is $14\frac{1}{2}$.

The settlement date must be moved forward for 74 days.

Day number 108 (18 April) plus 74 equals 182 and that is 1 July 2013. This date must be moved forward for $14\frac{1}{2}$ years. The maturity date is therefore 1 January 2028.

6.

$$\begin{aligned}
S &= (1+i)Rs_{\overline{n}|i} \\
S &= 250\,000 \\
i &= 9{,}4\% \div 12 \\
n &= 9
\end{aligned}$$

$$\begin{aligned}
250\,000 &= (1+i)Rs_{\overline{9}|0{,}094\div 12} \\
(1+i)R &= 26\,918{,}73 \\
R &= 26\,709{,}50
\end{aligned}$$

Francois' friend James monthly deposits equal R26 709,50.

7.

$$\begin{aligned}
P(8/12/2015) &= \frac{5}{2}a_{\overline{6}|0{,}12\div 2} + 100\left(1+\frac{0{,}12}{2}\right)^{-6} \\
&= 82{,}78936
\end{aligned}$$

The price on 8/12/2015 is R82,78936%. To this a coupon must be added because it is a cum interest case. The price is therefore R85,28936% (2,5 + 82,78936).

This amount must be discounted back for 33 days.

$$\begin{aligned}
\text{The all-in price} &= 85{,}28936\left(1+\frac{0{,}12}{2}\right)^{-33/184} \\
&= 84{,}40269
\end{aligned}$$

The all-in price is R84,40269%. The accrued interest must be deducted from this amount.

$$\begin{aligned}
\text{The clean price} &= 84{,}40269 - 2{,}06849 \\
&= 82{,}33420
\end{aligned}$$

The clean price is R82,33420%.

8.

$$\begin{aligned}
S &= \left(R+\frac{Q}{i}\right)s_{\overline{n}|i} - \frac{nQ}{i} \\
R &= 6\,500 \\
Q &= 1\,700 \\
i &= 10\% \\
n &= 20
\end{aligned}$$

$$\begin{aligned}
S &= \left(6\,500+\frac{1\,700}{0{,}10}\right)s_{\overline{20}|0{,}10} - \frac{20\times 1\,700}{0{,}10} \\
&= 1\,345\,962{,}49 - 340\,000 \\
&= 1\,005\,962{,}49
\end{aligned}$$

An amount of R1 005 962,48 was received.

9.

$$\begin{aligned}
\text{Amount paid} &= nR + Q\left(\frac{n(n-1)}{2}\right) \\
&= 20 \times 6\,500 + 1\,700\left(\frac{20 \times 19}{2}\right) \\
&= 130\,000 + 323\,000 \\
&= 453\,000
\end{aligned}$$

Interest earned is the amount received for the policy, R1 005 962,49, minus the actual amount paid of R453 000 and that equals R552 962,49.

10.

$$\begin{aligned}
P &= Ra_{\overline{n}|i} \\
P &= 225\,000 \\
R &= 36\,000 \\
n &= 15
\end{aligned}$$

$$\begin{aligned}
225\,000 &= 36\,000a_{\overline{15}|i} \\
i &= 13{,}65\%
\end{aligned}$$

The internal rate of return is 13,65%.

11. The arithmetic mean equals 15,36.

12. The regression line is represented by

$$y = 718{,}35 - 29{,}26x$$

13.

$$\begin{aligned}
S &= P\left(1 + \frac{j_m}{m}\right)^{tm} \\
S &= 100\,000 \\
j_m &= 8\% \\
m &= 12 \\
t &= 11
\end{aligned}$$

$$\begin{aligned}
100\,000 &= P\left(1 + \frac{0{,}08}{12}\right)^{11\times 12} \\
P &= 41\,599{,}60
\end{aligned}$$

The amount in the account was R41 599,60.

14.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ S &= 41\,599{,}60 \\ n &= 10 \times 2 \\ i &= 9{,}25\% \div 2 \\ \\ 41\,599{,}60 &= Rs_{\overline{10\times2}|0{,}0925\div2} \\ R &= 1\,308{,}77 \end{aligned}$$

Grandma will pay R1 308,77 every six months in this account.

### 5.2.7 Solution: additional exercise 7

1.

$$\begin{aligned} S &= P(1+rt) \\ P &= 16\,000 \\ S &= 16\,570{,}08 \\ t &= 341 - 52 = \frac{289}{365} \\ 16\,570{,}08 &= 16\,000\left(1 + r \times \frac{289}{365}\right) \\ r &= \left(\frac{16\,570{,}05}{16\,000} - 1\right) \div \frac{289}{365} \\ &= 0{,}045 \end{aligned}$$

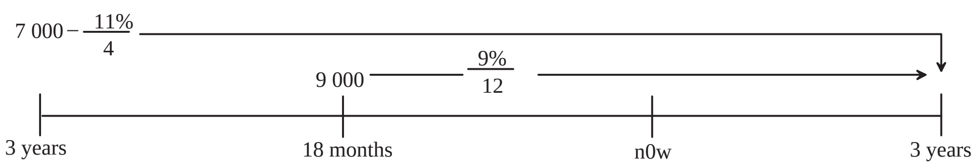
The simple interest rate is 4,5%.

2.

$$\begin{aligned} J_\alpha &= 100(e^c - 1) \\ 16{,}13 &= 100(e^c - 1) \\ 0{,}1613 + 1 &= e^c \\ \ln 1{,}1613 &= c \ln e \\ c &= 0{,}1495 \end{aligned}$$

The continuous rate is 14,95%.

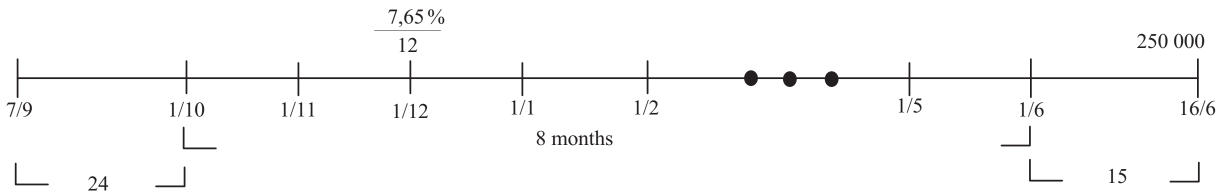
3.



$$\begin{aligned} S &= P(1+\tfrac{j_m}{m})^{tm} + P(1+\tfrac{j_m}{m})^{tm} \\ &= 7\,000\left(1+\tfrac{0,11}{4}\right)^{6\times 4} + 9\,000\left(1+\tfrac{0,09}{12}\right)^{4,5\times 12} \\ &= 13\,423,38 + 13\,473,35 \\ &= 26\,896,73 \end{aligned}$$

The amount that Jack owes Jill is R26 896,73.

4.




$$\begin{aligned} S &= P(1+rt)\left(1+\frac{j_m}{m}\right)^{tm}(1+rt) \\ 250\,000 &= P\left(1+0{,}0765\times\frac{15}{365}\right)\left(1+\frac{0{,}0765}{12}\right)^{\frac{8}{12}\times\frac{12}{1}}\left(1+0{,}0765\times\frac{24}{365}\right) \\ 249\,216{,}50 &= P\left(1+\frac{0{,}0765}{12}\right)^{\frac{8}{12}\times\frac{12}{1}}\left(1+0{,}0765\times\frac{24}{365}\right) \\ 236\,863{,}47 &= P\left(1+0{,}0765\times\frac{24}{365}\right) \\ P &= 235\,677{,}98 \end{aligned}$$

Ian deposit R235 677,98.

5.

$$\begin{aligned} S &= P\left(1+\frac{jm}{m}\right)^{tm} \\ 250\,000 &= P\left(1+\frac{0{,}0765}{12}\right)^{\left(\frac{8}{12}+\frac{15+24}{365}\right)\times\frac{12}{1}} \\ P &= 235\,679{,}96 \end{aligned}$$

Ian deposit R235 679,96.

6.

$$\begin{aligned}
S &= \left(R + \frac{Q}{i}\right) s_{\overline{n}|i} - \frac{nQ}{i} \\
R &= 6\,000 \\
Q &= 1\,500 \\
n &= 30 \\
i &= 9{,}7\% \\
S &= \left(6\,000 + \frac{1\,500}{0{,}097}\right) s_{\overline{30}|0{,}097} - \frac{30 \times 1\,500}{0{,}097} \\
&= 3\,336\,150{,}21 - 463\,917{,}53 \\
&= 2\,872\,232{,}69
\end{aligned}$$

Paul can expect to receive R2 872 232,69.

7.

$$\begin{aligned}
\text{Amount paid} &= nR + \frac{Q(n(n-1))}{2} \\
&= 30 \times 6\,000 + \frac{1\,500(30 \times 29)}{2} \\
&= 180\,000 + 652\,500 \\
&= 832\,500
\end{aligned}$$

Paul paid R832 500 for the policy.

8.

$$\begin{aligned}
IRR &= \frac{A}{(1+i)} + \frac{B}{(1+i)^2} + \frac{C}{(1+i)^3} + \frac{D}{(1+i)^4} + \frac{E}{(1+i)^5} - Z \\
&= \frac{75\,000}{(1+i)} + \frac{190\,000}{(1+i)^2} + \frac{40\,000}{(1+i)^3} + \frac{150\,000}{(1+i)^4} + \frac{180\,000}{(1+i)^5} - 500\,000 \\
&= 0{,}0778
\end{aligned}$$

The internal rate of return (IRR) is 7,78%.

9.

$$\begin{aligned}
A &= 2\,080{,}54 - 2\,014{,}27 \\
&= 66{,}27
\end{aligned}$$

The interest due at month end (A) is R66,27.

10.

$$\begin{aligned}
B &= 6\,141{,}56 - 2\,030{,}64 \\
&= 4\,110{,}92
\end{aligned}$$

The outstanding principal at month beginning (B) is R4 110,92.

11. We must first determine the applicable interest rate.

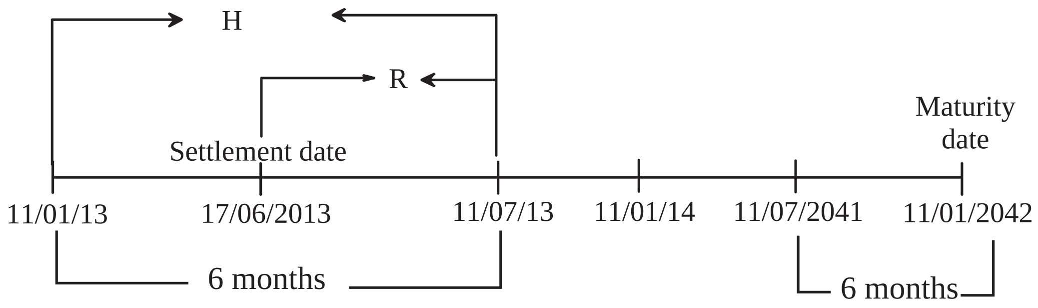
$$\begin{aligned} I &= Prt \\ 16{,}77 &= 2\,063{,}78 \times r \times \frac{1}{12} \\ r &= 0{,}0975 \end{aligned}$$

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= 2\,080{,}54 s_{\overline{150}|0{,}0975\div 12} \\ &= 180\,000{,}00 \end{aligned}$$

The original amount was R180 000,00.

The answers differs due to rounding off.

12.




$$\begin{aligned} \text{Years} &= 11/01/42 - 11/01/14 \\ &= 28 \end{aligned}$$

We must now multiply with to and add one to get $n$.

$$\begin{aligned} &= 28 \times 2 + 1 = 57 \\ R &= 192 - 168 = 24 \\ H &= 192 - 11 = 181 \end{aligned}$$

$$\begin{aligned} P(11/07/13) &= da_{\overline{n}|z} + 100(1 + z = z)^{-n} \\ &= \frac{9{,}91}{2} a_{\overline{57}|0{,}0747\div 2} + 100\left(1 + \frac{0{,}0747}{2}\right)^{-57} \\ &= 128{,}62450 + 4{,}955 \quad \text{(cum interest)} \\ &= 133{,}57950 \end{aligned}$$

$$\begin{aligned}\text{All-in price} &= 133{,}57950\left(1+\frac{0{,}0747}{2}\right)^{-\frac{24}{181}} \\ &= 132{,}93158\end{aligned}$$

The All-in price is R132,93158%.

13.

$$\begin{aligned}\text{Accrued interest} &= \frac{H-R}{365}\times C \\ &= \frac{181-24}{365}\times 9{,}91 \\ &= 4{,}26266\end{aligned}$$

$$\begin{aligned}\text{Clean price} &= \text{All-in price} - \text{Accrued interest} \\ &= 132{,}93158 - 4{,}26266 \\ &= 128{,}66892\end{aligned}$$

The clean price is R128,66892%.

14.

$$y = 3{,}17477 + 0{,}65407x$$

The equation is $y = 0{,}65407x + 3{,}17477$

15.

$$r = 0{,}90828$$

The correlation coefficient is 0,90828.

# Chapter 6

# Notes on the SHARP EL-738 calculator

**General**

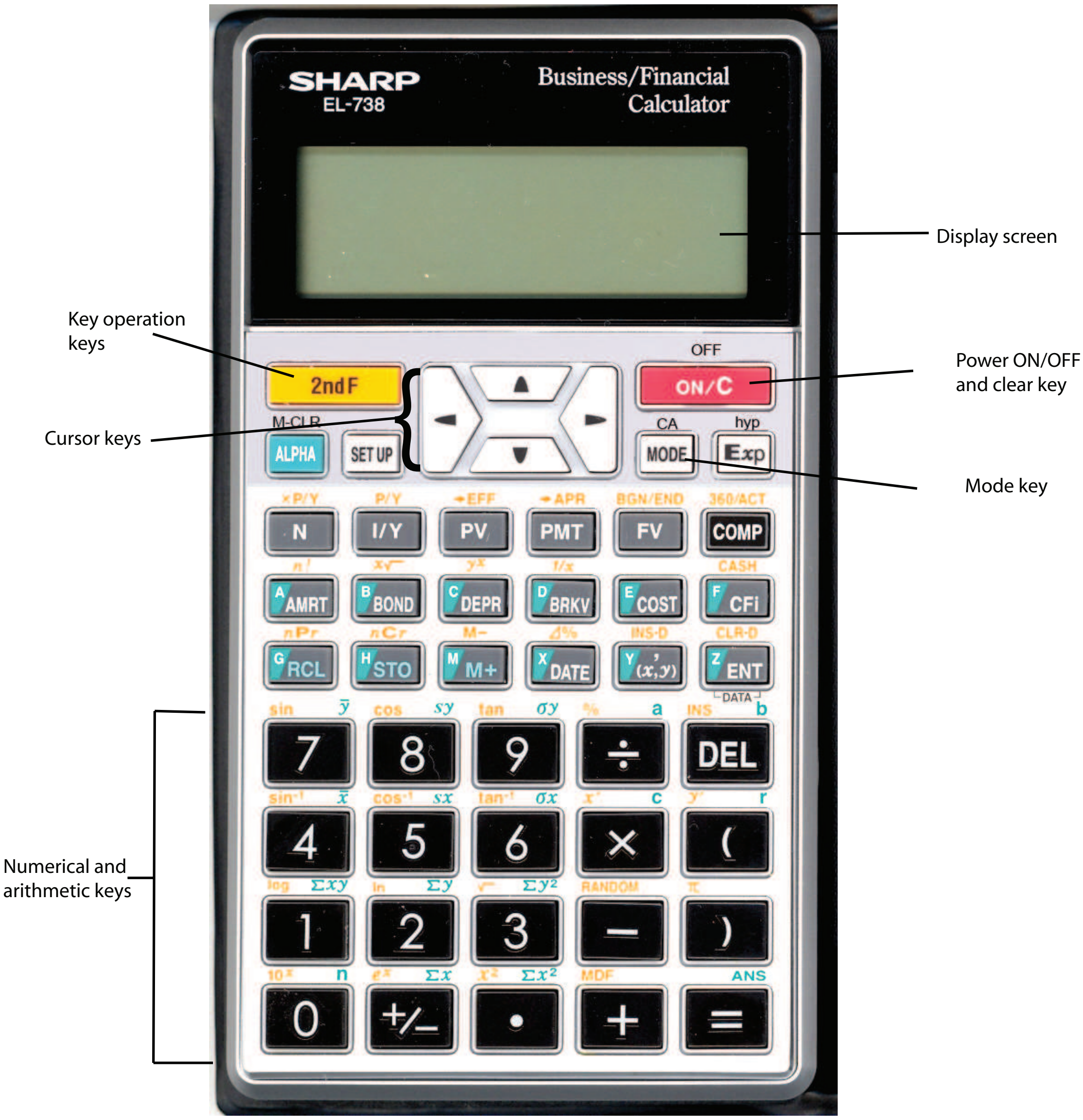
The SHARP EL-738 calculator is recommended for this module. (It replaces the SHARP EL-733A.) The advantage of this calculator is that it can do basic calculations, financial calculations and statistical calculations. You may, however, use any other **financial calculator**, but assistance will only be provided to SHARP EL-738 and Hewlett Packard 10BII users.

It is recommended that you purchase the SHARP EL-738 as you can use it in your further studies.

Most of the keys can perform two functions.

To perform a function written on the key, you simply press the key. To perform a function written on the surface just above the key, first press the orange **2ndF** key to activate it to perform the function when pressed.

## 6.1 Normal Calculation Mode



## 6.2 Switch on your calculator

Before using your calculator for the first time, reset (initialise) it. Press the RESET switch located on the back of the calculator with the tip of a ball-point pen.

After resetting the calculator, the initial display of the NORMAL mode appears.


**NOTE:**

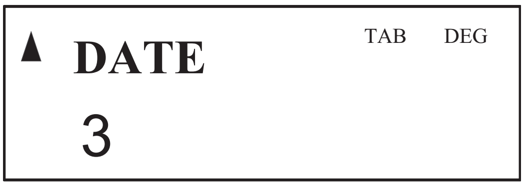
Pressing **2ndF** **M-CLR** 1 **=** will also erase all stored data in the memory and restore the calculator's default setting.

Note that I will not write the numbers 0, ...., 9 in blocks. All the other functions will be written in blocks.

## 6.3 The SET UP menu

Press the **SET UP** key to display the SET UP menu.


appears on the screen. Press the **►** arrow three times and




will appear on the screen.

A menu item can be selected by using the **◄** **►** keys (the selected number will blink). Press the **=** key.

To set the number of decimals, press **SET UP** 0 0.

appears on the screen.

Press 2 to select two decimals

If you want four decimals press **SET UP** 0, 0, 4.

```
DIG(0-9)?          TAB   DEG
```

```
                   TAB   DEG

                        0.00
```

(However, we will use two decimals. Press **SET UP** 0, 0, 2.)

NOTE: The calculator uses a decimal point (0.00) where we use the decimal comma (0,00).

## 6.4 Selecting a MODE

Press **MODE**.

The menu display appears

```
                   TAB   DEG
NORMAL          STAT
   0               1
```

Press 0.

appears on the screen

**Normal mode**

The NORMAL mode allows you to perform financial, arithmetic or scientific calculations.

### 6.4.1 Calculator keys

The keys are classified according to the work they do.

The following keys are worth mentioning:

- ON: **ON/C**

  Last key, first row. To switch on the calculator. The **ON/C** key also clears the screen. To preserve the batteries, the calculator turns itself off after about 10 minutes.


- OFF: OFF / ON/C

  The orange function on the red ON/C key. Press 2ndF ON/C to switch your calculator off.

- NUMERIC KEYS: 1, 2, 3 .... 9, 0

  These keys are used to enter numbers.

- MULTIPLICATION X

  Second last key, third last row.

- DIVISION ÷

  Second last key, sixth row.

- EQUAL =

  Last key, last row.

- CLEAR ON/C

  Last key, first row.

- BRACKETS ( )

  Last key, third last row and second last row.

  Use the ( and ) keys to place parentheses around parts of expressions. The closing parenthesis ) may be omitted.

- NEGATIVE +/-

  Second key, last row.

  This key is used to enter a negative number or change the sign of a number, while the − key is used for the operation of subtraction. Note the different ways in which subtraction, with the long dash, and the sign of the number, with a small dash, are displayed.

  For example: 3 – 2 and 3+(-2).

- DELETE: DEL

  If you made a mistake, press DEL (last key, sixth row) to erase the number and then enter the correct number to continue.

  If you want to change a number or sign after you have pressed = use the ◄ cursor to move to the place where you want to change it. Enter the new number or sign, then press DEL and continue.

- INSERT: **INS**

  Use the ◄ cursor to move to the place where you want to insert a number. Press **2ndF** **INS** (sixth row, last key) and enter the number. The cursor will flicker after the inserted number.

- TO THE POWER key $y^x$.

  The **2ndF** third key, fourth row

  **Example:**

  Calculate $2^3$.

  Enter the base number first − press 2.

  Then press **2ndF** $y^x$ 3 **=**

  The answer is 8,00.

  If the power consists of more than one term, use brackets for the power.

  **Example:**

  Calculate $\left(3^2\right)^4$

  Press **(** 3 **2ndF** $y^x$ 2 **)** **2ndF** $y^x$ 4 **=**

  The answer is 6 561,00.

  **Example:**

  Calculate $5^{2/3}$

  Press 5 **2ndF** $y^x$ **(** 2 ÷ 3 **)** **=**

  The answer is 2,92.

- SQUARE: $(x^2)$

  Use the power key.

  **Example:**

  Calculate $4^2$.

  Press 4 **2ndF** $y^x$ 2 **=**

  The answer is 16.

- SQUARE ROOT: $\sqrt{x}$

  Use the $\sqrt[x]{\ }$ key. **2ndF** second key, fourth row.

  **Example:**

  Calculate $\sqrt{64}$. $\sqrt{64}$ means $\sqrt[2]{64}$.

  Press 2 $F$ $\sqrt[x]{\ }$ 64 **=**

  The answer is 8.

  **Example:**

  Calculate $\sqrt[3]{64}$.

Press 3 `2ndF` $\sqrt[x]{\ }$ 64 `=`

The answer is 4.

- LOGARITHM to the base e: ln

  **Example:**

  Calculate ln 3.

  Press `2ndF` `ln`, (second key, second last row) 3 `=`

  The answer is 1,10.

  **Example:**

  Calculate $\ln\left(\frac{1\,253}{1479}\right)$.

  Press `2ndF` ln `(` 1 253 `÷` 1 479 `=`

  The answer is −0,17.

- THE EXPONENTIAL FUNCTION: $e^x$ – The inverse of ln.

  **Example:**

  Calculate $e^{1,10}$.

  Press `2ndF` `e^x` (2nd key, last row)

  1.10 `=`

  The answer is 3.

- MEMORY: M+

  The calculator has 11 temporary memories (A-H and X-Z), one independent memory (M) and one last answer memory (ANS).

  Temporary memory

  To store a value, press `STO` and the variable in which you want to store it.

  **Example:** Store 17 in A.

  Press 17 `STO` (fifth row, second key) `A` (first key, fourth row). appears on the screen.


  If you want to recall the value stored in A, press `RCL` (first key, fifth row) `A`. appears on the screen.

  Store a value in the independent memory M+.

  **Example:** Store 19, 21 and 25 in M+.

  Press 19 `M+` (third key, fifth row).


21 [**M+**]

25 [**M+**]

To find the answer, press [**RCL**] [**M+**].

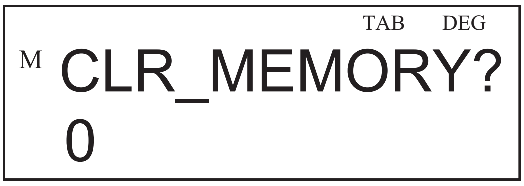
The answer is 65.

To clear the register.

press [**2ndF**] [**M-CLR**].


appears on the screen.

Press 0.



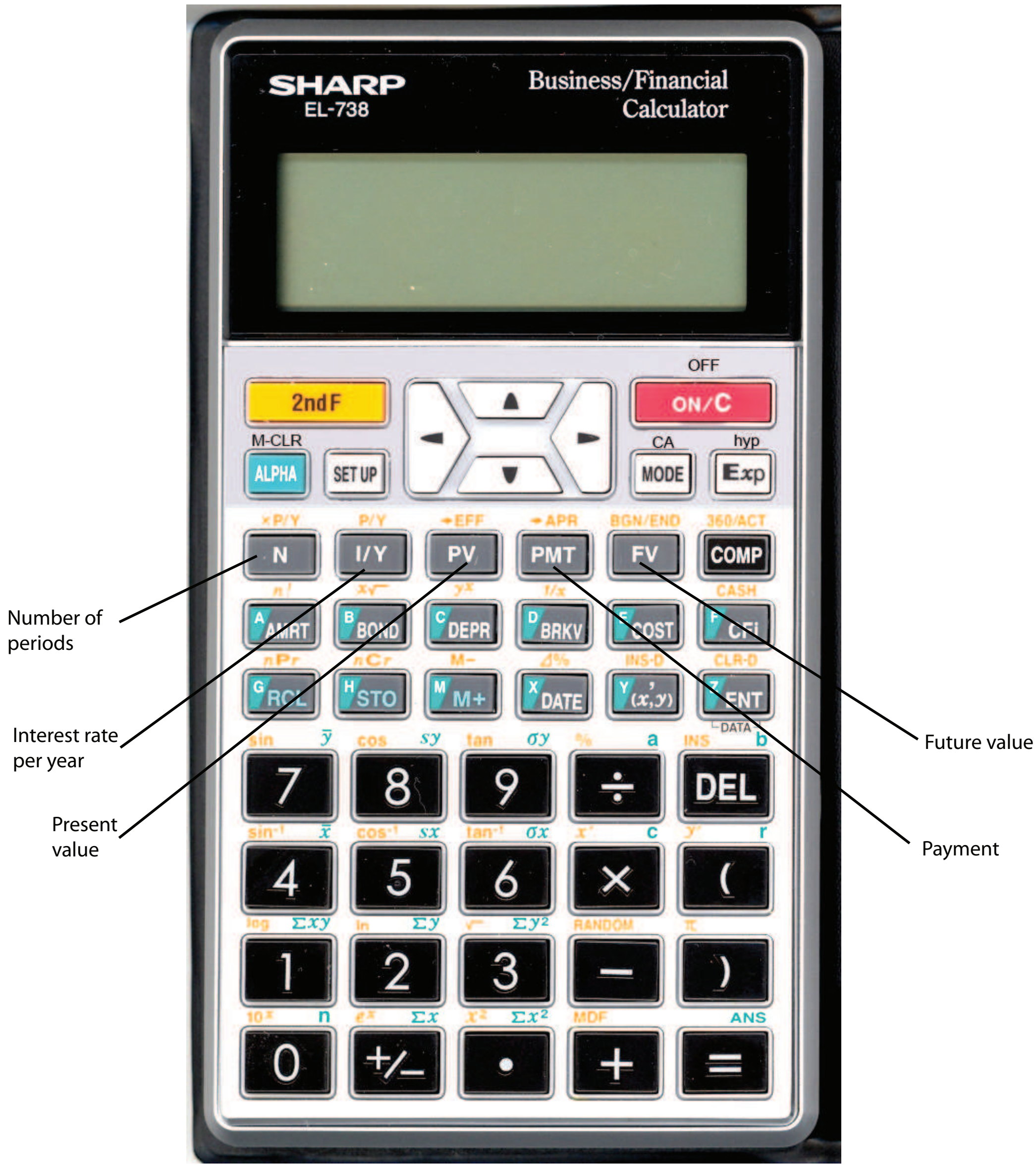

appears on the screen.

Press 0. appears on the screen.


- ERROR

  If ERROR 1 appears on the screen after you did a calculation press the [**◄**] key and the cursor will flicker where you made the mistake, press [**DEL**] and continue by pressing [**=**].

# Financial Calculator Mode



**Normal mode**

The NORMAL mode allows us to use the financial keys. The financial keys **N**, **1/Y**, **PV**, **PMT**, **FV** can only be used when the exponent in the applicable formula consists of a single number (not a product or sum of numbers).

Before using the financial keys, first clear the register by pressing **2ndF** **M-CLR** 0, 0.

## 6.5 Interest rates

### 6.5.1 Simple interest

$$I = Prt$$

- Determine the amount of interest received if R1 200 is invested for 4 years at 14% simple interest per year.

$$\begin{aligned} I &= Prt \\ &= 1\,200 \times 14\% \times 4 \\ &= 1\,200 \times 0{,}14 \times 4 \\ &= 672{,}00 \end{aligned}$$

  The interest received is R672,00. We cannot use the financial keys because there is no exponent in the formula.

  Key in as

  1 200 **×** 0.14 **×** 4 **=**

  The answer is 672,00.

$$S = P\,(1 + rt)$$

- Determine the accumulated amount for if R2 400 is invested for 42 months at a 9% simple interest rate per year.

$$\begin{aligned} S &= 2\,400\left(1 + 9\% \times \tfrac{42}{12}\right) \\ &= 2\,400\left(1 + 0{,}09 \times \tfrac{42}{12}\right) \\ &= 3\,156{,}00. \end{aligned}$$

  The accumulated amount is R3 156,00.

  Key in as

  2 400 **(** 1 **+** 0.09 **×** 42 **÷** 12 **)** **=**

  The answer is 3 156,00.

- Determine the simple interest rate if R3 600 accumulates to R5 760 in five years' time.

$$\begin{aligned} S &= P(1+rt) \\ 5\,760 &= 3\,600(1+r\times 5) \\ 1+5r &= \tfrac{5\,760}{3\,600} \\ 5r &= \tfrac{5\,760}{3\,600}-1 \\ r &= \left(\tfrac{5\,760}{3\,600}-1\right)\div 5 \\ &= 0{,}12 \end{aligned}$$

The simple interest rate is 12%.

Key in as

( 5 760 ÷ 3 600 − 1 ) ÷ 5 =

The answer is 0,12, that is, 12%.

### 6.5.2 Simple discount

$$P = S(1-dt)$$

- Determine the present value of a promissory note that is worth R2 500 15 months later, and the applicable discount rate is 10,24% per year.

$$\begin{aligned} P &= S(1-dt) \\ P &= 2\,500\left(1-0{,}1024\times\tfrac{15}{12}\right) \\ &= 2\,180{,}00 \end{aligned}$$

The present value is R2 180,00.

Key in as

2 500 ( 1 − 0.1024 × 15 ÷ ) 12 =

The answer is 2 180,00.

- Determine the time under consideration (in months) if a simple interest rate of 11,76% is equivalent to a 10,25% simple discount rate.

By manipulating

$$S = P(1+rt) \text{ and } P = S(1-dt)$$

we get

$$r = \frac{d}{1-dt}$$

and

$$t = \left(1-\frac{d}{r}\right)\div d.$$

Substituting the values, we get

$$\begin{aligned} t &= \left(1 - \tfrac{0{,}1025}{0{,}1176}\right) \div 0{,}1025 \\ &= 1{,}25. \end{aligned}$$

The time under consideration is 1,25 years, that is, 15 months.

Key in as

**(** 1 **−** 0.1025 **÷** 0.1176 **)** **÷** 0.1025 **=**

The answer is 1,25, that is, 15 months.

### 6.5.3 Compound interest

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm} \text{ or } S = P\,(1+i)^n$$

We use our financial keys to do the calculations because there is only one exponent in the formula:

$$S = P\,(1+i)^n$$

**NB: The interest rate must be entered into the calculator as a percentage and *NOT* as a decimal because the calculator has been preprogrammed to automatically divide the interest rate by a hundred. Remember that it is convention to enter either the present value or future value as a negative amount.**

- Calculate the future value if R5 000 is invested for five years at 15% per year compounded monthly.

$$\begin{aligned} S &= P\,(1+i)^n \\ &= 5\,000\left(1 + \tfrac{0{,}15}{12}\right)^{5\times 12} \\ &= 10\,535{,}91 \end{aligned}$$

The future value is R10 535,91.

Key in as

**2ndF** **CA** (to clear the register).

First enter the number of compounding periods.

**2ndF** **P/Y** (second key, third row) 12 **ENT** (sixth key, fifth row)

**ON/C**

**+/−** 5 000 **PV**

15 **I/Y**

5 **2ndF** **×P/Y** (first key, third row) **N**

To check if you have entered the correct values press **RCL** and the financial key that you want to check. If the value is incorrect, enter the new value, press the financial key and continue.

COMP FV

The answer is 10 535,91.

- Determine the time under consideration if R5 000 is invested at 15% per year, compounded half yearly, and the accumulated amount is R10 000.

$$\begin{aligned} S &= P(1+i)^n \\ 10\,000 &= 5\,000\left(1+\tfrac{0,15}{2}\right)^{t\times 2} \\ t &= 4,79 \end{aligned}$$

The time under consideration is 4,79 years.

Key in as

2ndF CA

2ndF P/Y 2 ENT

ON/C

+/− 10 000 FV

5 000 PV

15 I/Y

COMP N

N = 9.5844 appears on the screen. Because the number of compounding periods is half yearly, divided the answer by two.

Press ÷ 2 =.

4.79 appears on the screen.

### 6.5.4 Effective rate

$$J_{\mathit{eff}} = 100\left(\left(1+\frac{j_m}{m}\right)^m - 1\right)$$

- Determine the effective rate for a nominal rate of 14% per year, compounded quarterly.

$$\begin{aligned} J_{\mathit{eff}} &= 100\left(\left(1+\tfrac{0,14}{4}\right)^4 - 1\right) \\ &= 14,75. \end{aligned}$$

The effective rate is 14,75%.

Key in as

100 ( ( 1 + 0.14 ÷ 4 ) 2ndF $y^x$ 4 − 1 ) =

The answer is 14,75%.

**OR**

**2ndF** **CA**

4 $x,y$ (fifth key, fifth row)

14 **2ndF** **→ EFF** (third key, third row)

14.75 appears on the screen.

- Determine the nominal rate per year, compounded monthly for an effective rate of 19,56%.

$$\begin{aligned} J_{eff} &= 100\left(\left(1+\frac{j_m}{m}\right)^m - 1\right) \\ 19{,}56 &= 100\left(\left(1+\frac{j_m}{12}\right)^{12} - 1\right) \\ j_m &= 18{,}00 \end{aligned}$$

The nominal rate is 18,00%.

Key in as

**2ndF** **CA**

12 $x,y$ 19.56 **2ndF** **→ APR** (fourth key, third row)

17.998 appears on the screen, that is, 18%.

## 6.6 Converting interest rates

### 6.6.1 Nominal rates

$$i = n\left(\left(1+\frac{j_m}{m}\right)^{m\div n} - 1\right)$$

- Convert 15% compounded every two months to compounded half yearly.

$$\begin{aligned} i &= 2\left(\left(1+\frac{0{,}15}{6}\right)^{6\div 2} - 1\right) \\ &= 0{,}1538 \end{aligned}$$

The new rate compounded half yearly is 15,38%.

Key in as

2 **(** **(** 1 **+** 0.15 **÷** 6 **)** **2ndF** $y^x$ **(**

6 **÷** 2 **)** **−** 1 **)** **=** **×** 100 **=**

The answer is 15,38%.

### 6.6.2 Continuous compounding

$$c = m \ln \left(1 + \frac{j_m}{m}\right)$$

- Convert 15%, compounded every two months, to continuous compounding.

$$\begin{aligned} c &= 6 \ln \left(1 + \tfrac{0{,}15}{6}\right) \\ &= 0{,}1482 \end{aligned}$$

The continuous compounding rate is 14,82%.

Key in as

6 [2ndF] [ln] [(] 1 [+] 0.15 [÷] 6 [)] [×] 100 [=]

The answer is 14,82%.

$$i = m\left(e^{c/m} - 1\right)$$

- Convert 14,82% continuous compounding to a nominal rate compounded half yearly.

$$\begin{aligned} i &= 2\left(e^{0{,}1482 \div 2} - 1\right) \\ &= 0{,}1538 \end{aligned}$$

The nominal interest rate is 15,38%.

Key in as

2 [(] [2ndF] [$e^x$] [(] 0.1482 [÷] 2 [)] [−] 1 [)] [=] [×] 100 [=]

The answer is 15,38%.

$$J_\alpha = 100\left(e^c - 1\right)$$

- Convert 8% continuous compounding to an effective interest rate.

$$J_\alpha = 100\left(e^{0{,}08} - 1\right)$$

The effective rate is 8,33%.

Key in as

100 [(] [2ndF] [$e^x$] 0.08 [−] 1 [)] [=]

The answer is 8,33%.

- Convert an effective rate of 11,92% to a continuous compounding rate.

$$\begin{aligned} J_\alpha &= 100\left(e^c - 1\right) \\ 11{,}92 &= 100\left(e^c - 1\right) \\ e^c &= 0{,}1192 + 1 \\ c \ln e &= \ln 1{,}1192 \\ c &= 11{,}26 \end{aligned}$$

The continuous compounding rate is 11,26%.

Key in as

[2ndF] [ln] 1.1192 [×] 100 [=]

The answer is 11,26%.

$$S = Pe^{ct}$$

- Determine the accumulated amount if R2 400 is invested for five years at 8% per year compounded continuously.

$$\begin{aligned} S &= 2\,400e^{0{,}08\times 5} \\ &= 3\,508{,}38 \end{aligned}$$

The accumulated amount is R3 580,38.

Key in as

2 400 [(] [2ndF] [$e^x$] [(] 0.08 [×] 5 [)] [=]

The answer is 3 580,38.

- Determine the continuous compounding rate if R12 000 accumulates to R17 901,90 after five years.

$$\begin{aligned} S &= Pe^{ct} \\ 17\,901{,}90 &= 12\,000e^{c\times 5} \\ \tfrac{17\,901{,}90}{12\,000} &= e^{5c} \\ \ln\left(\tfrac{17\,901{,}90}{12\,000}\right) &= 5c\ln e \\ c &= \ln\left(\tfrac{17\,901{,}90}{12\,000}\right) \div 5 \\ &= 0{,}08 \end{aligned}$$

The continuous compounding rate is 8%.

Key in as

[2ndF] [ln] [(] 17 901.90 [÷] 12 000 [)] [÷] 5 [=]

[×] 100 [=]

The answer is 8%.

## 6.7 Annuities

### 6.7.1 Present value

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R\left[\tfrac{(1+i)^n-1}{i(1+i)^n}\right] \end{aligned}$$

- Calculate the present value of R1 600 quarterly payments for five years at an interest rate of 20% per year, compounded quarterly.

$$\begin{aligned} P &= 1\,600a_{\overline{5\times4}|0{,}20\div4} \\ &= 19\,939{,}54. \end{aligned}$$

The present value is R19 939,54.

Key in as

**2ndF** **CA**

**2ndF** **P/Y** 4 **ENT**

**ON/C**

**+/−** 1 600 **PMT**

5 **2ndF** **×P/Y** **N**

20 **I/Y**

**COMP** **PV**

The answer is 19 939,54.

### 6.7.2 Future value

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R\left[\frac{(1+i)^n-1}{i}\right]. \end{aligned}$$

- Determine the future value of R400 monthly payments made for five years at 16% interest per year, compounded monthly.

$$\begin{aligned} S &= 400s_{\overline{5\times12}|0{,}16\div12} \\ &= 36\,414{,}21 \end{aligned}$$

The future value is R36 414,21.

Key in as

**2ndF** **CA**

**2ndF** **P/Y** 12 **ENT**

**ON/C**

**+/−** 400 **PMT**

5 **2ndF** **×P/Y** **N**

16 **I/Y**

**COMP** **FV**

The answer is 36 414,21.

### 6.7.3 Annuity due

If the words *begin* immediately, in *advance* and in the *beginning* appear in the sentence, an annuity due calculation is involved.

$$\begin{aligned} S &= (1+i)\,Rs_{\overline{n}|i} \\ P &= (1+i)\,Ra_{\overline{n}|i}. \end{aligned}$$

- Determine the future value after five years of R400 payments made at the *beginning* of a month in an account earning 16% interest per year, compounded monthly.

$$\begin{aligned} S &= (1+i)\,Rs_{\overline{n}|i} \\ &= (1+i)\,400s_{\overline{5\times12}|0{,}16\div12} \\ &= 36\,899{,}73 \end{aligned}$$

The future value is R36 899,73.

Key in as

**2ndF** **CA**

**2ndF** **BGN/END** (fifth key, third row)

**2ndF** **P/Y** 12 **ENT**

**ON/C**

+/− 400 **PMT**

16 **I/Y**

5 **2ndF** **×P/Y** **N**

**COMP** **FV**

The answer is 36 899,73.

**NB: PRESS 2ndF BGN/END AGAIN TO CANCEL THE BEGIN FUNCTION – IF YOU DO NOT DO IT ALL THE ANSWERS THAT FOLLOW WILL BE INCORRECT.** (NOTE: If the BGN/END function is cancelled it will disappear from the screen)

### 6.7.4 Increasing annuity

$$S = \left(R + \frac{Q}{i}\right) s_{\overline{n}|i} - \frac{nQ}{i}$$

- An endowment policy with yearly payments of R3 600 matures in 20 years' time. Calculate the future value of this policy if the yearly payments increase by R360 per year and an interest rate of 13% per year is applicable.

$$\begin{aligned} S &= \left(3\,600 + \tfrac{360}{0{,}13}\right) s_{\overline{20}|0{,}13} - \tfrac{20\times360}{0{,}13} \\ &= 515\,569{,}03 - 55\,384{,}62 \\ &= 460\,184{,}42 \end{aligned}$$

The future value is R460 184,42.

Key in as

**2ndF** **CA**

**2ndF** **P/Y** 1 **ENT**

**ON/C**

3 600 **+** 360 **÷** 0.13 **=** **×** **+/−** 1 **=** **PMT**

20 **2ndF** **×P/Y** **N**

13 **I/Y**

**COMP** **FV**

FV = 515 569,03 appears on the screen.

Press **−** **(** 360 **×** 20 **÷** 0.13 **=**

460 184,42 is the answer.

### 6.7.5 Amortisation

- Draw up an amortisation schedule for a loan of R5 000 which is repaid in annual payments over five years at an interest rate of 15% per year.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 5\,000 &= Ra_{\overline{5}|0,15} \\ R &= 1\,491{,}58 \end{aligned}$$

Key in as

**2ndF** **CA**

**2ndF** **P/Y** 1 **ENT**

**ON/C**

**+/−** 5 000 **PV**

5 **2ndF** **×P/Y** **N**

15 **I/Y**

**COMP** **PMT**

1 491.58 appears on the screen.

Press **AMRT** (fourth row, first key) 1

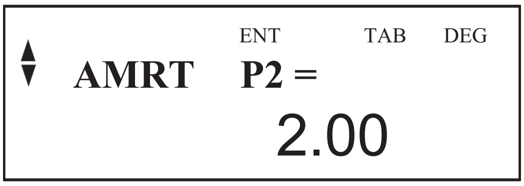
**▼** (Down arrow) 1 **ENT**

Press **▼** BALANCE = −4 258.42 appears on the screen.

Press ▼ $\sum$ PRINCIPAL = 741.58 appears on the screen.

Press ▼ $\sum$ INTEREST = 750.00 appears on the screen.

Press [ ▼ 2 ENT ] twice



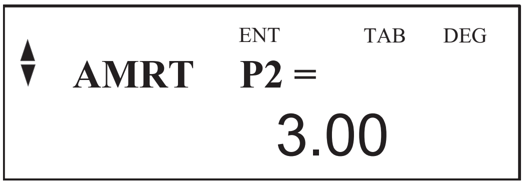

appears on the screen.

Press ▼ BALANCE = −3 405.60

Press ▼ $\sum$ PRINCIPAL = 852.82

Press ▼ $\sum$ INTEREST = 638.70

Press [ ▼ 3 ENT ] twice




appears on the screen.

Press ▼ BALANCE = −2 424.86

Press ▼ $\sum$ PRINCIPAL = 980.74

Press ▼ $\sum$ INTEREST = 510.84

Press [ ▼ 4 ENT ] twice

Press ▼ BALANCE = −1 297.01

Press ▼ $\sum$ PRINCIPAL = 1 127.83

Press ▼ $\sum$ INTEREST = 363.73

Press [ ▼ 5 ENT ] twice

Press ▼ BALANCE = 0.02

Press ▼ $\sum$ PRINCIPAL = 1 297.03

Press ▼ $\sum$ INTEREST = 194.55

- A loan of R135 000 must be repaid over a 20-year period in monthly instalments. The applicable interest rate is 18% per year, compounded monthly.

  Determine the outstanding balance, interest due and principal repaid after 235 payments made.

First determine the monthly payments.

$$\begin{aligned} P &= a_{\overline{n}|i} \\ 135\,000 &= Ra_{\overline{20\times 12}|0{,}18\div 12} \\ R &= 2\,083{,}47 \end{aligned}$$

Key in as

**2ndF** **CA**

**2ndF** **P/Y** 12 **ENT**

**ON/C**

**+/−** 135 000 **PV**

18 **I/Y**

20 **2ndF** **×P/Y** **N**

**COMP** **PMT**

The answer is 2 083,47.

Press **AMRT** 1 **ENT**

**▼** 1 **ENT**

Only press **▼** BALANCE = −134 941.53

Only press **▼** $\sum$ PRINCIPAL = 58.47

Only press **▼** $\sum$ INTEREST = 2 025.00

Now press [ **▼** 235 **ENT** ] twice

Press **▼** BALANCE −9 964.50

Press **▼** $\sum$ PRINCIPAL 1 905.42

Press **▼** $\sum$ INTEREST 178.05

## 6.8 Internal Rate of Return and Net Present Value

$$I_{\text{out}} = \frac{A}{1+i} + \frac{B}{(1+i)^2} + \ldots + \frac{N}{(1+i)^n}$$

- An investment with an initial outlay of R120 000 returns a constant cash flow of R24 000 per year for 10 years. Determine the internal rate of return (IRR) of this investment

$$120\,000 = \frac{24\,000}{1+i} + \frac{24\,000}{(1+i)^2} + \ldots + \frac{24\,000}{(1+i)^{10}}$$

Clear the memory by pressing

**2ndF** **M-CLR** 0, 0

**+/−** 120 000 **DATA** (last key, fifth row).

24 000 **DATA**
Keep on pressing DATA until


appears on the screen

**ON/C**

**2ndF** **CASH** (sixth key, fourth row)

**2ndF** **CA** **COMP**


appears on the screen. This is the **IRR**.

## 6.9 Internal Rate of Return and Net Present Value

$$NPV = \frac{A}{1+i} + \frac{B}{(1+i)^2} \cdots \frac{N}{(1+i)^n}$$

- An investment of R120 000 generates three successive cash inflows of R60 000, R48 000 and R35 000 respectively. Determine the IRR. Determine the NPV if the cost of capital is 8%.

$$0 = \frac{60\,000}{1+i} + \frac{48\,000}{(1+i)^2} + \frac{35\,000}{(1+i)^3} - 120\,000$$

Clear the memory by pressing

**2ndF** **M-CLR** 0, 0

**+/−** 120 000 **DATA** (last key, fifth row). appears on the screen

| |
|---|
| TAB DEG |
| **DATA SET : CF** |
| 0.00 |

60 000 **DATA**

48 000 **DATA**

35 000 **DATA**

**ON/C**

**2ndF** **CASH** (sixth key, fourth row)

**2ndF** **CA** **COMP**

| |
|---|
| TAB DEG |
| ▼ **RATE (I/Y) =** |
| 10.27 |

appears on the screen. This is the **IRR**.

Press 8 **ENT**

**▼** **COMP**

| |
|---|
| TAB DEG |
| ▲ **NET_PV =** |
| 4 491.95 |

appears on the screen. This is the **NPV**.

## 6.10 MIRR

$$MIRR = \left[ \left( \frac{C}{PV_{\text{out}}} \right)^{\frac{1}{n}} - 1 \right]$$

- Calculate the MIRR of an investment over a four-year period if the PV (*present value*) of the cash outflows equals R88 475 and the FV (*future value*) of the cash inflows equals R191 400.

$$\begin{aligned} MIRR &= \left[\left(\tfrac{191\,400}{88\,475}\right)^{\frac{1}{4}} - 1\right] \\ &= 21{,}28 \end{aligned}$$

The MIRR is 21,28%.

Key in as

**(** 191 400 **÷** 88 475 **)** **2ndF** **$y^x$** **(** 1 **÷** 4 **)** **−** 1 **)** **×** 100 **=**

The answer is 21,28%.

## 6.11 Bonds

$$P = da_{\overline{n}|z} + 100\,(1+z)^{-n}$$

- Consider Bond XYZ

  Coupon rate: 13% per year
  Maturity date: 15 July 2022
  Yield to maturity: 15,9% per year
  Settlement date: 24 May 2007

  Determine the present value

$$\begin{aligned} P &= \tfrac{13}{2} a_{\overline{30}|0{,}159\div 2} + 100\left(1+\tfrac{0{,}159}{2}\right)^{-30} \\ &= 73{,}52215 + 10{,}07675 \\ &= 83{,}59890 \end{aligned}$$

The present value is R83,59890%.

Key in as

**SET UP** 0, 0, 5

**2ndF** **CA**

**2ndF** **P/Y** 1 **ENT**

**ON/C**

13 **÷** 2 **×** 1 **+/−** **PMT**

30 **N**

15.9 **÷** 2 **=** **I/Y**

**COMP** **PV**

73.52215 appears on the screen.

**M+**

**2ndF** **CA**

+/− 100 **FV**

15.9 ÷ 2 **=** **I/Y**

30 **N**

**COMP** **PV**

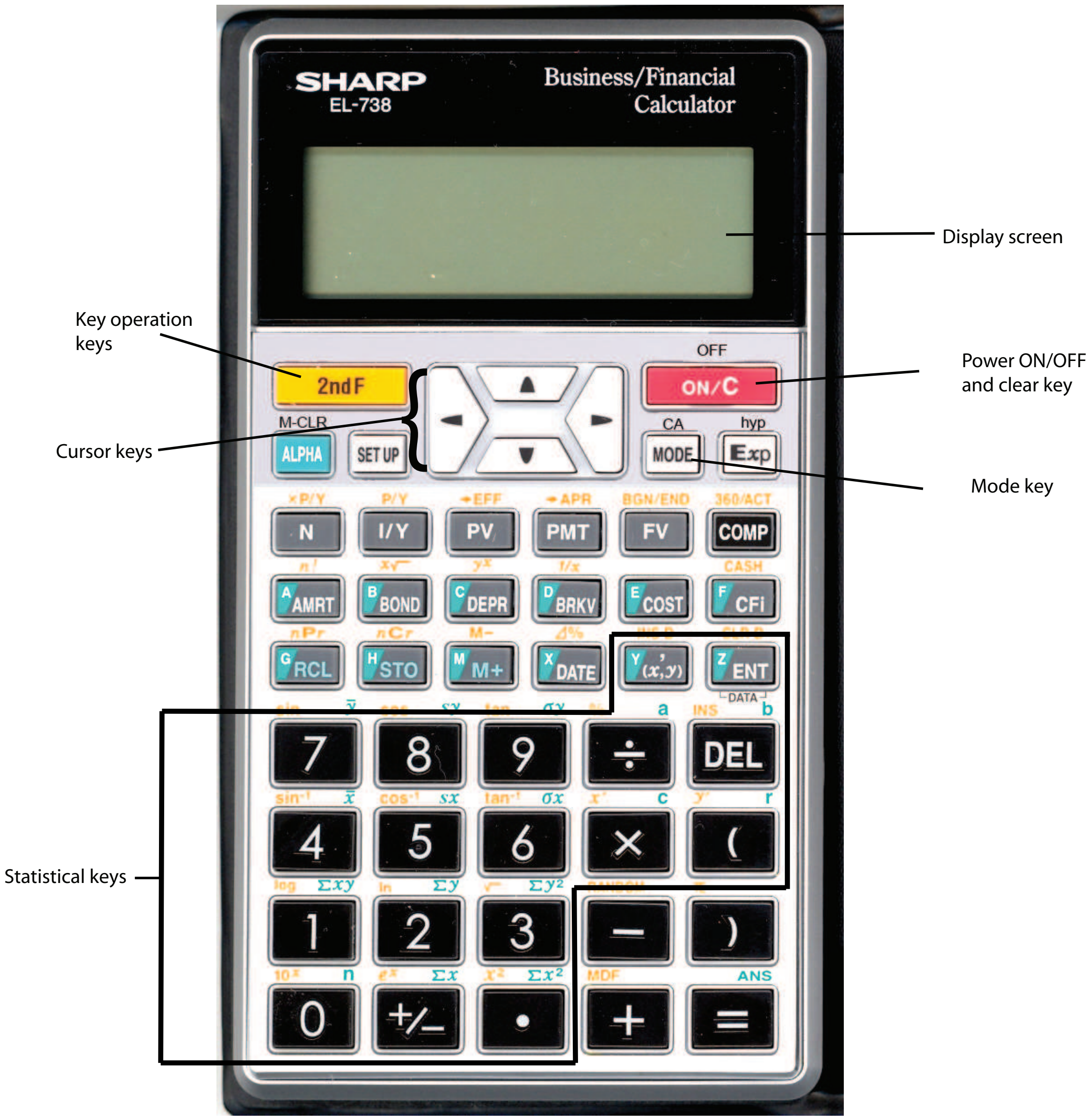
The answer is 10.07675

**+** **RCL** **M+** **=**

The answer is 83,59890.

Change the SET UP back to two decimals. Press **SET UP** 0, 0, 2.

## 6.12 Statistical mode



Given a data set, the calculator's STAT function can be used to calculate certain statistical values such as the average (mean), standard deviation and the equation of a linear line.

Change to the stat mode.

Press **MODE** 1, 0

```
                TAB  DEG  STAT
Stat 0
                0.00
```

appears on the screen.

## 6.13 Mean

- Determine the mean of the following values: 25; 30; 26; 15; 40; 35

  Key the data in

  25 DATA

  The calculator displays

  ```
                  TAB  DEG  STAT
  DATA SET =
                  1.00
  ```

  This means that it accepted the first data point. Keep on entering the data until the last one. 6 should be displayed.

  30 DATA

  26 DATA

  15 DATA

  40 DATA

  35 DATA

  Calculate the mean.

Press **ON/C**

TAB DEG STAT
Stat 0
0.00

appears on the screen.

Press **RCL** (first key, fifth row) $\bar{x}$ (first key, seventh row)

TAB DEG STAT
$\bar{x}$ =
28.50

appears on the screen.

The mean is 28,50.

## 6.14 Standard deviation

- Determine the standard deviation of the above data.

  Without re-entering the data, press **ON/C** **RCL** $sx$ (second key, seventh row).

  The standard deviation is 8,69.

## 6.15 Linear line

- Determine the equation of a linear line

$$y = bx + a$$

Press **MODE** 1 1

TAB DEG STAT
Stat 1
0.00

appears on the screen.

- Determine the equation for the straight line passing through the points (1 ; 3) and (3 ; 7).

  Key in as

  1 [$x,y$] - (fifth key, fifth row) 3 **[DATA]**

  3 [$x,y$] 7 **[DATA]**

  **[RCL]** **[a]** (fourth key, sixth row)

  $a = 1$ appears on the screen.

  **[RCL]** **[b]** (fifth key, sixth row)

  b = 2 appears on the screen.

  The equation for the straight line is

$$y = 2x + 1$$

## 6.16 Regression line

- Determine the equation of the regression line that represents the relationship between the two variables $x$ and $y$.

| $x$ | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 14 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 2 | 4 | 4 | 5 | 7 | 8 | 9 |

  Clear all data

  **[2ndF]** **[CA]**


  appears on the screen.

  Key in as

  1 [$x,y$] 1 **[DATA]**

  3 [$x,y$] 2 **[DATA]**

  4 [$x,y$] 4 **[DATA]**

  6 [$x,y$] 4 **[DATA]**

  8 [$x,y$] 5 **[DATA]**

  9 [$x,y$] 7 **[DATA]**

11 **x,y** 8 **DATA**

14 **x,y** 9 **DATA**

**RCL** **a**

a = 0.55

**RCL** **b**

b = 0.64

The equation is $y = 0{,}64x + 0{,}55$.

## 6.17 Correlation coefficient

- Determine the correlation coefficient of the above data.

  WITHOUT RE-ENTERING THE DATA ONLY

  Press **RCL** **r** (seventh row, fifth key)

  $r = 0.98$.

  The correlation coefficient is 0,98.

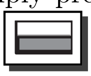
# Chapter 7

# Notes on the Hewlett Packard 10BII calculator

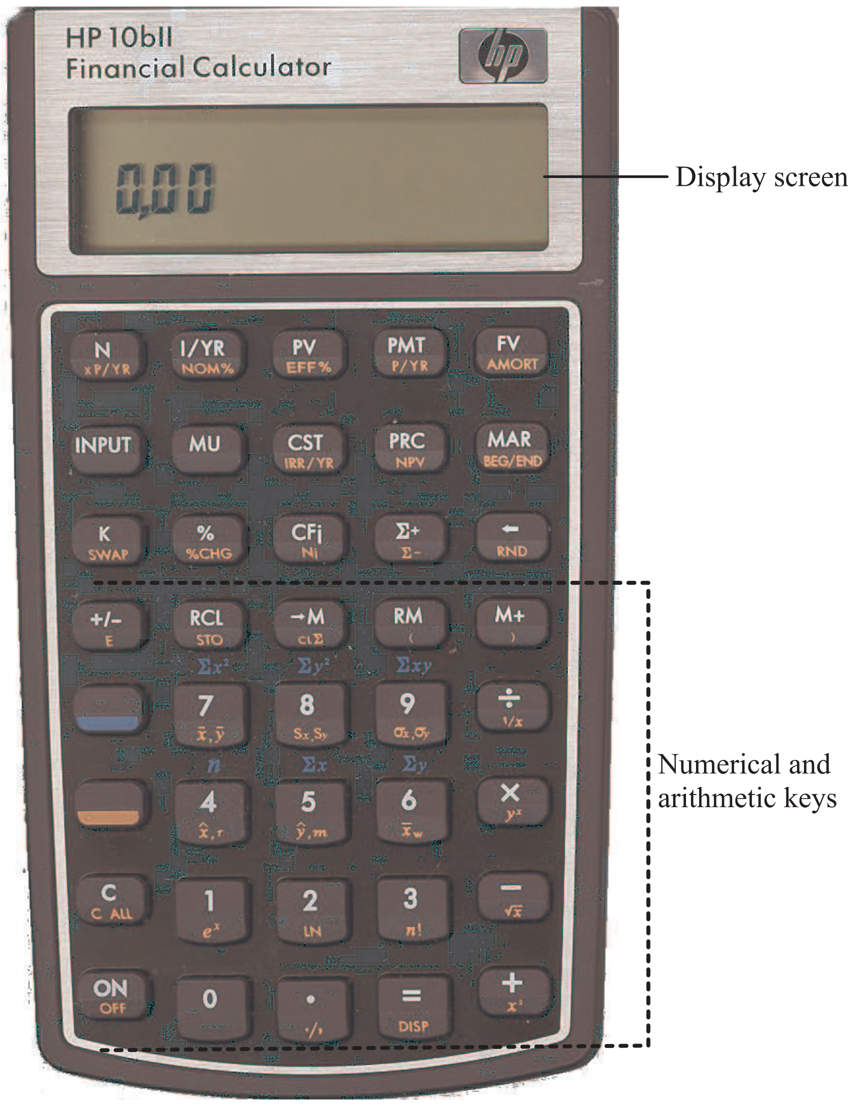
**General**

The HEWLETT PACKARD 10BII calculator is recommended as an alternative calculator for this module. The advantage of this calculator is that it can do basic calculations, financial calculations and statistical calculations.

Most of the keys can perform two functions.

To perform a function written on the key, you simply press the key. To perform a function written on the surface just above the key, first press the orange  key to activate it to perform the function when pressed.

### Calculations



## 7.1 Calculator keys

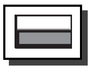
The keys are classified according to the work they do.

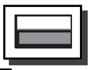
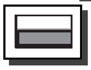
The following keys are worth mentioning:

- ON: **ON OFF**

  Last key, first row. To switch on the calculator. To preserve the batteries, the calculator turns itself off after about 10 minutes.

- OFF: **ON OFF**

  The orange function on the **ON** key. Press  **ON/C** to switch your calculator off.

- DISPLAY:  **DISP** (fourth key, last row) To specify the number of displayed decimal places. Press  **DISP** and then the number of digits you wish to display.

- NUMERIC KEYS: 1, 2, 3 .... 9, 0

  These keys are used to enter numbers.

- MULTIPLICATION **X**

  Last key, sixth row.

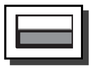
- DIVISION **÷**

  Last key, fifth row.

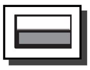
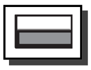
- EQUAL **=**

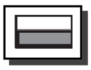
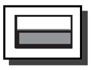
  Fourth key, last row.

- CLEAR **C ALL**

  First key, second last row (to clear the screen or to clear the last number entered)  **C ALL**. To clear the register.

- BRACKETS **(** **)**

   **(**  **)** fourth and fifth keys, fourth row.

  Use the  **(** and  **)** keys to place parentheses around parts of expressions. The closing parenthesis **)** may be omitted.

- NEGATIVE **+/-**

  First key, fourth row.

This key is used to enter a negative number or change the sign of a number, while the − key is used for the operation of subtraction. Note the different ways in which subtraction, with the long dash, and the sign of the number, with a small dash, are displayed.

For example: $3 - 2$ and 3+(-2).

- DELETE: ←

  If you want made a mistake, press ← to erase the number and then enter the correct number to continue.

- TO THE POWER key $y^x$.

  The fifth key, sixth row

  **Example:**

  Calculate $2^3$.

  Enter the base number first − press 2.

  Then press $y^x$ 3 =

  The answer is 8,00.

  If the power consists of more than one term, use brackets for the power.

  **Example:**

  Calculate $\left(3^2\right)^4$

  Press ( 3 $y^x$ 2 ) $y^x$ 4 =

  The answer is 6 561,00.

  **Example:**

  Calculate $5^{2/3}$

  Press 5 $y^x$ ( 2 ÷ 3 ) =

  The answer is 2,92.

- SQUARE: $(x^2)$

  The last key, last row

  **Example:**

  Calculate $4^2$.

  Press 4 $x^2$

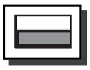
  The answer is 16.

- SQUARE ROOT: $\sqrt{x}$

  The fifth key, second last row.

  **Example:**

  Calculate $\sqrt{64}$. $\sqrt{64}$ means $\sqrt[2]{64}$.

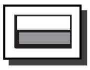
Press 64  $\sqrt{x}$

The answer is 8.

- LOGARITHM to the base e: ln

  **Example:**

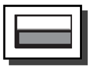
  Calculate ln 3.

  Press 3  **LN** third key, second last row

  The answer is 1,10.

  **Example:**

  Calculate $\ln\left(\frac{1\,253}{1479}\right)$.

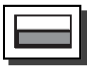
  Press 1 253 ÷ 1 479 **=**  **LN**

  The answer is −0,17.

- THE EXPONENTIAL FUNCTION: $e^x$ – The inverse of ln.

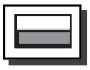
  **Example:**

  Calculate $e^{1,10}$.

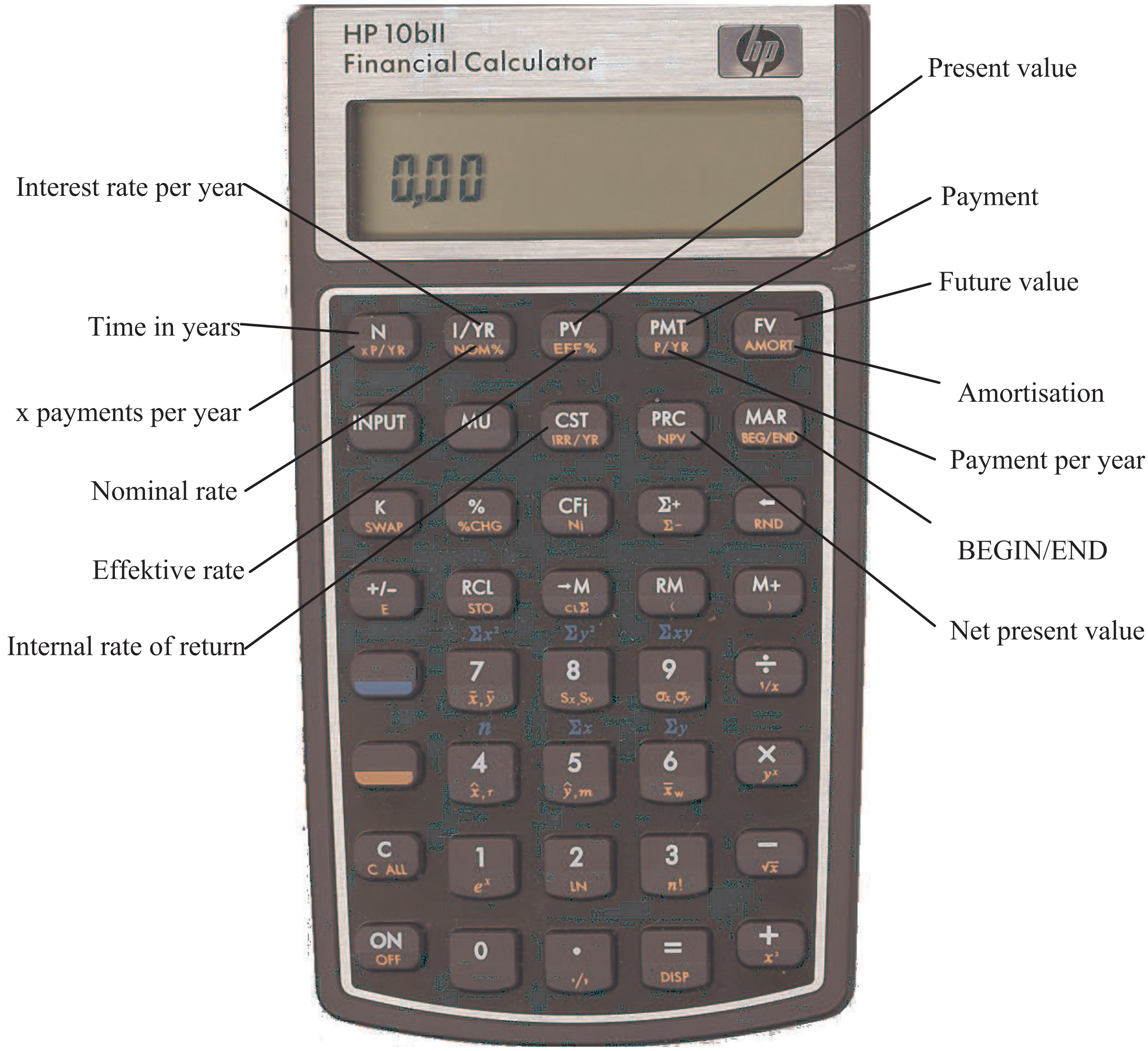
  Press 1.10  $e^x$ second key, second last row.

  The answer is 3.

- MEMORY: →M, RM and M+ (third, fourth and fifth keys, fourth row). These keys perform memory operations. In most cases, it is unnecessary to clear the M register, since **→M** replaced the previous contents. To add a series of numbers to the M register press the first number followed by **→M** then continue by entering the following numbers and **M+** after each number. To subtract a number from the M register press the number followed by **+/-** and **M+**. To get the answer press **RM**.

- ERROR: An error message will appear on the screen if data entered are incorrect. Clear the register by pressing  **C ALL** and re-enter the data.

# Financial Calculations



**Financial calculations**

## 7.2 Interest rates

### 7.2.1 Simple interest

$$I = Prt$$

- Determine the amount of interest received if R1 200 is invested for 4 years at 14% simple interest per year.

$$\begin{aligned} I &= Prt \\ &= 1\,200 \times 14\% \times 4 \\ &= 1\,200 \times 0{,}14 \times 4 \\ &= 672{,}00 \end{aligned}$$

The interest received is R672,00. We cannot use the financial keys because there is no exponent in the formula.

Key in as

1 200 [×] 0.14 [×] 4 [=]

The answer is 672,00.

$$S = P\,(1 + rt)$$

- Determine the accumulated amount if R2 400 is invested for 42 months at a 9% simple interest rate per year.

$$\begin{aligned} S &= 2\,400\left(1 + 9\% \times \tfrac{42}{12}\right) \\ &= 2\,400\left(1 + 0{,}09 \times \tfrac{42}{12}\right) \\ &= 3\,156{,}00. \end{aligned}$$

The accumulated amount is R3 156,00.

Key in as

2 400 [×] [▭] [(] 1 [+] [▭] [(] 0.09 [×] 42 [÷] 12 [▭] [)] [▭] [)] [=]

The answer is 3 156,00.

- Determine the simple interest rate if R3 600 accumulates to R5 760 in five years' time.

$$\begin{aligned} S &= P\,(1 + rt) \\ 5\,760 &= 3\,600\,(1 + r \times 5) \\ 1 + 5r &= \tfrac{5\,760}{3\,600} \\ 5r &= \tfrac{5\,760}{3\,600} - 1 \\ r &= \left(\tfrac{5\,760}{3\,600} - 1\right) \div 5 \\ &= 0{,}12 \end{aligned}$$

The simple interest rate is 12%.

Key in as

[■] [(] 5 760 [÷] 3 600 [−] 1 [■] [)] [÷] 5 [=]

The answer is 0,12, that is, 12%.

### 7.2.2 Simple discount

$$P = S\,(1 - dt)$$

- Determine the present value of a promissory note that is worth R2 500 15 months later, and the applicable discount rate is 10,24% per year.

$$\begin{aligned} P &= S\,(1 - dt) \\ P &= 2\,500\left(1 - 0{,}1024 \times \tfrac{15}{12}\right) \\ &= 2\,180{,}00 \end{aligned}$$

The present value is R2 180,00.

Key in as

2 500 [×] [■] [(] 1 [−] [■] [(] 0.1024 [×] 15 [÷] 12 [■] [)] [■] [)] [=]

The answer is 2 180,00.

- Determine the time under consideration (in months) if a simple interest rate of 11,76% is equivalent to a 10,25% simple discount rate.

By manipulating

$$S = P\,(1 + rt) \text{ and } P = S\,(1 - dt)$$

we get

$$r = \frac{d}{1 - dt}$$

and

$$t = \left(1 - \frac{d}{r}\right) \div d.$$

Substituting the values, we get

$$\begin{aligned} t &= \left(1 - \tfrac{0{,}1025}{0{,}1176}\right) \div 0{,}1025 \\ &= 1{,}25. \end{aligned}$$

The time under consideration is 1,25 years, that is, 15 months.

Key in as

[▭] [ ( ] 1 [ − ] [▭] [ ( ] 0.1025 [ ÷ ] 0.1176 [▭] [ ) ] [▭] [ ) ] [ ÷ ] 0.1025 [ = ]

The answer is 1,25, that is, 15 months.

### 7.2.3 Compound interest

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm} \text{ or } S = P\,(1+i)^n$$

We use our financial keys to do the calculations because there is only one exponent in the formula:

$$S = P\,(1+i)^n$$

**NB: The interest rate must be entered into the calculator as a percentage and *NOT* as a decimal because the calculator has been preprogrammed to automatically divide the interest rate by a hundred. Remember that it is convention to enter either the present value, or future value as a negative amount.**

- Calculate the future value if R5 000 is invested for five years at 15% per year compounded monthly.

$$\begin{aligned} S &= P\,(1+i)^n \\ &= 5\,000\left(1+\tfrac{0{,}15}{12}\right)^{5\times 12} \\ &= 10\,535{,}91 \end{aligned}$$

The future value is R10 535,91.

Key in as

[▭] [ C ALL ] (to clear the register).

First enter the number of compounding periods.

12 [▭] [ P/YR ] (Fourth key, first row)

5 000 [ +/− ] [ PV ]

15 [ I/YR ]

5 [▭] [ N ]

To check if you have entered the correct values press [ RCL ] and the financial key that you want to check. If the value is incorrect, enter the new value, press the financial key and continue.

[ FV ]

The answer is 10 535,91.

- Determine the time under consideration if R5 000 is invested at 15% per year, compounded half yearly, and the accumulated amount is R10 000.

$$\begin{aligned} S &= P\,(1+i)^n \\ 10\,000 &= 5\,000\left(1+\tfrac{0{,}15}{2}\right)^{t\times 2} \\ t &= 4{,}79 \end{aligned}$$

The time under consideration is 4,79 years.

Key in as

■ C ALL

2 ■ P/YR

10 000 +/− FV

5 000 PV

15 I/YR

N

N = 9.5844 appears on the screen. Because the number of compounding periods is half yearly, divided the answer by two.

Press ÷ 2 =.

4.79 appears on the screen.

### 7.2.4 Effective rate

$$J_{\mathit{eff}} = 100\left(\left(1+\frac{j_m}{m}\right)^m - 1\right)$$

- Determine the effective rate for a nominal rate of 14% per year, compounded quarterly.

$$\begin{aligned} J_{\mathit{eff}} &= 100\left(\left(1+\tfrac{0{,}14}{4}\right)^4 - 1\right) \\ &= 14{,}75. \end{aligned}$$

The effective rate is 14,75%.

Key in as

100 × ■ ( ■ ( 1 + ■ ( 0.14 ÷ 4 ■ ) ■ $y^x$ 4 ■ ) − 1 ■ ) =

The answer is 14,75%.

**OR**

**C ALL**

4 **P/YR**

14 **NOM %**

**EFF %**

14.75 appears on the screen.

- Determine the nominal rate per year, compounded monthly for an effective rate of 19,56%.

$$\begin{aligned} J_{eff} &= 100\left(\left(1+\tfrac{j_m}{m}\right)^m - 1\right) \\ 19{,}56 &= 100\left(\left(1+\tfrac{j_m}{12}\right)^{12} - 1\right) \\ j_m &= 18{,}00 \end{aligned}$$

The nominal rate is 18,00%.

Key in as

**C ALL**

12 **P/YR**

19.56 **EFF %**

**NOM %**

17.998 appears on the screen, that is, 18%.

## 7.3 Converting interest rates

### 7.3.1 Nominal rates

$$i = n\left(\left(1+\frac{j_m}{m}\right)^{m\div n} - 1\right)$$

- Convert 15% compounded every two months to compounded half yearly.

$$\begin{aligned} i &= 2\left(\left(1+\tfrac{0{,}15}{6}\right)^{6\div 2} - 1\right) \\ &= 0{,}1538 \end{aligned}$$

The new rate compounded half yearly is 15,38%.

Key in as

2 [×] [■] [(] [■] [(] 1 [+] [■] [(] 0.15 [÷] 6 [■] [)] [■] [$y^x$] [■] [(] 6 [÷] 2 [■] [)] [−] 1 [■] [)] [=] [×] 100 [=]

The answer is 15,38%.

### 7.3.2 Continuous compounding

$$c = m \ln\left(1 + \frac{j_m}{m}\right)$$

- Convert 15%, compounded every two months, to continuous compounding.

$$\begin{aligned} c &= 6 \ln\left(1 + \tfrac{0,15}{6}\right) \\ &= 0,1482 \end{aligned}$$

The continuous compounding rate is 14,82%.

Key in as

0.15 [÷] 6 [=] [+] 1 [=] [■] [LN] [×] 6 [=] [×] 100 [=]

The answer is 14,82%.

$$i = m\left(e^{c/m} - 1\right)$$

- Convert 14,82% continuous compounding to a nominal rate compounded half yearly.

$$\begin{aligned} i &= 2\left(e^{0,1482 \div 2} - 1\right) \\ &= 0,1538 \end{aligned}$$

The nominal interest rate is 15,38%.

Key in as

0.1482 [÷] 2 [=] [■] [$e^x$] [−] 1 [=] [×] 2 [=] [×] 100 [=]

The answer is 15,38%.

$$J_\alpha = 100\left(e^c - 1\right)$$

- Convert 8% continuous compounding to an effective interest rate.

$$J_\alpha = 100\left(e^{0,08} - 1\right)$$

The effective rate is 8,33%.

Key in as

0.08 [■] [$e^x$] [−] 1 [=] [×] 100 [=]

The answer is 8,33%.

- Convert an effective rate of 11,92% to a continuous compounding rate.

$$\begin{aligned} J_\alpha &= 100\,(e^c - 1) \\ 11{,}92 &= 100\,(e^c - 1) \\ e^c &= 0{,}1192 + 1 \\ c \ln e &= \ln 1{,}1192 \\ c &= 11{,}26 \end{aligned}$$

The continuous compounding rate is 11,26%.

Key in as

1.1192 [ ] [LN] [×] 100 [=]

The answer is 11,26%.

$$S = Pe^{ct}$$

- Determine the accumulated amount if R2 400 is invested for five years at 8% per year compounded continuously.

$$\begin{aligned} S &= 2\,400e^{0{,}08\times 5} \\ &= 3\,580{,}38 \end{aligned}$$

The accumulated amount is R3 580,38.

Key in as

0.08 [×] 5 [=] [ ] [$e^x$] [×] 2 400 [=]

The answer is 3 580,38.

- Determine the continuous compounding rate if R12 000 accumulates to R17 901,90 after five years.

$$\begin{aligned} S &= Pe^{ct} \\ 17\,901{,}90 &= 12\,000e^{c\times 5} \\ \tfrac{17\,901{,}90}{12\,000} &= e^{5c} \\ \ln\left(\tfrac{17\,901{,}90}{12\,000}\right) &= 5c \ln e \\ c &= \ln\left(\tfrac{17\,901{,}90}{12\,000}\right) \div 5 \\ &= 0{,}08 \end{aligned}$$

The continuous compounding rate is 8%.

Key in as

17 901.90 [÷] 12 000 [=] [ ] [LN] [÷] 5 [=]

[×] 100 [=]

The answer is 8%.

## 7.4 Annuities

### 7.4.1 Present value

$$
\begin{aligned}
P &= Ra_{\overline{n|}i} \\
&= R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]
\end{aligned}
$$

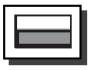
- Calculate the present value of R1 600 quarterly payments for five years at an interest rate of 20% per year, compounded quarterly.

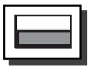
$$
\begin{aligned}
P &= 1\,600a_{\overline{5\times4|}0{,}20\div4} \\
&= 19\,939{,}54.
\end{aligned}
$$

The present value is R19 939,54.

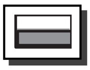
REMEMBER TO ENTER AN AMOUNT (THAT IS THE PAYMENT, OR FUTURE VALUE OR THE PRESENT VALUE) AS A NEGATIVE.

Key in as

 **C ALL**

4  **P/YR**

1 600 **+/−** **PMT**

5  **N**

20 **I/Y**

**PV**

The answer is 19 939,54.

### 7.4.2 Future value

$$
\begin{aligned}
S &= Rs_{\overline{n|}i} \\
&= R\left[\frac{(1+i)^n-1}{i}\right].
\end{aligned}
$$

- Determine the future value of R400 monthly payments made for five years at 16% interest per year, compounded monthly.

$$
\begin{aligned}
S &= 400s_{\overline{5\times12|}0{,}16\div12} \\
&= 36\,414{,}21
\end{aligned}
$$

The future value is R36 414,21.

Key in as

**C ALL**

12 **P/YR**

400 +/− **PMT**

16 **I/YR**

5 **N**

**FV**

The answer is 36 414,21.

### 7.4.3 Annuity due

If the words *begin* immediately, in *advance* and in the *beginning* appear in the sentence, an annuity due calculation is involved.

$$\begin{aligned} S &= (1+i)\,Rs_{\overline{n}|i} \\ P &= (1+i)\,Ra_{\overline{n}|i}. \end{aligned}$$

- Determine the future value after five years of R400 payments made at the *beginning* of a month in an account earning 16% interest per year, compounded monthly.

$$\begin{aligned} S &= (1+i)\,Rs_{\overline{n}|i} \\ &= (1+i)\,400s_{\overline{5\times12}|0{,}16\div12} \\ &= 36\,899{,}73 \end{aligned}$$

The future value is R36 899,73.

Key in as

**C ALL**

**BEG/END** (fifth key, second row)

12 **P/YR**

400 +/− **PMT**

16 **I/YR**

5 **N**

**FV**

The answer is 36 899,73.

**NB: PRESS BEG/END AGAIN TO CANCEL THE BEGIN FUNCTION – IF YOU DO NOT DO IT ALL THE ANSWERS THAT FOLLOW WILL BE INCORRECT.**
(NOTE: If the BEG/END function is cancelled it will disappear from the screen)

### 7.4.4 Increasing annuity

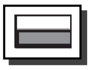
$$S = \left(R + \frac{Q}{i}\right) s_{\overline{n}|i} - \frac{nQ}{i}$$

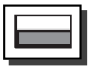
- An endowment policy with yearly payments of R3 600 matures in 20 years' time. Calculate the future value of this policy if the yearly payments increase by R360 per year and an interest rate of 13% per year is applicable.

$$\begin{aligned} S &= \left(3\,600 + \tfrac{360}{0{,}13}\right) s_{\overline{20}|0{,}13} - \tfrac{20\times 360}{0{,}13} \\ &= 515\,569{,}03 - 55\,384{,}62 \\ &= 460\,184{,}42 \end{aligned}$$

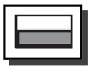
The future value is R460 184,42.

Key in as

 [C ALL]

1  [P/YR]

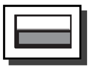
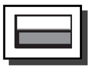
360 [÷] 0.13 [=] [+] 3 600 [=] [+/−] [PMT]

20  [N]

13 [I/YR]

[FV]

FV = 515 569.03 appears on the screen.

Press [−]  [(] 360 [×] 20 [÷] 0.13  [)] [=]

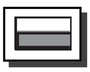
460 184,42 is the answer.

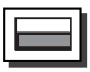
### 7.4.5 Amortisation

- Draw up an amortisation schedule for a loan of R5 000 which is repaid in annual payments over five years at an interest rate of 15% per year.

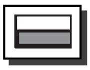
$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ 5\,000 &= Ra_{\overline{5}|0{,}15} \\ R &= 1\,491{,}58 \end{aligned}$$

Key in as

 [C ALL]

1  [P/YR]

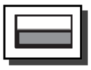
5 000 [+/−] [PV]

5  [N]

15 [I/YR]

[PMT]

1.491.58 appears on the screen.

Press  [AMORT] (fifth key, first row)

AMORT PER
**1 - 1**

appears on the screen.

Press [=]

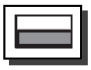
AMORT PRIN 741.58 appears on the screen.

Press [=]

AMORT INT 750.00 appears on the screen.

Press [=]

AMORT BAL −4258.42 appears on the screen.

Press  [AMORT]

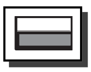
AMORT PER
**2 - 2**

appears on the screen.

Press [=] [PRIN] 852.82 appears

Press [=] [INT] 638.76 appears

Press [=] [BAL] −3405.60 appears

Press  [AMORT]

Press [=] [PRIN] 980.74

Press [=] [INT] 510.84

Press [=] [BAL] −2424.86

Press [■] [AMORT]

Press [=] [PRIN] 1 127.85

Press [=] [INT] 363.73

Press [=] [BAL] −1297.01

Press [■] [AMORT]

Press [=] [PRIN] 1 297.03

Press [=] [INT] 194.55

Press [=] [BAL] 0.02

- A loan of R135 000 must be repaid over a 20-year period in monthly instalments. The applicable interest rate is 18% per year, compounded monthly.

  Determine the outstanding balance, interest due and principal repaid after 235 payments made.

  First determine the monthly payments.

$$\begin{aligned} P &= a_{\overline{n}|i} \\ 135\,000 &= Ra_{\overline{20\times 12}|0,18\div 12} \\ R &= 2\,083,47 \end{aligned}$$

  Key in as

  [■] [C ALL]

  12 [■] [P/YR]

  135 000 [+/−] [PV]

  18 [I/YR]

  20 [■] [N]

  [PMT]

  2 083.47 appears on the screen.

  Press 235 [INPUT] [■] [AMORT]

  235-235 appears on the screen.

  ONLY Press [=] [PRIN] 1 905,42

  ONLY Press [=] [INT] 178,05

  ONLY Press [=] [BAL] −9 964,49

## 7.5 Internal Rate of Return and Net Present Value

$$NPV = \frac{A}{1+i} + \frac{B}{(1+i)^2} \cdots \frac{N}{(1+i)^n}$$

- An investment with an initial outlay of R120 000 returns a constant cash flow of R24 000 per year for 10 years. Determine the internal rate of return (IRR) of this investment

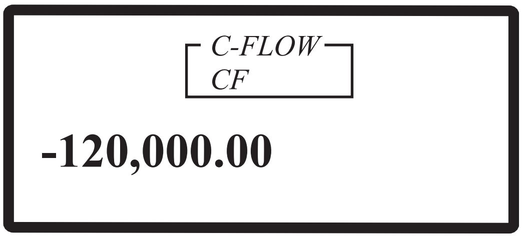
$$120\,000 = \frac{24\,000}{1+i} + \frac{24\,000}{(1+i)^2} + \ldots + \frac{24\,000}{(1+i)^{10}}$$

Clear the memory by pressing

C ALL

1 P/YR

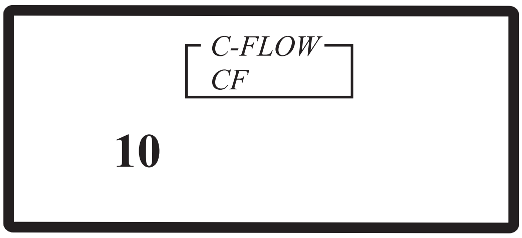
120 000 +/− $CF_j$ (third key third row)




appears on the screen

24 000 $CF_j$

Keep on pressing $CF_j$ until




and

appears on the screen IRR/YR 15.10 appears on the screen

## 7.6 Internal Rate of Return and Net Present Value

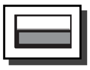
$$NPV = \frac{A}{1+i} + \frac{B}{(1+i)^2} \cdots \frac{N}{(1+i)^n}$$

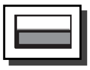
| C-FLOW CF |
| --- |
| **24,000.00** |

- An investment of R120 000 generates three successive cash inflows of R60 000, R48 000 and R35 000 respectively. Determine the IRR. Determine the NPV if the cost of capital is 8%.

$$0 = \frac{60\,000}{1+i} + \frac{48\,000}{(1+i)^2} + \frac{35\,000}{(1+i)^3} - 120\,000$$

Clear the memory by pressing

 **C ALL**

1  **P/YR**

120 000 [+/−] $\text{CF}_j$ (third key third row)

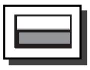
| C-FLOW CF |
| --- |
| **-120.000,00** |

appears on the screen

60 000 $\text{CF}_j$

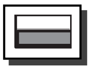
48 000 $\text{CF}_j$

35 000 $\text{CF}_j$

 **IRR/YR**

10.27 appears on the screen and

Press 8 **I/YR**

 **NPV**

4 491.95 appears on the screen and that is the NPV

## 7.7 MIRR

$$MIRR = \left[\left(\frac{C}{PV_{\text{out}}}\right)^{\frac{1}{n}} - 1\right]$$

- Calculate the MIRR of an investment over a four-year period if the PV (*present value*) of the cash outflows equals R88 475 and the FV (*future value*) of the cash inflows equals R191 400.

$$\begin{aligned} MIRR &= \left[\left(\tfrac{191\,400}{88\,475}\right)^{\frac{1}{4}} - 1\right] \\ &= 21{,}28 \end{aligned}$$

The MIRR is 21,28%.

Key in as

▭ [(] 191 400 [÷] 88 475 [=] ▭ [)] ▭ [$y^x$] ▭ [(] 1 [÷] 4 [=] [−] 1 ▭ [)] [=]
[×] 100 [=]

The answer is 21,28%.

## 7.8 Bonds

$$P = da_{\overline{n}|z} + 100\,(1+z)^{-n}$$

- Consider Bond XYZ

| | |
|---|---|
| Coupon rate: | 13% per year |
| Maturity date: | 15 July 2022 |
| Yield to maturity: | 15,9% per year |
| Settlement date: | 24 May 2007 |

Determine the present value

$$\begin{aligned} P &= \tfrac{13}{2} a_{\overline{30}|0{,}159\div 2} + 100\left(1 + \tfrac{0{,}159}{2}\right)^{-30} \\ &= 73{,}52215 + 10{,}07675 \\ &= 83{,}59890 \end{aligned}$$

Key in as

▭ [DISP] (fourth key, last row) 5

▭ [C ALL]

1 ▭ [P/YR]

13 [÷] 2 [=] [+/−] [PMT]

30 ▭ [N]

15.9 [÷] 2 [=] [I/YR]

[PV]

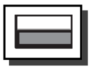
73.52215 appears on the screen.

NOW ONLY CLEAR THE SCREEN BY PRESSING

C

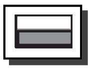
100 +/− FV

15.9 ÷ 2 = I/YR

30  N

PV

The answer is 83,59890.

Change the DISP back to two decimals. Press  DISP 2.

## 7.9 Statistical calculations


Statistical keys

Given a data set, the calculator can be used to calculate certain statistical values such as the average (mean), standard deviation and the equation of a linear line.

Clearing statistical data.

Clear the statistical registers before entering new data. If you don't clear the register, the data currently stored is automatically included in the summation calculations. To clear the statistical registers, press **CL∑**, (third key, fourth row).

## 7.10 Mean

- Determine the mean of the following values: 25; 30; 26; 15; 40; 35

  **CL∑**

  Key the data in

  25 ∑+ (fourth key, third row)

  The calculator displays

  **1,00**

  This means that it accepted the first data point. Keep on entering the data until the last one. 6 should be displayed.

  30 ∑+ (fourth key, third row)

  26 ∑+

  15 ∑+

  40 ∑+

  35 ∑+

  Calculate the mean by pressing $\bar{x}$,$\bar{y}$ (second key, fifth row).

  The mean is 28,50.

## 7.11 Standard deviation

- Determine the standard deviation of the above data.

  Without re-entering the data, press $Sx$,$Sy$ (third key, fifth row).

  The standard deviation is 8,69.

## 7.12 Linear line

- Determine the equation of a linear line

$$y = bx + a$$

- Determine the equation for the straight line passing through the points (1 ; 3) and (3 ; 7).

  CL$\sum$

  Key in as

  1 INPUT - (first key, second row) 3 $\sum+$ (fourth key third row)

  3 INPUT 7 $\sum+$

  To calculate the value of a

  Press 0 $\hat{y}$,$m$ (third key, sixth row)

  1 appears on the screen.

  Press SWAP (first key, third row) to calculate to

  2 appears on the screen.

  The equation for the straight line is

$$y = 2x + 1$$

## 7.13 Regression line

- Determine the equation of the regression line that represents the relationship between the two variables $x$ and $y$.

| $x$ | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 14 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 2 | 4 | 4 | 5 | 7 | 8 | 9 |

  Clear all data

  CL $\sum$

  Key in as

  1 INPUT 1 $\sum+$

  3 INPUT 2 $\sum+$

  4 INPUT 4 $\sum+$

  6 INPUT 4 $\sum+$

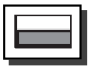
8 **INPUT** 5 $\sum +$

9 **INPUT** 7 $\sum +$

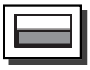
11 **INPUT** 8 $\sum +$

14 **INPUT** 9 $\sum +$

8 appears on the screen. This means that 8 data points have been accepted.

0  $\hat{y},m$

0.55 is displayed

 **SWAP**

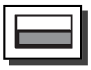
0.64 is displayed.

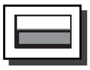
The equation is $y = 0{,}64x + 0{,}55$.

## 7.14 Correlation coefficient

- Determine the correlation coefficient of the above data.

  WITHOUT RE-ENTERING THE DATA ONLY

  Press  $\hat{x},r$ (second key, sixth row)

   **SWAP**

  0.98 is displayed

  The correlation coefficient is 0,98.