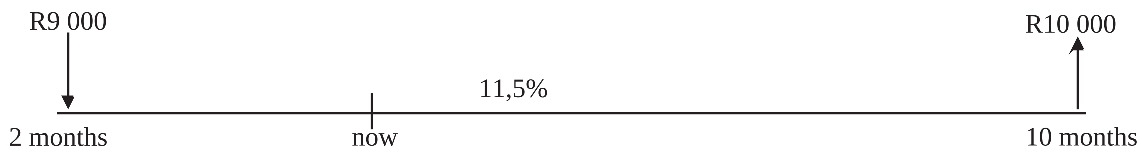
BASIC SOLUTIONS FOR DSC1630 EXAMINATION QUESTION PAPER OCT/NOV 2012

1. Identification: Simple interest rate



Savings worth at month 10 – move R9 000 from 2 month's ago to 10 month's in future:

$$\begin{aligned} S &= P(1+rt) \\ &= 9\,000\left(1+0{,}115\times\frac{12}{12}\right) \\ &= 9\,000(1{,}115\ldots) \\ &= \text{R}10\,035{,}00. \end{aligned}$$

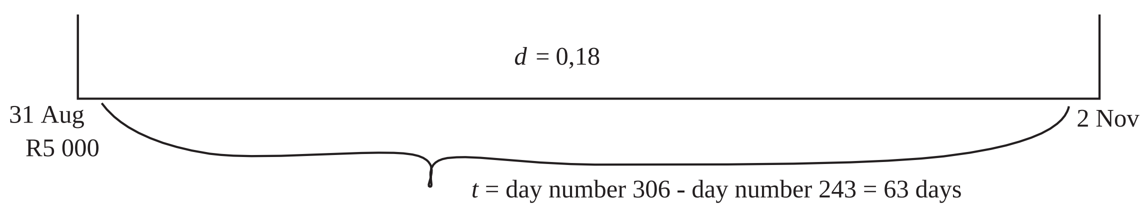
Money Short:

R10 035,00 − R10 500 = −465,00

She shorts R465,00.

Option [3]

2. Identification: Simple discount rate



$$\begin{aligned} P &= S(1-dt) \\ 5\,000 &= S\left(1-0{,}18\times\frac{63}{365}\right) \\ \frac{5\,000}{\left(1-0{,}18\times\frac{63}{365}\right)} &= S \\ S &= \text{R}5\,160{,}32. \end{aligned}$$

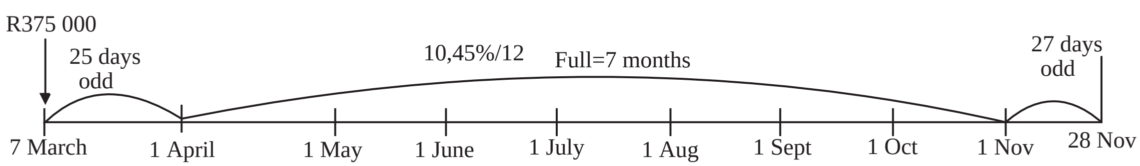
Option [2]

3. Identification: Compound interest

$$\begin{aligned}
j_m &= 12\% \\
m &= 12 \\
P &= P \\
S &= 2P \\
t &= ? \\
S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\
2P &= P(1+\frac{0{,}12}{12})^{t12} \\
\frac{2P}{P} &= \left(1+\frac{0{,}12}{12}\right)^{12t} \\
\ln\,2 &= 12t\ln\left(1+\frac{0{,}12}{12}\right) \\
\frac{\ln\,2}{\ln\left(1+\frac{0{,}12}{12}\right)} &= 12t \\
t &= \frac{\left(\frac{\ln\,2}{\ln(1+\frac{0{,}12}{12})}\right)}{12} \\
t &= 5{,}80506 \text{ years} \\
t &\approx 5{,}81.
\end{aligned}$$

Option [1]

4. Identification: Odd periodes – method given



$$\begin{aligned}
S &= P(1+rt)(1+r)^t(1+rt) \\
&= 375\,000\left(1+0{,}1045\times\frac{25}{365}\right)\left(1+\frac{0{,}1045}{12}\right)^{\frac{7}{12}\times\frac{12}{1}}\left(1+\frac{27}{365}\times 0{,}1045\right) \\
&= 404\,419{,}5870\ldots \\
&\approx \text{R}404\,419{,}59.
\end{aligned}$$

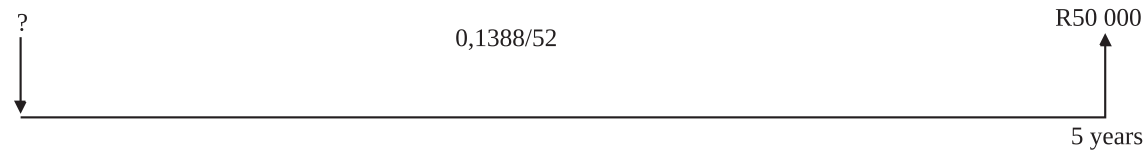
Option [4]

5. Identification: Fractional compounding

$$
\begin{aligned}
S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\
&= 375\,000\left(1+\frac{0{,}1045}{12}\right)^{\left(\frac{7}{12}+\frac{25}{365}+\frac{27}{365}\right)\times\frac{12}{1}} \\
&= \text{R}404\,415{,}85.
\end{aligned}
$$

Option [4]

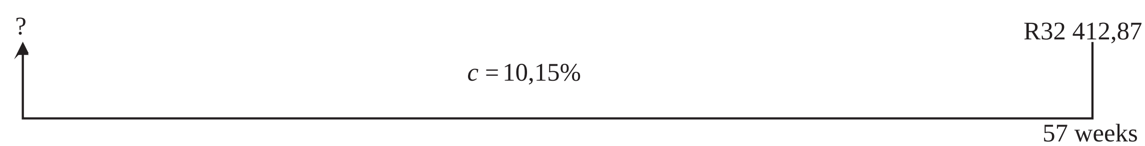
6. Identification: Compound interest



$$
\begin{aligned}
S &= P(1+j_m/m)^{tm} \\
50\,000 &= P\left(1+\frac{0{,}1388}{52}\right)^{5\times 52} \\
P &= \text{R}25\,001{,}79.
\end{aligned}
$$

Option [2]

7. Identification: Continuous compounding rate



$$
\begin{aligned}
S &= Pe^{ct} \\
P &= S/e^{ct} \\
&= \frac{32\,412{,}87}{e^{(0{,}1015\times\frac{57}{52})}} \\
&= \text{R}29\,000{,}00.
\end{aligned}
$$

Option [1]

8. Identification: Equivalent compound interest rate

$$
\begin{aligned}
j_n &= n\left(\left(1+\frac{j_m}{m}\right)^{\frac{m}{n}}-1\right)\\
j_{52} &= 52\left(\left(1+\frac{0{,}149}{4}\right)^{\frac{4}{52}}-1\right)\\
&= 0{,}14650\\
&\approx 14{,}65\%.
\end{aligned}
$$

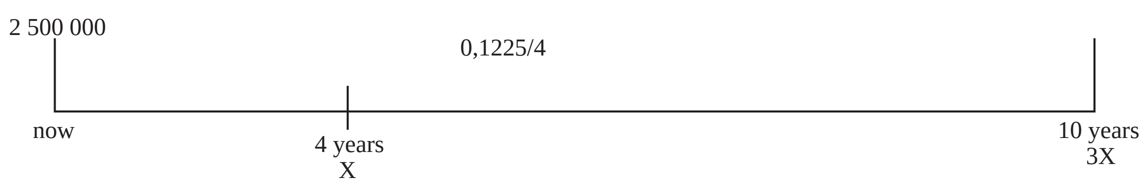
Option [1]

9. Identification: Effective interest rate

$$
\begin{aligned}
j_{eff} &= 100\left(\left(1+\frac{j_m}{m}\right)^{m}-1\right)\\
&= 100\left(\left(1+\frac{0{,}165}{6}\right)^{6}-1\right)\\
&= 17{,}67684\\
&\approx 17{,}677\%.
\end{aligned}
$$

Option [4]

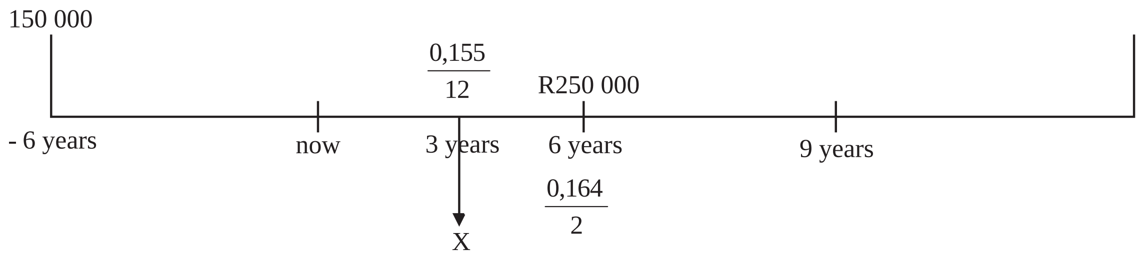
10. Identification: Re-scheduling of debt – equation of value.



$$
\begin{aligned}
2\,500\,000\left(1+\frac{0{,}1225}{4}\right)^{10\times 4} &= X\left(1+\frac{0{,}1225}{4}\right)^{6\times 4}+3X\\
2\,500\,000\left(1+\frac{0{,}1225}{4}\right)^{10\times 4} &= X\left[\left(1+\frac{0{,}1225}{4}\right)^{24}+3\right]\\
2\,500\,000\left(1+\frac{0{,}1225}{4}\right)^{10\times 4} &= 5{,}06261X\\
2\,500\,000\left(1+\frac{0{,}1225}{4}\right)\Big/ 5{,}06261 &= X\\
X &= \text{R}1\,650\,412{,}32\\
3X &= \text{R}4\,951\,236{,}95.
\end{aligned}
$$

Option [4]

11. Identification: Re-scheduling of debt – time value of money.



$$\begin{aligned} X &= 150\,000\left(1+\frac{0{,}155}{12}\right)^{9\times 12} + 250\,000\left(1+\frac{0{,}164}{2}\right)^{-(2\times 3)} \\ &= 599\,863{,}8759 + 155\,803{,}23 \\ &= \text{R}755\,667{,}10. \end{aligned}$$

Option [4]

12. Identification: Payments paid indefinitely – Perpetuity

$$\begin{aligned} PMT &= 3\,500 \quad i = 0{,}112/12 \\ P &= R/i \\ &= 3\,500 \Big/ (0{,}112/12) \\ &= \text{R}375\,000. \end{aligned}$$

Option [4]

13. Identification: Equal payments in equal time intervals plus compound interest rate – annuity but time intervals of payments not equal to compounding periods thus change compound interest rate from quarterly to monthly.

$$
\begin{aligned}
j_n &= n\left(\left(1+\frac{j_m}{m}\right)^{\frac{m}{n}}-1\right)\\
&= 12\left(\left(1+\frac{0{,}0775}{4}\right)^{\frac{4}{12}}-1\right)\\
&= 0{,}07700.
\end{aligned}
$$

Thus

$$
\begin{aligned}
S &= Rs_{\overline{n}|i}\\
&= 1\,200s_{\overline{10\times 12}|\frac{0{,}077}{12}}\\
&= \text{R}215\,899{,}01.
\end{aligned}
$$

Option [2]

14. Identification: Maths manipulation of equation.

$$
\begin{aligned}
S &= Pe^{ct}\\
S \div P &= e^{ct}\\
\ln(S \div P) &= \ln e^{ct}\\
\ln(S \div P) &= ct\ln e\\
\frac{\ln(S \div P)}{t} &= c
\end{aligned}
$$

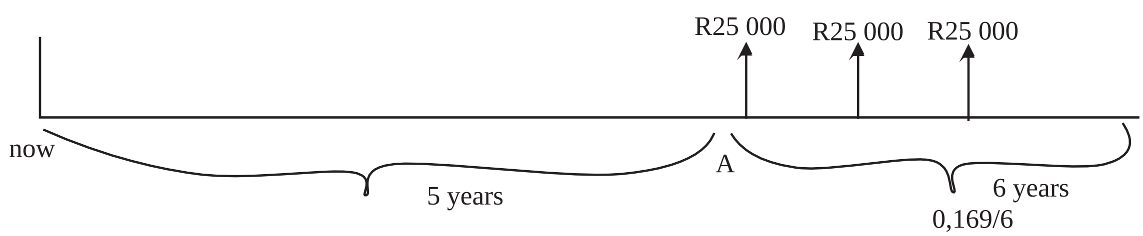
Option [5]

15. Identification: Payments that are made on equal time periods but payments increase each time period with a constant amount – increasing annuity.

$$
\begin{aligned}
S &= \left(R+\frac{Q}{i}\right)s_{\overline{n}|i}-\frac{nQ}{i}\\
&= \left(3\,600+\frac{360}{0{,}10}\right)s_{\overline{20}|0{,}10}-\frac{20(360)}{0{,}1}\\
&= \text{R}340\,379{,}99\\
&\approx \text{R}340\,380.
\end{aligned}
$$

Option [2]

16. Identification: Payment being postponed – deffered annuity.



$$
\begin{aligned}
A &= Ra_{\overline{n}\rceil i} \\
&= 25\,000a_{\overline{6\times 6}\rceil 0{,}169/6} \\
&= \text{R}561\,047{,}91.
\end{aligned}
$$

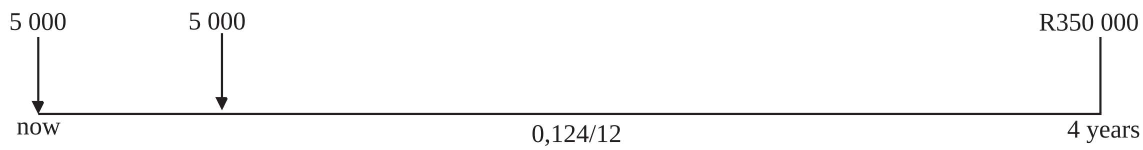
Option [4]

17. Identification: Moving money back in time – time value of money and compound interest.

$$
\begin{aligned}
P &= S/(1+j_m/m)^{tm} \\
&= \frac{\text{R}561\,047{,}91}{(1+\frac{0{,}169}{6})^{5\times 6}} \\
&= \text{R}243\,834{,}05.
\end{aligned}
$$

Option [4]

18. Identification: Equal amount's deposited in equal time periods + payments made immediately − annuity due.



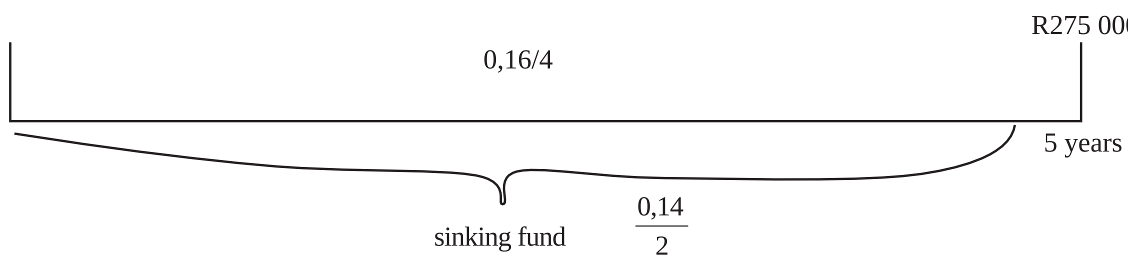
$$
\begin{aligned}
S &= (1+i)Rs_{\overline{n}\rceil i} \\
&= (1+\frac{0{,}124}{12})5\,000s_{\overline{4\times 12}\rceil \frac{0{,}124}{12}} \\
&= \text{R}311\,882{,}75.
\end{aligned}
$$

Amount needed still:

$$
\begin{aligned}
&= 350\,000 - 311\,882{,}75 \\
&= \text{R}38\,117{,}25.
\end{aligned}
$$

Option [1]

19. Identification: Sinking fund.



$$
\begin{aligned}
S &= Rs_{\overline{n}|i} \\
275\,000 &= Rs_{\overline{5\times 2}|\frac{0,14}{2}} \\
R &= \text{R}19\,903{,}81 \\
R &\approx \text{R}19\,904.
\end{aligned}
$$

Option [3]

20. Identification: Amortisation schedule.

$$
\begin{aligned}
A &= \text{payment} \\
\text{Interest} + \text{principal repaid} &= 3081{,}86 + 1119{,}21 \\
&= \text{R}4\,201{,}07.
\end{aligned}
$$

Option [5]

21. Identification: Internal rate of return.

    Using your calculator:

$$
\begin{aligned}
IRR &= 15{,}23893\% \\
&\approx 15{,}24\%.
\end{aligned}
$$

Option [5]

22. Identification: Equal payments in equal time intervals – annuity or armotisation

$$
\begin{aligned}
A &= Ra_{\overline{n}|i} \\
&= 5\,311{,}69a_{\overline{20\times 12}|\frac{0,0975}{12}} \\
&= 559\,999{,}54 \\
&\approx \text{R}560\,000.
\end{aligned}
$$

Option [3]

23. Identification: Percentage calculation.

$$
\begin{aligned}
560\,000 &= 80\% \\
? &= 100\% \\
\frac{560\,000}{1} \times \frac{100}{80} &= \text{R}700\,000.
\end{aligned}
$$

Option [4]

24. Identification: Total real cost

$$\text{Total real cost} = PV \text{ of annuity using inflation rate} - PV \text{ of annuity}$$
$$PV = 5\,311{,}69 a_{\overline{20\times 12}|\frac{0{,}0467}{12}}$$
$$= 827\,543{,}12.$$

$$\text{Real cost} = 827\,543{,}12 - 560\,000$$
$$= \text{R}267\,543{,}13.$$

Option [2]

25. Identification: Correlation coefficient.
Using your calculator:

$$r = -0{,}98185.$$

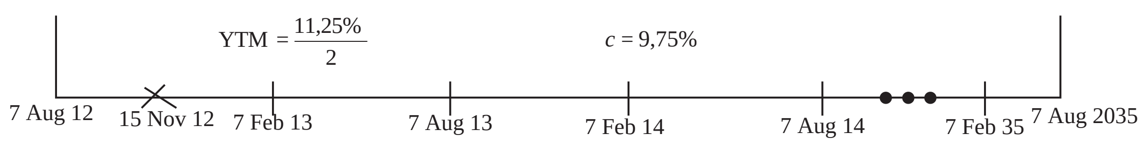
Option [2]

26. Identification: Slope of regression line.
Using your calculator:

$$\text{intercept } a = 1\,021{,}13$$
$$\text{slope } = b = -207{,}59$$

Option [1]

27. Identification: Bond



$$n = 22{,}5 \text{ years}$$
$$= 22 \times 2 + 1$$
$$= 44 + 1 = 45$$

$$\begin{aligned} P &= da_{\overline{n}|z} + 100(1+z)^{-n} \\ &= \frac{9{,}75}{2} a_{\overline{45}|0{,}1125/2} + 100\left(1+\frac{0{,}1125}{2}\right)^{-45} \\ &= 87{,}80282 \end{aligned}$$

Add coupon as number of days > 10 days between settlement date and next coupon date.

$$\begin{aligned} &87{,}80282 + 4{,}8785 \\ &= 92{,}67782\%. \end{aligned}$$

Move value to 15 Nov' 12: $R =$ 15 Nov' 12 −7 Feb' 13= 84
$H =$ 7 Aug' 12 −7 Feb' 13= 184

Thus all-in-price is:

$$\begin{aligned} &92{,}67782\left(1+\frac{0{,}1125}{2}\right)^{\frac{-84}{184}} \\ &= 90{,}39112\%. \end{aligned}$$

Option [4]

28. Identification: Bond

$$\begin{aligned} 107{,}55174 &= da_{\overline{29}|0{,}135} + 100\left(1+\frac{0{,}135}{2}\right)^{-2} \\ 107{,}55174 - 100\left(1+\frac{0{,}135}{2}\right)^{-29} &= da_{\overline{29}|0{,}135} \\ 92{,}50885 &= da_{\overline{29}|0{,}135} \end{aligned}$$

This looks like an annuity formule. thus use your calculator's financial mode with $92{,}50885 = PV$; $N = 29$; $P/Y = 2$; $I/Y = 13{,}5$ and solve for your payment which is d. $c/2 = d = 7{,}35\%$.

Option [2]

29. Identification: Profitability index Typing error in answers – Question ignored in October/November 2012 exams.

$$\begin{aligned} PI &= \frac{NPV + \text{initial}}{\text{initial}} \\ 1{,}0514 &= \frac{25\,700 + x}{x} \\ 1{,}0514x &= 25\,700 + x \\ 0{,}0514x &= 25\,700 \\ x &= 500\,000. \end{aligned}$$

30. Identification: NPV; PI and IRR

Investment A	**Investment B**
$NPV < 0$ reject	$NPV > 0$ accept
$PI < 1$ reject	$PI > 1$ accept
$IRR < K$ reject	$IRR > K$ accept

Accept Invest B

Option [2]