

# DSC1630

**Mrs Adéle Immelman**

**Hazelwood Campus, Room 4-28**

**Tel: +27 12 4294735**

**E-mail: [immelf@unisa.ac.za](mailto:immelf@unisa.ac.za)**

**Appointments and Telephone**

**08:00 – 13:30: Monday till Friday**

**Telephone only**

**13:30 – 16:00: Monday till Thursday**

**Mrs Edinah Mudimu**

**Hazelwood Campus, Room 4-35**

**Tel: +27 726800137**

**E-mail: [mudime@unisa.ac.za](mailto:mudime@unisa.ac.za)**

# **INTEREST**

- **Money that you earn or pay for the use of money.**
- **Always a percentage of the money deposited or borrowed.**
- **Different types of interest – depends on the way interest is calculated.**

# Chapter 2

- **Simple Interest**
- **Simple Discount**

# SIMPLE INTEREST

- Interest calculated once
- % of present/principal value

$$I = Prt$$

**I** = Interest received/paid - money

**P** = Principal/present value

**r** = Interest rate **per year**

**t** = Time **in years** - if not change it to a fraction of a year -> ÷ number of periods in one year

- Interest + principal paid in future

$$S = P + I$$

$$S = P + Prt$$

$$S = P(1 + rt)$$

**S** = Accumulated (future) amount

**ID: Simple interest or simple interest rate**

**Calculator: Use normal mode**

# Question 1

**An amount of R4 317,26 was borrowed on 5 May at a simple interest rate of 15% per year. Determine the value of the loan, on 16 August of the same year.**



**ID: simple interest rate**

$$S = P (1 + rt)$$

**Given:**

$$P = 4\,317,26$$

$$r = 15\%$$

$$t = 5 \text{ May till } 16 \text{ Aug}$$

*count days or use*

*table value in guide*

*appendix C*

*row  $\Rightarrow$  day*

*column  $\Rightarrow$  month*

*intersection  $\Rightarrow$  date value*

$$= 228 - 125$$

$$= \frac{103}{365}$$

$$S = P(1 + rt)$$

$$= 4317,26(1 + 0,15 \times \frac{103}{365})$$

$$= R4500$$

**Calculator: Use normal mode**

## Question 2

**Determine the amount of money received as interest in the previous question.**



$$I = Prt$$

$$= 4317,26 \times 0,15 \times \frac{103}{365}$$

$$= 182,74$$

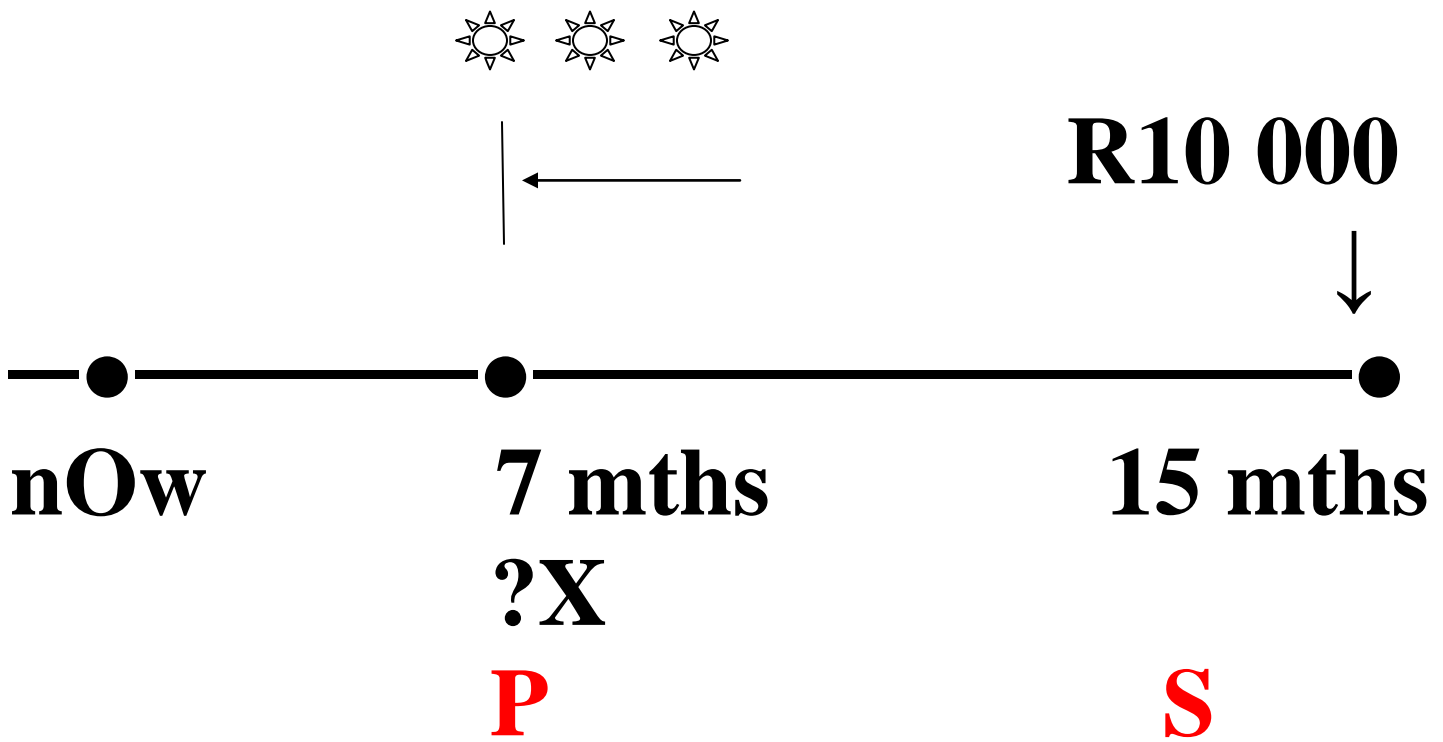
**An amount of R182,74 was received.**

**Calculator: Use normal mode**



# Question 3

Fifteen months from *now* Jenny has to pay Jonas R10 000. She decides to pay him back earlier. A simple interest rate of 13% per year is applicable. Determine the amount that Jenny will have to pay Jonas seven months from *now*.



$$S = P(1 + rt)$$

$$10\,000 = P \left( 1 + 0,13 \times \frac{8}{12} \right)$$

$$\frac{10\,000}{\left( 1 + 0,13 \times \frac{8}{12} \right)} = P$$

$$P = 9\,202,45$$

**Jenny must pay Jonas R 9 202,45.**

**Calculator: Use normal mode**

# SIMPLE DISCOUNT

- Interest is calculated on  $S$  (FV)
- % of FV
- Interest deducted from  $S$

$$P = S - D$$

$$P = S - Sdt$$

$$P = S(1 - dt)$$

$P$  = Present value

$S$  = Future value

$d$  = discount rate

$t$  = time in years

**ID: Simple discount/rate**

**Calculator: use normal mode**

## Question 4

**A bank's discount rate is 12%. You must pay the bank R5 000 in six months' time. Determine the amount of money that you will receive now.**



**ID: discount rate**

$$P = S(1 - dt)$$

$$S = 5\ 000$$

$$d = 0,12$$

$$t = \frac{6}{12}$$

$$P = 5\ 000 \left( 1 - 0,12 \times \frac{6}{12} \right)$$
$$= 4\ 700$$

**You will receive R4 700.**

## Question 5

An amount of money is invested for 281 days at a simple discount rate of 10,9%. Determine the equivalent simple interest rate.



**ID:**

simple  $i$  + discount rate  $d \Rightarrow$  **conversion**

$$P = S(1 - dt) \text{ and}$$

$$S = P(1 - rt) \text{ The two equations}$$

is equal if

$$r = \frac{d}{1 - dt} \quad p 151 \text{ guide}$$

$$= \frac{0,109}{1 - 0,109 \times 281 \div 365}$$

$$= 0,119$$

The equivalent simple interest rate is 11,9%.

# Chapter 3

- **Compound interest**
- **Odd periods and fractional compounding**
- **Effective rate**
- **Continuous compounding**
- **Conversions**
- **Time value of money**
- **Equation of value**

# COMPOUND INTEREST

- Only one principle value
- Interest calculations are done more than once according to the number of compounding periods
- Earn interest on interest

$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm} \quad \text{or} \quad S = P (1 + i)^n$$

$j_m$  – nominal interest rate **per year**

$m$  – # compound periods **in 1 year**

$t$  – **years or fraction of year!!!**

$tm$  – also known as N on your calculator

**ID: compound interest + one principle**

**Calculator: finance mode or normal**

## Question 6

Susan deposits R15 000 into a new savings account. The amount of money that she will have in the bank after three years, if interest is compounded monthly at 8% per year, equals?



**ID: compounded interest/one principle**

$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$

with  $P = 15\ 000$  ;  $j_m = 0,08$

$m = 12$  ;  $t = 3$

$$\begin{aligned} S &= 15\ 000 \left( 1 + \frac{0,08}{12} \right)^{3 \times 12} \\ &= R19\ 053,56 \end{aligned}$$



# SHARP:

2ndf CA

2ndf P/Y 12 ENT ON/CL

3 × 12 = N or 3 2ndf ×P/Y N

± 15000 PV

8 I/Y

Comp FV

# HP:

ORANGE C ALL

12 ORANGE P/YR

3 × 12 = N or 3 ORANGE ×P/YR N

15000 ± PV

8 I/YR

FV

## Question 7

**Jack invested R17 000 into an account earning 13,89% interest per year, compounded weekly. Determine the accumulated amount after five years.**



**ID: compounded interest/one principle**

$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$

**with  $P = 17\ 000$**

$$j_m = 0,1389$$

$$m = 52$$

$$t = 5$$

$$\begin{aligned} S &= 17\ 000 \left( 1 + \frac{0,1389}{52} \right)^{5 \times 52} \\ &= 34\ 014,52 \end{aligned}$$

**The accumulated amount is  
R34 014,52.**

## Question 8

Harry's parents think that they will need R170 000 to pay for his university fees in 15 years' time. They invest money at 7,71% per year, compounded quarterly. Determine the amount that they need to invest now.



**ID: compounded interest/one principle**

$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$

**Given:**

**FV = 170 000; t = 15;  $j_m = 7,71$ ; m = 4**

**PV = ?**

$$170\ 000 = P \left( 1 + \frac{0,0771}{4} \right)^{15 \times 4}$$

**Do not change the formula – your calculator fin mode can solve it as it is!!!!!!! Enter the known values then COMP unknown**

**SHARP:**

**P/Y: 4**

**N: 15 × 4**

**FV: ± 170 000**

**I/Y: 7.71**

**Comp PV**

**HP:**

**P/YR: 4**

**N: 15 × 4**

**FV: ± 170 000**

**I/YR: 7.71**

**PV**

**R54 071,27 must be invested.**

# **Odd periods and fractional compounding**

**Odd period: Time period is not a full compounding period but smaller**

**Different methods to deal with it:**

**1. use simple interest for odd periods and compound interest for full periods -Important !!!! Time line**

**2. fractional compounding**

**Use compounding formula but the time period is calculated as the full period plus the odd period expressed as a fraction of the compounding period. – Page 37 guide**

**ID: odd periods or method given**

## Question 9

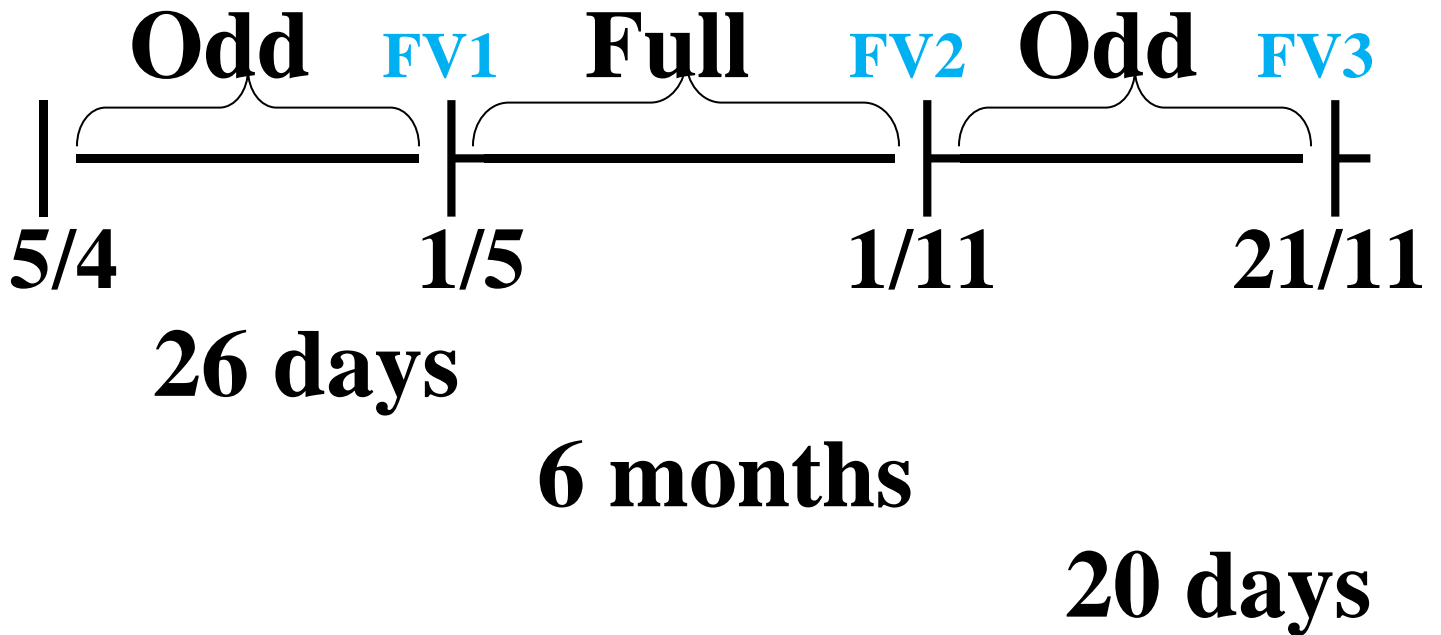
**On 5 April Sam invested R75 000 in an account paying 8,37% interest per year, compounded monthly. Interest is credited on the first day of every month. Sam wants to move into his new shop on 21 November of the same year. Determine the amount of money that Sam will have available on 21 November of the same year if simple interest is used for odd period calculations and compound interest for the full period.**



**ID: odd periods or method given**

**Draw time line first:**

**R75 000**



**Odd:  $S = P (1 + rt)$**

**Full:  $S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$**



**Method 1: Calc each period separately**  
**Don't round off until end**

$$\begin{aligned}S(FV1) &= P(1 + rt) \\ &= 75\,000 \left( 1 + 0,0837 \times \frac{26}{365} \right) \\ &= 75447,16438\end{aligned}$$

$$\begin{aligned}S(FV2) &= 75447,16438 \left( 1 + \frac{0,0837}{12} \right)^{\left( \frac{6}{12} \times \frac{12}{1} \right)} \\ &= 78660,20122\end{aligned}$$

$$\begin{aligned}S(FV3) &= 78660,20122 \left( 1 + 0,0837 \times \frac{20}{365} \right) \\ &= 79\,020,96\end{aligned}$$

**OR**

**Method2: Enter whole calculation once**

$$\begin{aligned} S &= P(1+rt) \left( 1 + \frac{j_m}{m} \right)^{tm} (1+rt) \\ &= 75\,000 \left( 1 + 0,0837 \times \frac{26}{365} \right) \times \\ &\quad \left( 1 + \frac{0,0837}{12} \right)^{\left( \frac{6 \times 12}{12 \times 1} \right)} \left( 1 + 0,0837 \times \frac{20}{365} \right) \\ &= 79\,020,96 \end{aligned}$$

**Sam will have R79 020,96 available.**

## Question 10

**Determine the amount that Sam in Question 5 has available if fractional compounding is used for the full term.**



## **ID: Fractional compounding**

**Use compounding formula but the time period is calculated as the full period plus the odd period expressed as a fraction of the compounding period.**

$$\begin{aligned} S &= P \left( 1 + \frac{j_m}{m} \right)^{tm} \\ &= 75\,000 \left( 1 + \frac{0,0837}{12} \right)^{\left( \frac{26+20}{365} + \frac{6}{12} \right) \times \frac{12}{1}} \\ &= 79\,020,29 \end{aligned}$$

**Sam will have R79 020,29 available.**

# **EFFECTIVE RATE**

- **Effective interest rate is the rate that you in effect will receive in one year.**
- **You actually earn more as the specified compound interest rate because of the compounding effect.**

**For example:**

**If you invest R100 for 3 months at 12% per year, compounded monthly:**

$$\begin{aligned} S &= P \left( 1 + \frac{j_m}{m} \right)^{tm} \\ &= 100 \left( 1 + \frac{,12}{12} \right)^{\frac{3}{12} \times \frac{12}{1}} \\ &= 103,03 \end{aligned}$$

**Thus the interest earned is  
R100 – R103,03 = R3,03**

**Now if you calculate the interest  
rate earning of R3,03 it is**

$$I = Prt$$

$$3,03 = 100 \times r \times \frac{3}{12}$$

$$r = 12,12\%$$

**Thus although the rate is 12% you earn 12,12% effectively.**

**Formula:**

$$J_{eff} = 100 \left( \left( 1 + \frac{j_m}{m} \right)^m - 1 \right)$$

**ID: effective interest rate**

## Question 11

**Determine the effective rate for a nominal rate of 18,75% per year, compounded every three months.**



**ID: effective interest rate**

$$\begin{aligned} J_{eff} &= 100 \left( \left( 1 + \frac{j_m}{m} \right)^m - 1 \right) \\ &= 100 \left( \left( 1 + \frac{0,1875}{4} \right)^4 - 1 \right) \\ &= 20,11\% \end{aligned}$$

**The effective rate is 20,11%.**



# CONTINUOUS COMPOUNDING

Compounding periods almost an infinite number of times

$$S = Pe^{ct}$$

$c, t$  - yearly if not change!

**ID: continuous compounding or continuous rate**

**Note:** The mathematical constant  $e$  is a unique real number. The number  $e$  is of considerable importance in mathematics. The numerical value of  $e$  truncated to 20 decimal places is 2,71828182845904523536. Some calculators have a specific key (for example  $e^x$ ) to calculate the value of the power of  $e$ . You should be able to call up the specified value of  $e$  by entering  $e^1$  into your calculator.

## Question 12

**R2 800 accumulates to R3 798 in 42 months, determine the continuous compounding rate.**



**ID: continuous compounding**

$$S = Pe^{ct}$$

$$3798 = 2800e^{c \times \frac{42}{12}}$$

**We need to solve c. Need to have it on its own on one side.**

$$\frac{3798}{2800} = e^{c \times \frac{42}{12}}$$

**But  $c$  in the power – use  $\ln$  rule to bring it “down” we take  $\ln$  on both sides of the equation:**

$$\ln\left(\frac{3798}{2800}\right) = \ln e^{c \times \frac{42}{12}}$$

*Now  $\ln a^x = x \ln a$  is a  $\ln$  rule. Thus*

$$\ln\left(\frac{3798}{2800}\right) = \left(c \times \frac{42}{12}\right) \ln e$$

*But  $\ln e = 1$*

$$\ln\left(\frac{3798}{2800}\right) \div \frac{42}{12} = c$$

$$c = 8,7\%$$

## Conversions:

- Between nominal rate and continuous compounding rate  $c$ :

$$c = m \ln \left( 1 + \frac{j_m}{m} \right)$$

### Question 13

Determine the continuous compounding rate for a 12% nominal rate per year, compounded every three months.



$$\begin{aligned} c &= m \ln \left( 1 + \frac{j_m}{m} \right) \\ &= 4 \ln \left( 1 + \frac{0,12}{4} \right) \\ &= 11,824\% \end{aligned}$$

- **Continuous compounding and effective rate**

$$J_{\infty} = 100(e^c - 1)$$

## Question 14

**Determine the effective rate for a 9% continuous compounding rate.**



$$\begin{aligned} J_{\infty} &= 100(e^c - 1) \\ &= 100(e^{0,09} - 1) \\ &= 9,42\% \end{aligned}$$

- **Ordinary compounding but compounding periods differ**

**Convert a given compounding period to another compounding period and still get the same return.**

$$i = n \left( \left( 1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right)$$

**Tip: Remember  $n$  stands for new compounding periods and  $m$  the old compounding periods**

## Question 15

**An interest rate of 15,25% per year compounded half-yearly is equivalent to a quarterly compounded interest rate of?**



$$\begin{aligned} i &= n \left( \left( 1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right) \\ &= 4 \left( \left( 1 + \frac{0,1525}{2} \right)^{2 \div 4} - 1 \right) \\ &= 14,97\% \end{aligned}$$

# TIME VALUE OF MONEY

- **The same amount of money has a different value at different time periods because of interest.**
- **Moving money forward in time or calculating a FV value => we × by interest component. Money becomes more.**

$$S = P \left(1 + \frac{j_m}{m}\right)^{tm}$$

- **Moving money back in time or calculating a PV value => we ÷ interest component or × by interest component to the power of -1 .**

$$P = \frac{S}{\left(1 + \frac{j_m}{m}\right)^{tm}} \quad \text{or} \quad P = S \left(1 + \frac{j_m}{m}\right)^{-tm}$$



## Question 16

**R35 000 must be paid back in three years time from *now*.**

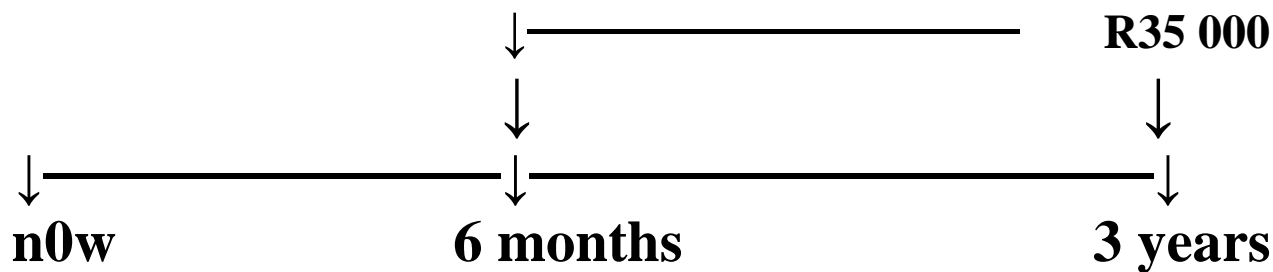
**Interest is calculated at 8,4% per year, compounded half yearly.**

**Calculate the amount that must be paid if the debt is paid**

**(a) six months from *now*.**

**(b) five years from *now*.**





$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$

$$S = 35\ 000$$

$$j_m = 0,084$$

$$m = 2$$

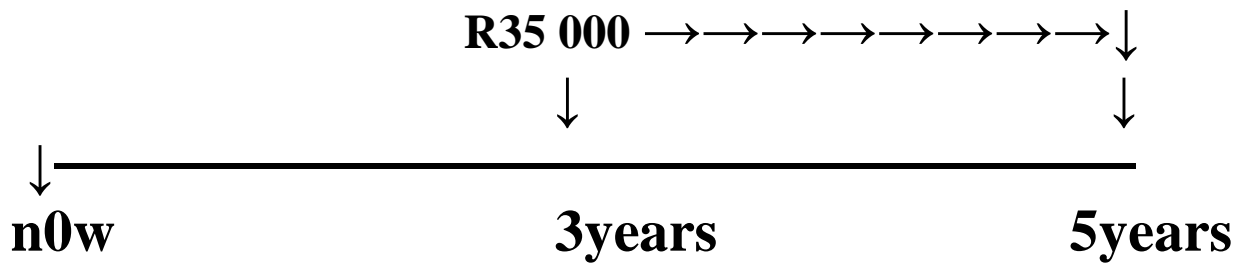
$$t = 2,5$$

$$P = S \left( 1 + \frac{j_m}{m} \right)^{-tm}$$

$$= 35\ 000 \left( 1 + \frac{0,084}{2} \right)^{-2,5 \times 2}$$

$$= 28\ 492,43$$

The amount that must be paid back six months from now is R28 492,43.



$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$

$$P = 35\ 000$$

$$j_m = 0,084$$

$$m = 2$$

$$t = 2$$

$$S = 35\ 000 \left( 1 + \frac{0,084}{2} \right)^{2 \times 2}$$

$$= 41\ 260,92$$

The amount that must be paid back five years from now is R41 260,92.

# EQUATION OF VALUE

Sometimes we want to reschedule the payments of our debts.

To calculate the new payments the  
Sum of obligations = Sum of payments.

**We can only compare money that is at the same time period** because of the time value of money => move all the moneys to the same date before we can add or subtract.

Moving money forward  $\times$  by interest component – calculating a FV value

Moving money back  $\div$  interest component or  $\times$  by interest component to the power of  $-1$  calculating a PV value

Use a time line

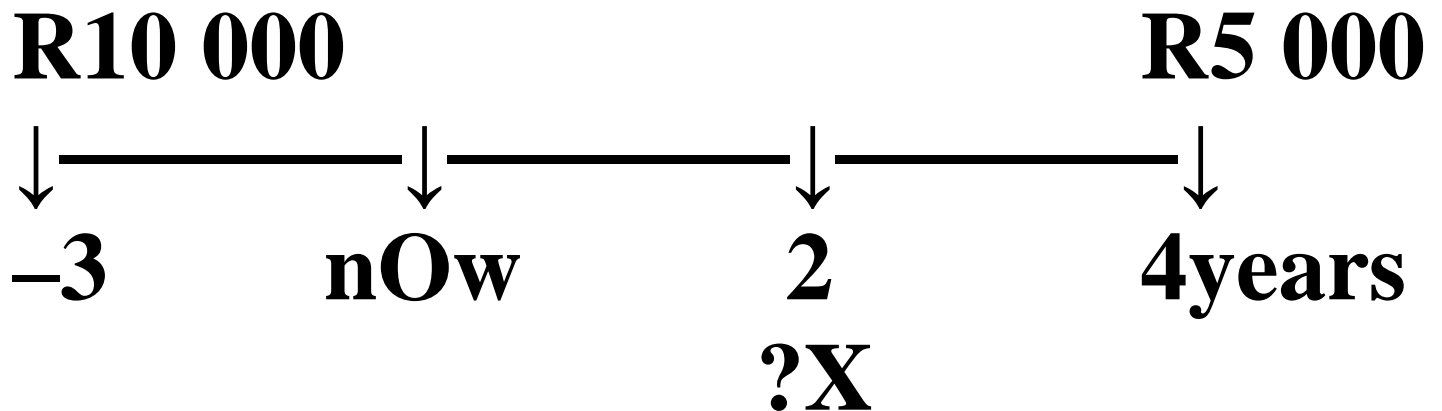
**ID: compound interest and more than one payment or debts but not equal size and not equal time periods**

## Question 17

**Three years ago Siphso borrowed R10 000 from Judith that is due two years from *now*. He must also pay her R5 000 four years from *now*. Siphso decides to settle his obligations with one payment two years from *now*. Determine the amount that Siphso will pay Judith two years from *now* if an interest rate of 9,5% per year, compounded quarterly is applicable.**



# ID: rescheduling + comp i



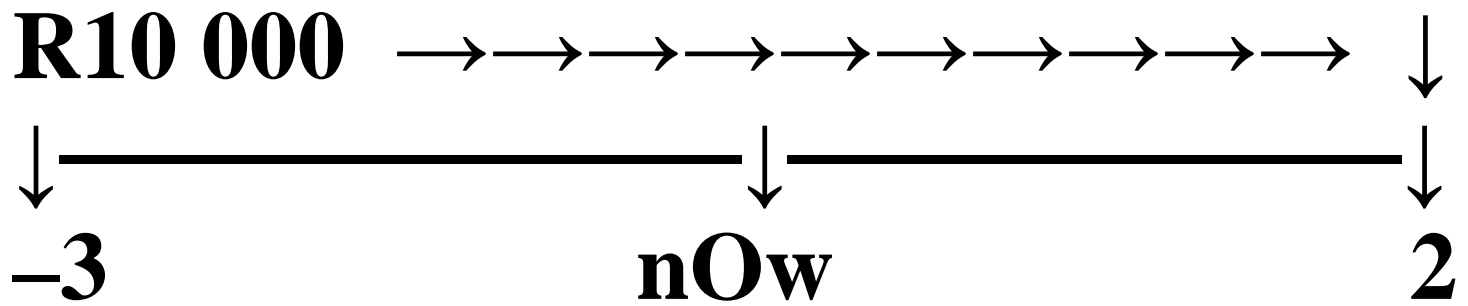
**What we owe we must pay back.**

**Thus**

**Sum of obligations = Sum of payments.**

**But we can only add moneys if they at the same time period because of time value of money. Thus we move all the money to year2.**

# Obligation 1: R10 000



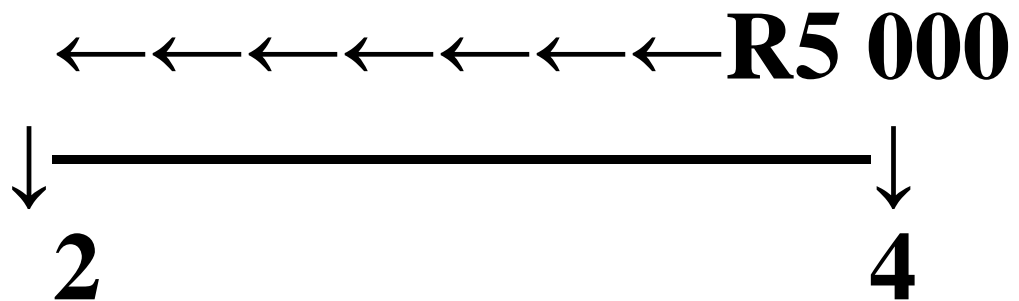
We must move the R10 000 forward to year two – (five years)

$$S = P(1 + i)^n$$

$$= 10\,000 \left( 1 + \frac{0,095}{4} \right)^{5 \times 4}$$

$$= 15\,991,10$$

## Obligation 2: R5 000



We must move the R5 000 that is due four years from *now*, back to year two.

$$S = P(1+i)^n$$

$$P = S(1+i)^{-n}$$

$$= 5\,000 \left( 1 + \frac{0,095}{4} \right)^{-2 \times 4}$$

$$= 4\,143,99$$



**Now the payment is say  $X$ . Thus**

**Sum of obligations = Sum of payments**

$$15\,991,10 + 4\,143,99 = X$$

**Sipho will pay Judith R20 135,09.**

## **Question 18**

**If  $S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$  write  $j_m$  in terms of  $P, S, t$  and  $m$ .**



# ID: Maths question

$$S = P \left( 1 + \frac{j_m}{m} \right)^{tm}$$

$$\left( \frac{S}{P} \right) = \left( 1 + \frac{j_m}{m} \right)^{tm}$$

$$\left( \frac{S}{P} \right)^{\frac{1}{tm}} = \left( 1 + \frac{j_m}{m} \right)$$

$$\frac{j_m}{m} = \left( \frac{S}{P} \right)^{\frac{1}{tm}} - 1$$

$$j_m = m \left( \left( \frac{S}{P} \right)^{\frac{1}{tm}} - 1 \right)$$

# Chapter 4

- **Annuities**
- **Ordinary Annuities**
- **Annuities due**

# ANNUITIES

- **More than one principal called payments/deposits/withdrawals**
- **Principals are of equal size and made in equal time intervals.**
- **Payments can be made in the beginning of the time interval or at the end or can be deferred for a period.**
- **Payment period = comp interest periods  
If not we convert the interest period**

## **ID:**

- 1. Equal size payments/deposit/withdrawal made in equal time intervals plus**
- 2. Compound interest.**
- 3. Comp i periods = payment periods**

# 1. Ordinary annuity

Payments made at **end of time period** or are not specified

**ID: Equal payments in equal time intervals plus compound interest plus payments made at end of each time period.**

- **Present value:**

$$P = Ra_{\overline{n}|i}$$
$$= R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\text{with } i = \frac{j_m}{m} \text{ and } n = tm$$

Remember  $a_{\overline{n}|i}$  is just a place holder for a formula!!!!!!!

**Calculator use fin mode. Ordinary mode takes long**

## Question 19

Determine the present value of equal payments of R1 500 per month for nine years at 7,5% per year compounded monthly.



**ID:**

- Equal payments in equal time intervals  
+ comp  $i$
- Nothing said = Payment end of period
- Comp  $i$  periods = payment periods  
=> Ordinary annuity

$$P = Ra_{\overline{n}|i}$$

$$\text{with } R = 1\ 500$$

$$n = 9 \times 12$$

$$i = 0,075 \div 12$$

$$P = 1\ 500a_{\overline{9 \times 12}|0,075 \div 12}$$

## SHARP:

2ndf CA

2ndf P/Y 12 ENT ON/CL

9 × 12 = N or 9 2ndf × P/Y N

± 1500 PMT

7.5 I/Y

Comp PV

## HP:

ORANGE C ALL

12 ORANGE P/YR

9 × 12 = N or 9 ORANGE × P/YR N

1500 ± PMT

7.5 I/YR

PV

**The present value is R117 545,50.**

- **Future value**

$$S = Rs_{\overline{n}|i}$$
$$= R \left[ \frac{(1+i)^n - 1}{i} \right]$$

*with  $i = \frac{j_m}{m}$  and  $n = tm$*

**Remember  $s_{\overline{n}|i}$  is just a place holder for a formula!!!!!!!!!!**



## Question 20

Determine the accumulated amount of equal monthly payments of R1 500 for nine years at an interest rate of 7,5% per year, compounded monthly.



**ID:**

- Equal payments in equal time intervals + comp  $i$
- Nothing said = Payment end of period
- Comp  $i$  periods = payment periods  
=> Ordinary annuity

$$S = Rs_{\overline{n}|i}$$

with  $R = 1\,500$ ;  $i = 0,075 \div 12$ ;  $n = 9 \times 12$

$$\begin{aligned} S &= 1\,500s_{\overline{9 \times 12}|0,075 \div 12} \\ &= 230\,378,79 \end{aligned}$$

**The future value is R230 378,79.**

## Question 21 and 22

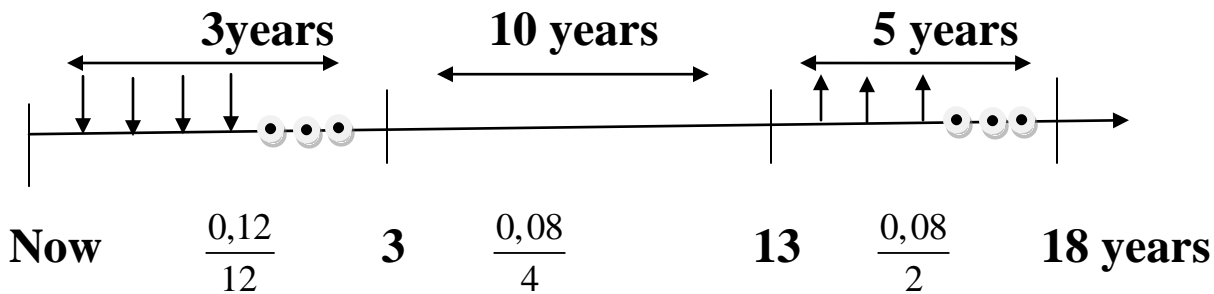
During a three-year period when her business was prospering, Patricia was able to deposit R3 000 at the end of every month into an account earning 12% interest per year, compounded monthly. At the end of the three year period Patricia decided to stop her payments into this account as the interest was lowered to 8% per year compounded quarterly. She left the money in the account for ten years. She then decided to exhaust this account by withdrawing equal amounts every six months for five years. The interest rate was still 8% per year but was now compounded semi-annually.

## Question 21

Determine the total amount of savings after the ten-year period.



## First draw a time line:



The question can be divided into 3 parts

Part 1:

ID:

- Equal payments in equal time intervals + comp  $i$
- Nothing said = Payment end of period
- Comp  $i$  periods = payment periods  
=> Ordinary annuity

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= 3000s_{\overline{3 \times 12}|0,12 \div 12} \\ &= 129\,230,64 \end{aligned}$$

## Part 2:

**This amount was left in the account for 10 years without adding any money. It however received compound interest of 8% per year, compounded quarterly.**

$$\begin{aligned} S &= P \left( 1 + \frac{j_m}{m} \right)^{tm} \\ &= 129\,230,64 \left( 1 + \frac{0,08}{4} \right)^{10 \times 4} \\ &= 285\,346,37 \end{aligned}$$

**The balance in the account is R285 346,37 after 13 years.**

## Question 22

Determine the amount that Patricia can withdraw every six months.



**Part 3:**

This R285 346,37 will be exhausted over a five year period in equal payments.

Do I use ani or sni?

Is R285 346,37 a PV or FV?

PV for the withdrawal – use ani formula

$$P = Ra_{\overline{n}|i}$$

$$285\,346,37 = Ra_{\overline{5 \times 2}|0,08 \div 2}$$

**NB<sup>10000</sup>: You do not have to change the formula if you need to calculate any other value except P and S. If you use the financial mode of your calculator you just enter the values of the formula that are known and then just compute the value you do not have**

**Using your SHARP calculator's financial mode:**

**P/Y=2; I/Y=8; N= 5x2; PV= ± 285 346,37**

**Comp PMT**

$$**R = 35180,62**$$

**Or HP:**

**P/YR=2; I/YR=8; N= 5x2; PV= ± 285 346,37**

**PMT**

**Patricia will be able to withdraw R35 180,62 every six months for five years.**

## Question 23

**Determine the accumulated value (approximated) of R500 payments made at the end of each month for a period of eight years. Interest is compounded semi-annually at 13,5% per year.**



**ID:**

- **Equal payments in equal time intervals  
+ comp i**
- **Payment end of period  
=>Ordinary annuity**
- **But the interest rate is semi-annually  
and payments periods monthly!**

**We must first convert the interest rate from semi-annually to monthly.**

$$\begin{aligned}
i &= n \left( \left( 1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right) \\
&= 12 \left( \left( 1 + \frac{0,135}{2} \right)^{2 \div 12} - 1 \right) \\
&= 0,13135\dots
\end{aligned}$$

**This is the new interest rate that the R500 payments will receive.**

$$\begin{aligned}
S &= Rs_{\overline{n}|i} \\
&= 500s_{\overline{8 \times 12}|0,13135\dots \div 12} \\
&= 84\,218,28
\end{aligned}$$

**The accumulated amount is approximately R84 000.**



## 2. Annuity due

- Payments made at beginning of period
- When the words “**begin, start immediately or in advance**” are used together with the payments in the sentence.
- Multiply the formulae with  $(1 + i)$

$$S = (1 + i) Rs_{\overline{n}|i}$$

$$P = (1 + i) Ra_{\overline{n}|i}$$

**ID: Equal payments in equal time intervals plus compound interest plus payments made at beginning of each time period. Comp i periods must be equal to payment periods.**

**Calculator use fin mode and Begin key**

## Question 24

Determine the present value of four equal payments of R2 000 per year at 8% per year where the payments are made at the *beginning* of each year.



**ID:**

- Equal payments in equal time intervals + comp  $i$
- Payment **beginning** of period
- interest rate period = payment period  $\Rightarrow$  annuity due

<b>R2 000</b>	<b>R2000</b>	<b>R2 000</b>	<b>R2 000</b>
↓	↓	↓	↓
<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>

$$P = (1 + i) 2000 a_{\overline{4}|0,08}$$
$$= 7\,154,19$$

**Calculator use fin mode and Begin mode key**  
**By using the Begin mode key the calculator**  
**know it is an annuity due calculation.**

**The present value is R7 154,19.**

**Remember to switch off the**  
**BEGIN key!**

**Thank you!**