Tutorial Letter 201/1/2015

Introductory Financial Mathematics
DSC1630

Semester 1

Department of Decision Sciences

Important Information:

This tutorial letter contains the solutions of Assignment 01.

Bar code

UNISA university of south africa

Learn without limits.

Dear student

The solutions to the questions of the compulsory Assignment 01 are included in this tutorial letter. Study them when you prepare for the examination. I have added comments on how to approach each problem as well as a section on when to use which formula plus a list of formulas similar to the one you will receive in the exam.

You will also find the key operations for the SHARP EL-738 and the HP10BII calculators as well as newer versions at the end of each answer. I hope it will help you to understand the workings of your calculator. **Just remember that these key operations are not the only method, these are just examples**. You can use any method as long as you get the same answer.

REMEMBER TO CLEAR ALL DATA FROM THE MEMORY BEFORE YOU ATTEMPT ANY CALCULATIONS.

- **SHARP EL-738 users**: Press 2ndF M-CLR 0 0 and 2ndF CA if you just want to clear the financial keys.
- **HP10BII users**: Press C ALL

Remember to please contact me via email, fax, telephone or appointment if you need help regarding the study material. Please note that only students with appointments will be assisted. My contact details and contact hours are:

Office: Hazelwood Campus, Room 4-28 **Tel: +27 12 4334691**
E-mail: immelmf@unisa.ac.za

08:00 until 13:30 - Monday till Friday: **Appointments and Telephone**
13:30 until 16:00 - Monday till Thursday: **Telephone only**

Please remember that there are also E-tutors available on myUnisa to help you with the study material and problems you might encounter.

Lastly everything of the best with Assignment 02. Please work through the self-evaluation exercises in Tutorial letter 101 **before** you attempt to answer the assignments.

Kind regards,

Mrs Adéle Immelman

1 Solution Summary

The following is a summary of the correct answers:

Q 1	Option 1	Q 9	Option 4
Q 2	Option 1	Q 10	Option 4
Q 3	Option 4	Q 11	Option 3
Q 4	Option 4	Q 12	Option 1
Q 5	Option 3	Q 13	Option 4
Q 6	Option 2	Q 14	Option 1
Q 7	Option 3	Q 15	Option 2
Q 8	Option 1		

2 How to approach a question

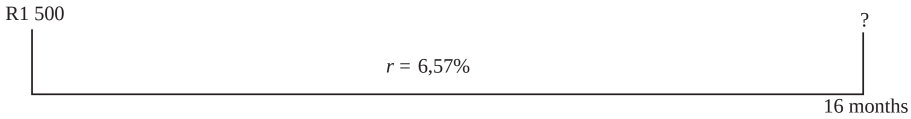
Below are a few hints to consider when approaching a problem.

1. Read through the question.
2. Draw a time line of the situation, where applicable.
3. Try to identify the type of problem asked. Identifying words, for example, the type of interest rate, or payment methods used, will help you to decide which formula to use.
4. Write down the formula.
5. Go back to the question and identify the given values and substitute them into the formula.
6. Manipulate the formula if necessary.
7. Use your calculator to solve the unknown.

3 Assignment 01 – Detailed Solution

1. First we identify the problem. By identifying the type of interest used it gives way to the type of formula used. This is a simple interest rate calculation as the term *simple interest* is found in the question. The formulas for simple interest calculations are $S = P(1 + rt)$ and $I = Prt$.

   The time line of the problem is:



   Given are the value now or present value (P) of R1 500; the simple interest rate of 6,57% and the time period of 16 months. The time period t must always be expressed as years. Thus we need to change the 16 months to a fraction of a year by dividing the 16 months by the number of months in a year, namely 12. Thus $t = \frac{16}{12}$. Now we need to determine the amount after 16 months or future value (S). Thus

$$\begin{aligned} S &= P(1+rt) \\ &= 1\,500\left(1+0{,}0657\times\frac{16}{12}\right) \\ &= 1\,631{,}40 \end{aligned}$$

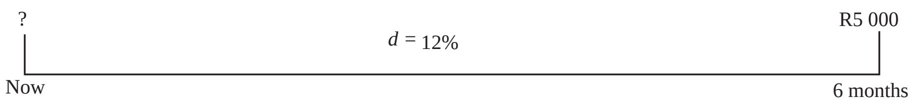
   The balance after 16 months is R1 631,40.

EL-738 and EL-738F	**HP10BII and HP10BII+**
2ndF CA *Use normal keys* *Enter as* $1\,500 \times (1 + 0.0657 \times 16 \div 12) =$ 1 631.40 *to two decimal places is displayed.*	C ALL *Use normal keys* *Enter as* $(1+$ $(0.0657 \times 16 \div 12$ $) =$ $\times 1\,500 =$ 1 631.40 *to two decimal places is displayed.*

[Option 1]

2. This is a simple discount calculation as the term *simple discount rate* is found in the question. The formula for simple discount is $P = S(1 - dt)$.

   The time line is:



Given are the future value of the loan (S) which is R5 000, the time period (t) which is six months from now and the discount rate (d) of 12%.

The time period that we use must always be in years. As the given time is in months we change it to a fraction of a year by dividing the months by the number of months in a year which is 12. Thus $t = \frac{6}{12}$.

Now we need to determine the amount you receive from the bank now or present value of the loan. Thus

$$\begin{aligned} P &= S(1 - dt) \\ &= 5\,000\left(1 - 0{,}12 \times \frac{6}{12}\right) \\ &= 4\,700 \end{aligned}$$

The amount you will receive from the bank now is R4 700.

EL-738 and EL-738F
2ndF CA
Use normal mode
$5\,000 \times (1 - 0.12 \times 6 \div 12) =$
4 700 *is displayed.*

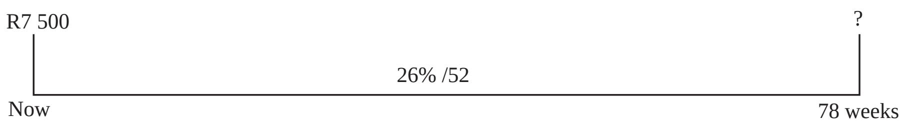
HP10BII and HP10BII+
▭ C ALL
Use normal mode
$0.12 \times 6 \div 12 = \pm + 1 =$
$\times 5\,000 =$
4 700 *to two decimal places is displayed.*
Or alternatively
$5\,000\times$ ▭ (1−
▭ (0.12 × 6 ÷ 12 ▭)
▭)=
4 700 *is displayed.*

[Option 1]

3. This is a compound interest calculation as the term *compounded weekly* is found in the question. The formula for compound interest calculations is:

$$S = P(1 + \frac{j_m}{m})^{tm}$$

First we draw a time line of the problem:



Now given are the principle value P as R7 500, the interest rate as 26%($j_m = 0{,}26$), compounded weekly ($m = 52$) and the time period (t) of 78 weeks. The time period that we use must always be in years. As the given time is in weeks we change it to a fraction of a year by dividing the weeks by the number of weeks in a year which is 52. Thus $t = \frac{78}{52}$. We need to determine the future value or S. Thus

$$
\begin{aligned}
S &= P(1+\frac{j_m}{m})^{tm} \\
S &= 7\,500\left(1+\frac{0{,}26}{52}\right)^{(\frac{78}{52}\times\frac{52}{1})} \\
&= 11\,066{,}59663 \\
&\approx 11\,066{,}60.
\end{aligned}
$$

The amount that Frieda will have to pay back is R11 066,60.

EL-738 and EL-738F

2ndF CA
Use financial mode
2ndF P/Y 52 ENT ON/C
± 7 500 PV
26 I/Y *Remember you do not enter the divide by 52 when entering the interest rates. The calculator does that automatically.*
78 ÷ 52 × 52 = N *or use* 78 ÷ 52 = 2ndF ×P/Y N
COMP FV
11 066.596... *is displayed.*

HP10BII and HP10BII+

▬ C ALL
Use financial mode
52 ▬ P/YR
7 500± PV
26 I/YR *Remember you do not enter the divide by 52 when entering the interest rates. The calculator does that automatically.*
78 ÷ 52 = ×52 =N *or use* 78 ÷ 52
▬ ×P/YR
FV
11 066.596... *is displayed.*

Or alternatively

EL-738 and EL-738F

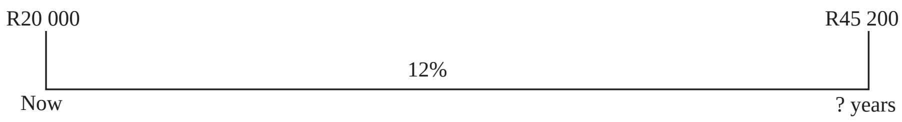
2ndF CA
Use normal mode
7 500(1 + 0.26 ÷ 52)
2ndF y^x *[on the DEPR key]*
(78 ÷ 52 × 52) =
11 066.596... *is displayed.*

HP10BII and HP10BII+

▬ C ALL
Use normal mode
0.26 ÷ 52 = +1 =
▬ y^x *[on the × key]*
▬ (78 ÷ 52 × 52 ▬) =
×7 500 =
11 066.596... *is displayed.*

[Option 4]

4. This is a simple interest rate calculation as the term *simple interest* is found in the question. The formula for a simple interest calculation is $S = P(1 + rt)$ or $I = Prt$. Now given are the present value (P) of R20 000, the future value (FV) of R45 200 and the simple interest rate (r) of 12%. We need to determine the time t under consideration. Drawing a time line:



$$
\begin{aligned}
S &= P(1 + rt) \\
45\,200 &= 20\,000(1 + 0{,}12 \times t) \\
\frac{45\,200}{20\,000} &= 1 + 0{,}12t \\
\frac{45\,200}{20\,000} - 1 &= 0{,}12t \\
1{,}26 &= 0{,}12t \\
\frac{1{,}26}{0{,}12} &= t \\
t &= 10{,}50
\end{aligned}
$$

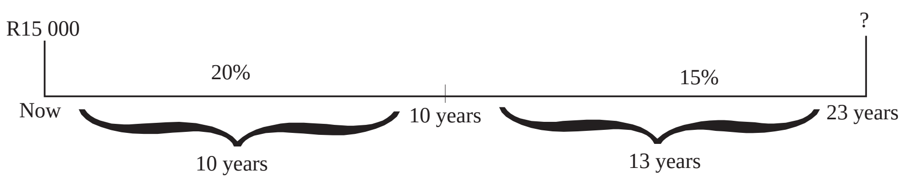
The time under consideration is 10,50 years.

EL-738 and EL-738F
2ndF CA
Use normal keys
$45\,200 \div 20\,000 = -1 = \div 0.12 =$
10.50 *is displayed.*

HP10BII and HP10BII+
C ALL
Use normal keys
$45\,200 \div 20\,000 = -1 = \div 0.12 =$
10.50 *is displayed.*

[Option 4]

5. No interest rate is specified but the painting increases with a certain percentage every year thus you earn increase on increase similar to compound interest where interest is calculated on interest thus the compound interest formula is applicable here. First the value increases by 20% each year for 10 years and then again by 15% each year for the remainder of $23 - 10$ that is 13 years. Drawing a time line:



The value after 10 years is:

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 15\,000\left(1+\frac{0{,}2}{1}\right)^{10\times 1} \\ &= 92\,876{,}04634\ldots \end{aligned}$$

Value after 23 years is:

$$\begin{aligned} & 92{,}876{,}04634\left(1+\frac{0{,}15}{1}\right)^{(23-10)\times 1} \\ = \; & 571\,446{,}5882 \end{aligned}$$

Or alternatively in one step:

$$\begin{aligned} S &= 15\,000(1+0{,}2)^{10}(1+0{,}15)^{(23-10)} \\ &= 571\,446{,}5882 \\ &\approx 571\,446{,}59. \end{aligned}$$

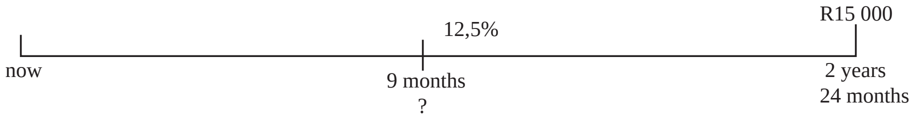
Miriam can expect to receive R571 446,59.

EL-738 and EL-738F
2ndF CA
Use normal keys
$15\,000(1+0.2)$ 2ndF y^x $10\times$
$(1+0.15)$ 2ndF y^x $(23-10) =$
571446.588... *is displayed.*

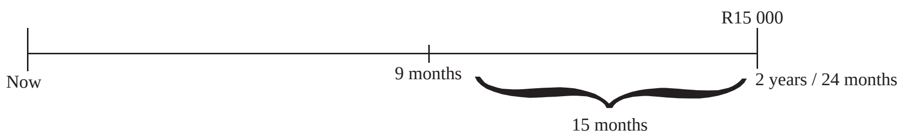
HP10BII and HP10BII+
▬ C ALL
Use normal keys
$1+0.15 =$ ▬ $y^x 13 =$
6.15279... *is displayed*
Store in memory for later use $\rightarrow$ *M*
$1+0.2 =$ ▬ $y^x 10 = \times 15000 =$
92.876.05 *is displayed.*
Multiply with memory
$\times$ RM=
571446.588... *is displayed.*

[Option 3]

6. This is a simple interest rate calculation as the word *simple interest* is found in the question. The formulas for a simple interest calculation are $S = P(1+rt)$ and $I = Prt$. Given are the value Sipho has to pay Alet back in two year's time from now, thus the future value (S) of R15 000 and the simple interest rate (r) namely 12,5%. It is asked to determine the value that Sipho has to pay Alet earlier in the loan term namely at month nine so that he will pay off his debt. First draw the time line. This gives you a better idea of the time under consideration. For example:



Now the loan of R15 000 that must be paid at year 2 must now be paid earlier than the date specified, namely at month nine. We need to determine the value of the R15 000 at month nine. Thus we move the R15 000 from where it is in the future (2 year) to month nine:



Remember the time period is the time that you move it from to where you move it to.

In this example you move it from the maturity date to month nine back in time. We need to determine the PV of the R15 000.

Thus the time applicable is 24 months minus 9 months or 15 months and not 9 months.

Thus:

$$\begin{aligned} S &= P(1+rt) \\ 15\,000 &= P\left(1+0{,}125\times\frac{15}{12}\right) \\ P &= \left(\frac{15\,000}{1+0{,}125\times\frac{15}{12}}\right) \\ P &= 12\,972{,}97. \end{aligned}$$

Sipho will pay Alet R12 972,97 nine months from now.

EL-738 and EL-738F

2ndF CA

Use normal keys

Enter as

15 000÷(1+0.125×15÷12) = 12 972.97 *to two decimal places is displayed.*

HP10BII and HP10BII+

C ALL

Use normal keys

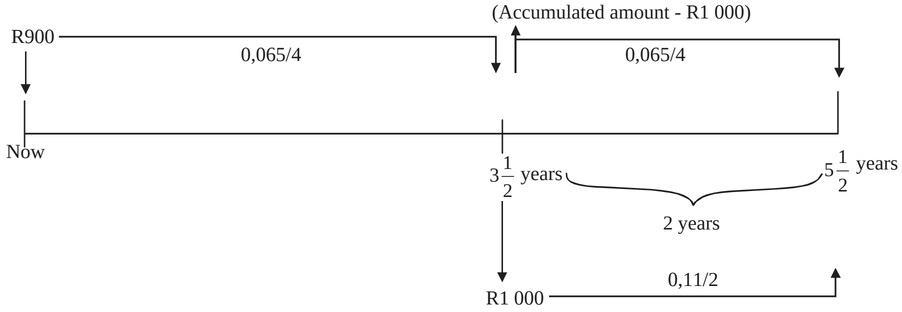
Enter as

15 000 ÷ (1 + (0.125 × 15 ÷ 12)) =

12 972.97 *to two decimal places is displayed.*

[Option 2]

7. This is a compound interest case as the terms *compounded quarterly* and *compounded semi-annually* are found in the wording of the problem. We make use of the formula $S = P(1+\frac{j_m}{m})^{tm}$. First we draw a time line:



First we calculate the value of R900 after three and a half years. Given are

$P = 900$, $j_m = 0{,}065$, $m = 4$ and $t = 3{,}5$. Now

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ &= 900\left(1+\frac{0{,}065}{4}\right)^{4\times 3{,}5} \\ &= 1\,127{,}85 \end{aligned}$$

Of the R1 127,85, R1 000 must be invested for two years at 11% compounded semi-annually and the remainder namely R1 127,85 – R1 000, that is R127,85 must be invested for two years at $6\frac{1}{2}$% compounded quarterly.

After two years R1 000 will accumulate to:

$$\begin{aligned} S &= P\left(1+\tfrac{j_m}{m}\right)^{tm} \\ &= 1\,000\left(1+\tfrac{0{,}11}{2}\right)^{2\times 2} \\ &= 1\,000\left(1+\tfrac{0{,}11}{2}\right)^{4} \\ &= 1\,238{,}82 \end{aligned}$$

After two years R127,85 will accumulate to:

$$\begin{aligned} S &= 127{,}85\left(1+\tfrac{0{,}065}{4}\right)^{4\times 2} \\ &= 127{,}85\left(1+\tfrac{0{,}065}{4}\right)^{8} \\ &= 145{,}45 \end{aligned}$$

His total accrued amount after two years of making the second deposit is R1 238,82 + R145,45 or R1 384,27.

EL-738 and EL-738F	**HP10BII and HP10BII+**
2ndF CA *Use financial mode. Calculate the FV of the 900:* ±900 PV 6.5 I/Y 4 × 3.5 = N 2ndF P/Y 4 ENT ON/C COMP FV 1 127.85 *is displayed.* *Subtract* R1 000 −1 000 = 127.85 *is displayed.* *Calculate the FV of* 127.85 2ndF CA ±127.85 PV 4 × 2 = N 6.5 I/Y 2ndF P/Y 4 ENT ON/C COMP FV 145.45 *is displayed. Store in memory* M+ *[on green keys]* *Calculate the FV of* 1 000: 2ndF CA ±1 000 PV 2 × 2 = N 11 I/Y 2ndF P/Y 2 ENT ON/C COMP FV 1 238.82 *is displayed.* *Calculate the total savings.* *Add the* 145.45 + RCL M+ [*on green keys*] 1 384.27 *is displayed.*	▬ C ALL *Use financial mode. Calculate the FV of the 900:* 900± PV 6.5 I/YR 4 × 3.5 = N 4 ▬ P/YR FV 1 127.85 *is displayed.* *Subtract the* R1 000 −1 000 = 127.85 *is displayed.* *Calculate the FV of the 127.85* ▬ C ALL 127.85 ±PV 4 × 2 = N 6.5 I/YR 4 ▬ P/YR FV 145.45 *is displayed. Store in memory* → *M [above 8 key]* *Calculate the FV of 1 000:* ▬ C ALL 1 000 ±PV 2 × 2 =N 11 I/YR 2 ▬ P/YR FV 1 238.82 *is displayed.* *Calculate the total savings.* *Add the* 145.45 +RM= [*above 9 key*] 1 384.27 *is displayed.*

[Option 3]

8. The terms *simple interest* and *simple discount* are found in the wording of the problem. Thus if it was just a simple interest rate calculation the formula used would have been $S = P(1+rt)$. If it was only a simple discount calculation the formula used would have been $P = S(1 - dt)$. Now as both of them are mentioned we need a formula which expresses the relationship between them. In the solution of Exercise 2.3.2 in the guide we derived a formula for the relationship between the simple interest rate and the simple discount rate as being: $r = \frac{d}{1-dt}$. Now given are the simple interest rate and simple discount rate and asked is the time period t. Thus we need to change the formula to have t as the subject of the formula. Now substituting the given values of r as 24% and d as 20,5% we determine t **in years** as:

$$\begin{aligned}
r &= \frac{d}{1-dt}\\
1-dt &= \frac{d}{r}\\
-dt &= \frac{d}{r}-1\\
dt &= 1-\frac{d}{r}\\
t &= \frac{1-\frac{d}{r}}{d}\\
t &= \frac{1-\frac{0{,}205}{0{,}24}}{0{,}205}\\
t &= 0{,}71138
\end{aligned}$$

Now as all the answers to the problem are given as days we need to change the years to days. Now there are 365 days in a year thus t **in days** is:

$$\begin{aligned}
t &= 0{,}71138 \times 365\\
&= 259{,}65\\
&\approx 260
\end{aligned}$$

The time under consideration is approximately 260 days.

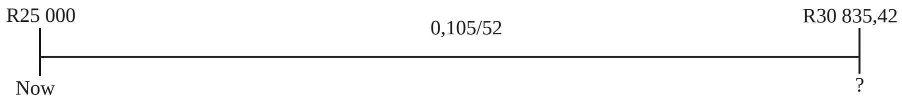
EL-738 and EL-738F	HP10BII and HP10BII+
2ndF CA	C ALL
Use normal keys	*Use normal keys*
Enter as	*Enter as*
$(1 - 0.205 \div 0.24) \div 0.205 =$	$1 -$ $(0.205 \div 0.24$ $) = \div 0.205 =$
0.71138 *years is displayed*	0.71138 *years is displayed*
To get days × *by* 365	*To get days* × *by* 365
×365 =	×365 =
259.65 *is displayed.*	259.65 *is displayed.*

[Option 1]

9. This is a compound interest calculation as the term *compounded weekly* is found in the question. The formula for compound interest calculations is:

$$S = P(1 + \frac{j_m}{m})^{tm}$$

First we draw a time line of the problem:



Now given are the values:

$$\begin{array}{rcl} S & = & \text{R30 835,42} \\ P & = & \text{R25 000,00} \\ j_m & = & 0{,}105. \\ m & = & 52 \\ t & = & ? \end{array}$$

Thus

$$\begin{array}{rcl} S & = & P\left(1 + \frac{j_m}{m}\right)^{tm} \\ 30\,835{,}42 & = & 25\,000\left(1 + \frac{0{,}105}{52}\right)^{52t} \\ \frac{30\,835{,}42}{25\,000} & = & \left(1 + \frac{0{,}105}{52}\right)^{52t} \\ \ln\left(\frac{30\,835{,}42}{25\,000}\right) & = & 52t\,\ln\left(1 + \frac{0{,}105}{52}\right) \\ \frac{\ln\left(\frac{30\,835{,}42}{25\,000}\right)}{\ln\left(1+\frac{0{,}105}{52}\right)} & = & 52t \\ 52t & = & 104 \\ t & = & 2 \end{array}$$

It will take $2 \times 52 = 104$ weeks for R25 000 to accumulate to R30 835,42.

EL-738 and EL-738F
2ndF CA
Use financial mode:
25 000± PV
30 835.42 FV
10.5 I/Y 2ndF P/Y 52 ENT ON/C
COMP N
104 *is displayed.*
Note: Remember N on your calculator is $t \times m$ thus number of compounding periods.

HP10BII and HP10BII+
CA
Use financial mode:
25 000± PV
30 835.42 FV
10.5 I/YR 52 P/YR N
104 *is displayed.*
Note: Remember N on your calculator is $t \times m$ thus number of compounding periods.

[Option 4]

10. This is a conversion between two types of interest rates as one is asked to express the *effective interest rate* in terms of the *nominal interest rate*. Now we have a formula for the effective interest rate in terms of the nominal interest rate j_m, namely $j_{eff} = 100((1+\frac{j_m}{m})^m - 1)$. Now given are $j_m = 16{,}5\%$ and $m = 6$ (compounded every second month). Thus

$$\begin{aligned} j_{eff} &= 100\left(\left(1+\frac{j_m}{m}\right)^m - 1\right) \\ &= 100\left(\left(1+\frac{0{,}165}{6}\right)^6 - 1\right) \\ &= 17{,}67684\%. \end{aligned}$$

The effective rate equals 17,677%.

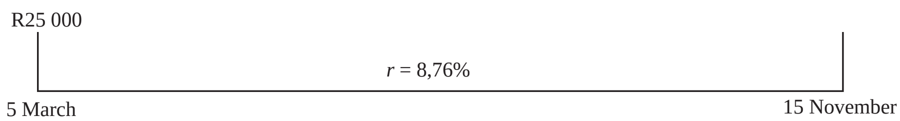
EL-738 and EL-738F
2ndF CA
6(*x*,*y*) [*on the key next to ENT*]
16.5 2ndF →EFF [*on the PV key*]
17.677 *is displayed if your calculator is set to 3 decimal places.*

HP10BII and HP10BII+
▬ C ALL
6 ▬ P/YR [*on PMT key*]
16.5 ▬ NOM% [*on PV key*]
▬ EFF% [*on I/YR key*]
17,677 *is displayed if your calculator is set to 3 decimal places.*

[Option 4]

11. This is a simple interest rate calculation as the term *simple interest* is found in the question. The formula for a simple interest calculation is $S = P(1 + rt)$ or $I = Prt$.

First we draw a time line:



Given are the value invested or present value (P) of R25 000; the simple interest rate (r) of 8,76% and the time period (t) of the period between 5 March and 15 November.

Now to calculate the number of days between 5 March and 15 November we make use of the number of days table at the back of your study guide Appendix C.

Now the rows in the table represent the days, the columns the months and where the two intersect we read off the number of the day.

Thus 5 March is day number 64 and 15 November day number 319. Thus the total number of days between 5 March and 15 November is $319 - 64 = 255$ days.

The time period t must always be expressed as years. Thus we need to change the 255 days to a fraction of a year by dividing the 255 days by the number of days in a year namely 365.

Thus $t = \frac{255}{365}$. Now we need to determine the interest earned over the period of time namely I. Thus

$$\begin{aligned} I &= Prt \\ &= 25\,000 \times 0{,}0876 \times \frac{(319-64)}{365} \\ &= 1\,530 \end{aligned}$$

The accumulated interest is R1 530.

Note that you could have used $S = P(1+rt)$ to calculate S and then subtracted P, to calculate I.

EL-738 and EL-738F
2ndF CA
Use normal mode
25 000 × 0.0876 × (319 − 64)
÷365 =
1 530 *is displayed.*

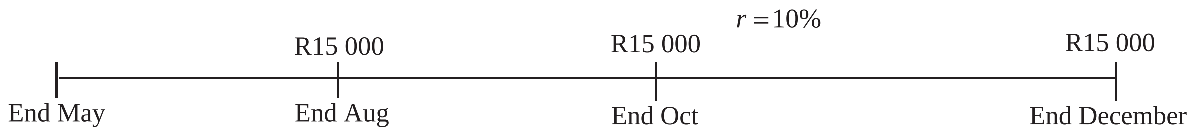
HP10BII and HP10BII+
C ALL
Use normal mode
319 − 64 = ÷365 =
×0.0876 × 25 000 =
1 530 *is displayed.*

[Option 3]

12. This is a simple interest calculation as the term *simple interest* is found in the question. The formula for simple interest calculations is

$$S = P(1 + rt).$$

Given are three values that Piet's debtors should pay him.



We need to determine the total amount that they owe Piet at the end of December. Now we can't just add the three values that are due at different time periods together because of the time value of money. Thus we need to move all the values to the same date, namely the end of December, before we can add them.

The first R15 000 must be moved from the end of August to the end of December, that is four months forward in time. The second R15 000 must be moved from the end of October to the end of December, that is two months forward in time. Lastly the R15 000 at the end of December does not move forward or backwards. Thus the total amount outstanding at the end of December is:

$$\begin{aligned} S &= P(1+rt) \\ &= 15\,000\left(1+0{,}10\times\frac{4}{12}\right)+15\,000\left(1+0{,}10\times\frac{2}{12}\right)+15\,000 \\ &= 15\,500+15\,250+15\,000 \\ &= 45\,750{,}00 \end{aligned}$$

Piet will receive R45 750 at the end of December.

EL-738 and EL-738F
2ndF CA
15 000(1 + 0.10 × 4 ÷ 12) + 15 000(1 + 0.10 × 2 ÷ 12) + 15 000 =
45 750 *is displayed.*

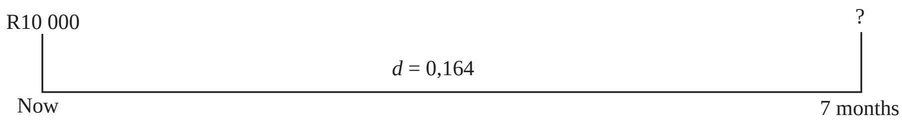
HP10BII and HP10BII+
C ALL
0.10 × 4 ÷ 12 = +1 × 15 000 =→M
0.10 × 2 ÷ 12 = +1 × 15 000 = M+
15 000 M+ RM
45 750 *is displayed.*

[Option 1]

13. This is a simple discount calculation as the term *discount rate* is found in the question. The formula for simple discount is

$$P = S(1-dt).$$

The time line is:



Given are the present value of the loan (P) which is R10 000, the time period (t) which equals seven months and the discount rate (d) of 16,4%. The time period (t) must always be in years. As the given time is in months we change it to a fraction of a year by dividing the months by the number of months in a year which is 12.

Thus

$$\begin{aligned} P &= S(1-dt) \\ 10\,000 &= S\left(1-0{,}164\times\frac{7}{12}\right) \\ S &= \frac{10\,000}{\left(1-0{,}164\times\frac{7}{12}\right)} \\ &= 11\,057{,}87 \end{aligned}$$

Aziza will have to pay R11 057,87 back.

EL-738 and EL-738F
2ndF CA 10 000 ÷ (1 − 0.164 × 7 ÷ 12) = 11 057.87 *to two decimal places is displayed.*

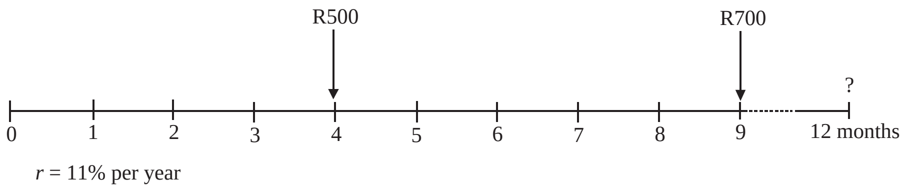
HP10BII and HP10BII+
C ALL 0.164 × 7 ÷ 12 = ±+ 1 =→M 10 000÷RM= 11 057.87 *to two decimal places is displayed.* *Or alternatively,* 10 000 ÷ (1− (0.164 × 7 ÷ 12))= 11 057.87 *is displayed.*

[Option 4]

14. This is a simple interest calculation as the term *simple interest* rate is found in the question. The formula for simple interest is

$$S = P(1 + rt).$$

First we draw a time line:

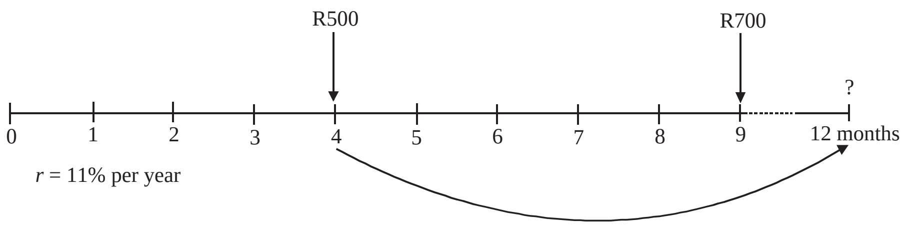


In this question Mario is *rescheduling* his payments towards his loans. We make use of the *equation of value* principal to calculate his new payments namely

the total of all the loans = total of all the payments.

But you cannot add money values at different time periods together. You must first move them to the same date. We use the date that the question specifies, namely month twelve as comparison date.

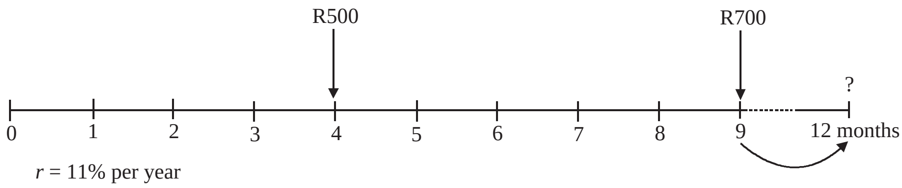
First we calculate the value of the R500 at month twelve. We move the R500 from month four to month twelve, thus eight months forward in time. We thus calculate a future value. Now $S = P(1 + j_m/m)^{tm}$ where t is the number of years and m the number of compounding periods.



We thus calculate:

R500's value at month 12: $500(1 + 0{,}11 \times \frac{8}{12}) = 536{,}67$.

The R700 must be moved forward in time from month nine to month twelve thus three months in forward.



R700's value at month 12: $700(1 + 0{,}11 \times \frac{3}{12}) = 719{,}25$.

Thus he owes R1 255,92 in total (536,67+719,25) at month 12. He has to make a single payment of R1 255,92 in 12 months' time.

EL-738 and EL-738F
2ndF CA *Calculate FV of R500* $500(1 + 0.11 \times 8 \div 12) =$ 536.67 *is displayed. Store for later use* M+ *Calculate FV of R700* $700(1 + 0.11 \times 3 \div 12) =$ 719.25 *is displayed.* *Add answer to memory* +RCL M+ = 1 255.92 *is displayed.*

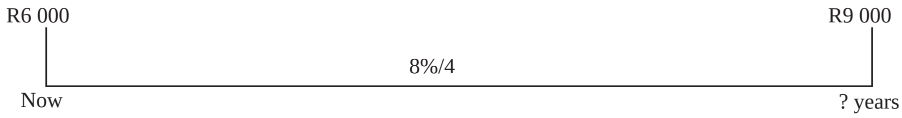
HP10BII and HP10BII+
C ALL *Calculate FV of R500* 1 + (0.11 × 8 ÷ 12) = ×500 = 536.67 *is displayed. Store for later use* →M *Calculate FV of R700* 1 + (0.11 × 3 ÷ 12) = ×700 = 719.25 *is displayed.* *Add answer to memory* +RM= 1 255.92 *is displayed.*

[Option 1]

15. This is a compound interest calculation as the term *compound interest* is found in the question and only one principle value is mentioned in the question. The formula for compound interest calculations is:

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm}$$

The time line is:



Now given is the principle or present value (P) of R6 000 that accumulates to the future value (S) of R9 000. The interest rate is given as 8%, compounded every three months, thus $j_m = 0{,}08$ and $m = 4$, as there are 4 three months periods in one year. We need to determine the number of years, thus t.

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ 9\,000 &= 6\,000\left(1+\frac{0{,}08}{4}\right)^{t4} \end{aligned}$$

Using our calculator's financial mode we determine

$$n = 4t = 20{,}4753$$

You must remember that when using your calculator it determines the number of compounding periods n which is $t \times m$. We must therefore divide the answer by four to obtain the number of years (t).

$$\begin{aligned} t &= 20{,}4753 \div 4 \\ &= 5{,}1188 \\ &\approx 5{,}12 \end{aligned}$$

OR alternatively,

solving it using normal mode of the calculator we first have to simplify the calculation

$$\begin{aligned} S &= P\left(1+\frac{j_m}{m}\right)^{tm} \\ 9\,000 &= 6\,000\left(1+\frac{0{,}08}{4}\right)^{t4} \\ \frac{9\,000}{6\,000} &= \left(1+\frac{0{,}08}{4}\right)^{t4} \end{aligned}$$

Take ln on both sides of the equation

$$\ln\left(\frac{9\,000}{6\,000}\right) = \ln\left(1+\frac{0{,}08}{4}\right)^{t4}$$

But $\ln a^x = x \ln a$, thus

$$\begin{aligned}
\ln\left(\frac{9\,000}{6\,000}\right) &= 4t\ln\left(1+\frac{0{,}08}{4}\right)\\
\frac{\ln\left(\frac{9\,000}{6\,000}\right)}{\ln(1+\frac{0{,}08}{4})} &= 4t\\
t &= \frac{\dfrac{\ln\left(\dfrac{9\,000}{6\,000}\right)}{\ln\left(1+\dfrac{0{,}08}{4}\right)}}{4}\\
t &= 5{,}1188\\
t &\approx 5{,}12
\end{aligned}$$

It will take 5,12 years.

EL-738 and EL-738F

2ndF CA
Use financial mode:
2ndF P/Y 4 ENT ON/C
8 I/Y
6 000± PV
9 000 FV
COMP N
20.4753 *is displayed. To calculate the years t we divide N by 4.*
÷4 =
5.12 *to two decimals is displayed.*

Use normal mode:
2ndF ln[*on the 2 key*](9 000 ÷ 6 000)÷
2ndF ln(1 + 0.08 ÷ 4) =
÷4 =
5.12 *is displayed.*

HP10BII and HP10BII+

C ALL
Use financial mode:
4 P/YR
8 I/Y
6 000± PV
9 000 FV
N
20.4753 *is displayed. To calculate the years t we divide N by 4.*
÷4 =
5.12 *to two decimals is displayed.*

Use normal mode:
0.08 ÷ 4 = +1 = LN
Store for later use
→M
9 000 ÷ 6 000 = LN [*on the 2 key*]
÷*RM* =
÷4 =
5.12 *is displayed.*

[Option 2]

4 WHEN TO USE WHICH FORMULA

Below are guidelines with regard to the identification and application of formulae (i.e. when to use which formula). Up to now you would have only used the first few. The rest will be used later.

4.1 Formulæ

4.1.1 Simple Interest

You go to the bank once and the calculation is done at the end of the investment time.

The words "**simple interest**" must be mentioned in the question.

$$I = Prt$$

Use this formula when you want to determine the interest received/paid.

$$S = P(1 + rt)$$

Use this formula when the accumulated amount is mentioned in the question.

4.1.2 Simple Discount

The words "**simple discount**" or "**discount rate**" must be mentioned in the question**.**

$$P = S(1 - dt)$$

The **relationship** between **simple interest** and **simple discount** is given as:

$$r = \frac{d}{1 - dt} \qquad \text{or} \qquad d = \frac{r}{1 + rt}$$

4.1.3 Compound Interest

You go to the bank once (one principle) and the calculations are done according to the number of compounding periods. The word "**compounded**" must be mentioned in the question.

$$S = P\left(1 + \frac{j_m}{m}\right)^{tm} \qquad \text{or} \qquad S = P(1 + i)^n$$

$\frac{j_m}{m}$ or i is the nominal interest rate per year divided by the number of compounding periods per year.

$tm = n$ where t is the total time expressed as years or a fraction of a year and m the number of compounding periods in one year.

4.1.4 Effective Rate

Refers to the rate that you will, in effect, receive/pay in one year. **Effective interest** rate will be mentioned in the question.

$$J_{eff} = 100\left(\left(1+\frac{j_m}{m}\right)^m - 1\right)$$

4.1.5 Continuous Compounding

When the rate at which compounding takes place tends to be infinity, the calculations of the interest are continuous.

$$c = m\ln\left(1+\frac{j_m}{m}\right)$$

This formula is used when a nominal interest rate with m compounded periods per year is converted to a continuous compounding rate.

$$S = Pe^{ct}$$

This formula is used when either the principal (present value) and/or the accumulated amount (future value) are given and the interest rate is continuous.

$$J_\infty = 100\left(e^c - 1\right)$$

This formula is used when the continuous rate is converted to an effective rate.

NB! Remember when a c is in the formula **continuous compounding** is applicable.

4.1.6 Convert Interest Rates

$$j_n = n\left(\left(1+\frac{j_m}{m}\right)^{m\div n} - 1\right)$$

This formula is used when you want to convert a given nominal interest rate j_m compounded m periods per year to n compounding periods per year and still get the same return.

4.1.7 Annuities

You go to the bank with **equal amounts** (payments or deposits) of money at the **same time intervals**. The applicable interest rate is **compounded**. Remember the compound interest rate periods must be the same as the payment periods.

4.1.8 Present Value

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R\left[\tfrac{(1+i)^n-1}{i(1+i)^n}\right] \end{aligned}$$

You use this formula when the balance in the account, at the end of the time, is zero or you are asked to calculate the present value or it is given.

4.1.9 Future Value

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R\left[\tfrac{(1+i)^n-1}{i}\right] \end{aligned}$$

You use this formula when, at the end of the time, there is a balance in the account or you are asked to calculate the future value or it is given.

4.1.10 Annuity Due

Payments are made at the beginning of the periods.

$$\begin{aligned} P &= (1+i)\,Ra_{\overline{n}|i} \\ S &= (1+i)\,Rs_{\overline{n}|i} \end{aligned}$$

Use this formula when the words "**begin, start immediately or in advance**" are mentioned in the sentence. Multiply the formulae with $(1+i)$ and use n in the formulae as the number of payments made.

4.1.11 Increasing Annuity

$$S = \left(R + \frac{Q}{i}\right) s_{\overline{n}|i} - \frac{nQ}{i}$$

The original payment R is **increased yearly with the same amount** Q.

$$A = nR + Q\frac{n\,(n-1)}{2}$$

The formula represents the actual amount paid for an increasing annuity where n is the number of payments made.

4.1.12 Perpetuity

$$P = \frac{R}{i}$$

You will receive the payment R **indefinitely**.

4.1.13 Deferred Annuity

With a deferred annuity, you are unable to start to repay your debt immediately. **Your first payment is a number of payment intervals after the end of the first interest period**. First calculate the present value of the payments made and then discount this amount back to now by using compound interest. Do not confuse the word "discount" with "simple discount rate". "Discount back" is just another term for "moving back". A deferred annuity can also apply in a future value annuity situation.

4.1.14 Amortisation

Use the present value formula for annuities and determine the payment.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R\left[\tfrac{(1+i)^n-1}{i(1+i)^n}\right] \\ R &= \frac{P}{\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]} \end{aligned}$$

4.1.15 Sinking Fund

Use the future value formula for annuities and determine the payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R\left[\tfrac{(1+i)^n-1}{i}\right] \\ R &= \frac{S}{\left[\frac{(1+i)^n-1}{i}\right]} \end{aligned}$$

4.1.16 Stocks

$$P = da_{\overline{n}|z} + 100\,(1+z)^{-n}$$

d is the half yearly coupon payments, z the half yearly interest rate and n the number of outstanding coupons.

4.1.17 Net Present Value

$$NPV = PV_{in} - I_{out}$$

4.1.18 Profitability Index

$$PI = \frac{\text{Present value of cash inflows}}{\text{Present value of cash outflows}}$$

OR

$$PI = \frac{\text{NPV} + \text{Outlays (initial investment)}}{\text{Outlays (initial investment)}}$$

FORMULÆ

$I = Prt$	$r = \dfrac{d}{1-dt}$
$S = P(1+rt)$	$S = (1+i)Rs_{\overline{n}\rceil i}$
$P = S(1-dt)$	$P = (1+i)Ra_{\overline{n}\rceil i}$
$S = P\left(1+\dfrac{j_m}{m}\right)^{tm}$	$P = da_{\overline{n}\rceil z} + 100(1+z)^{-n}$
$J_{eff} = 100\left(\left(1+\dfrac{j_m}{m}\right)^{m} - 1\right)$	$\dfrac{H-R}{365} \times c$
$S = Pe^{ct}$	$\dfrac{-R}{365} \times c$
$j_\infty = 100(e^c - 1)$	$MIRR = \left(\dfrac{C}{PV_{\text{out}}}\right)^{\frac{1}{n}} - 1$
$c = m\ln\left(1+\dfrac{j_m}{m}\right)$	$P = \frac{R}{i}$
$j_m = m\left(e^{\frac{c}{m}} - 1\right)$	$S = \left[R + \frac{Q}{i}\right] s_{\overline{n}\rceil i} - \frac{nQ}{i}$
$i = n\left(\left(1+\dfrac{j_m}{m}\right)^{\frac{m}{n}} - 1\right)$	$T_r = Ra_{\overline{n}\rceil r} - P$
$S = R\left(\dfrac{(1+i)^n - 1}{i}\right)$	$\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$
$S = Rs_{\overline{n}\rceil i}$	$\bar{x}_w = \dfrac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i}$
$P = Ra_{\overline{n}\rceil i}$	$\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
$P = R\left(\dfrac{(1+i)^n - 1}{i(1+i)^n}\right)$	$S = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$
$A = nR + Q\left[\dfrac{n(n-1)}{2}\right]$	$PI = \frac{NPV + \text{original investment}}{\text{original investment}}$