

Tutorial letter 201/1/2014

Introductory Financial Mathematics

DSC1630

Semester 1

Department of Decision Sciences

IMPORTANT INFORMATION:

This tutorial letter contains solutions to the assignments questions.

BAR CODE

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1 PREPARING FOR THE EXAMINATION

- (a) Make sure that you know **WHERE** you write **WHAT** and **WHEN** you write **WHAT!** Also make sure that you know the correct code.
- (b) **Try to work through all the assignments of semester one and two, self-evaluation and additional self-evaluation exercises and old examination papers available on myUnisa when you prepare for the examination. Do not hesitate to contact me if you have problems.**

2 USING A CALCULATOR

- (a) If you want to pass this module you need to use either a **FINANCIAL** or **PROGRAMMABLE** calculator. Re-read Tutorial letter 101 §1.6.3 p12.
- (b) Use your **CALCULATOR** for the calculations. The use of a programmable calculator in the examination is permissible. If you receive a letter from the University that states that a non-programmable calculator is acceptable in the examination **PLEASE** ignore it. It is permissible for DSC1630 students to use a programmable calculator and on the front page of your examination paper it is clearly indicated that a programmable calculator may be used.
- (c) You may take your programmable calculator, with formulæ already keyed in, into the examination hall. If any invigilator tells you to erase your programs, you should refuse and insist that they contact the examination department or the lecturer immediately.

In order to save the battery we suggest that you switch your calculator off from time to time while you write the examination.

- (d) Don't round off answers in the middle of a calculation. You must only round off your answer in the final step of a calculation.

3 EXAMINATION: MAY/JUNE 2014

- (a) It is a two hour paper consisting **ONLY** of multiple-choice questions.

The examination paper will cover **ALL** the chapters in the guide.

A **list of formulæ** (see page 38 of this tutorial letter) as well as a **table with the number of each day of the year** will be supplied.

There are 30 questions giving 100 marks in total. Marks will not be deducted for incorrect answers.

ALL the questions must be answered on the mark reading sheet. You **MUST** use an HB PENCIL. Carefully follow the instructions for completing the mark reading sheet. Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. Do not mark more than one option per question on the mark reading sheet.

The questions asked in the examination will be similar to those in the assignments and self-evaluation exercises. It is important that you also master the self-evaluation exercises in Tutorial Letter 101.

- (b) When answering the multiple-choice questions in the examination, don't just enter the data into your calculator. **Read the question attentively before you start to answer the question.** First write down the formula and/or draw a time line on the supplied rough work paper. Write down the values for the different variables. Check that your values correspond to those given in the question and only THEN enter these values into your calculator.
- (c) The given incorrect options were calculated with common mistakes that students make as I have found from experience. Remember that only ONE of the options is correct.
- (d) When you mark the option on your mark reading sheet, **please make 100% sure that they correspond to the number of the option of the answer that you obtained.** For example if the answer to question 1 corresponds to option [3], mark [3] for question 1 and not something else. I have students who failed the examination, because they marked the wrong option, even though they worked out the answer correctly.

Also make sure that you **mark the option corresponding to the question** that you have answered. Remember, the mark reading sheets are marked by computer and NOT by hand, so everything has to be correctly indicated on the mark reading sheet.

- (e) **REMEMBER: TO PASS YOUR FINAL MARK MUST BE AT LEAST 50%.**

You must OBTAIN AT LEAST 40% IN THE EXAMINATION BEFORE THE MARKS FOR THE COMPULSORY ASSIGNMENTS WILL COUNT TOWARDS YOUR FINAL MARK.

- (f) Final mark:

The semester mark consists of 10% of the mark obtained for Assignment 01 and 90% of the mark obtained for Assignment 02.

The final mark consists of 20% of the semester mark plus 80% of the examination mark.

For example:

Suppose you obtained 80 marks for Assignment 01, 60 marks for Assignment 02, and 40 marks in the examination.

Your semester mark will be calculated as follows:

10% of 80 will give you 8 marks ($\frac{10}{100} \times 80 = 8$).

90% of 60 will give you 54 marks ($\frac{90}{100} \times 60 = 54$).

Your total semester mark is thus 62 (54 + 8). This 62 will contribute to 20% of your final mark. You will therefore have 12,4 marks before you start to write the examination.

Your examination mark will be calculated as follows:

80% of 40 will give you 32 marks ($\frac{80}{100} \times 40 = 32$).

Your final mark will be 44,4 (32 + 12,4).

4 AVAILABILITY OF LECTURER

Remember to please contact us via email, fax, telephone or appointment if you need help regarding the study material. Please note that only students with appointments will be assisted. My contact details and contact hours are:

Mrs Adèle Immelman:

Office: Hazelwood Campus, Room 4-28 **Tel:** +27 12 4334691

E-mail: immelfm@unisa.ac.za

08:00 until 13:30 - Monday till Friday: **Appointments and Telephone**

13:30 until 16:00 - Monday till Thursday: **Telephone only**

5 SOLUTIONS FOR COMPULSORY ASSIGNMENTS

The solutions to the questions of the compulsory assignments are included in this tutorial letter. Study them when you prepare for the examination.

I have included the key operations for the SHARP EL-738 and the HP10BII. I hope it will help you to understand the workings of your calculator. Just remember that these key operations are not the only method, these are just examples. You can use any method as long as you get the same answer.

REMEMBER TO CLEAR ALL DATA FROM THE MEMORY BEFORE YOU ATTEMPT ANY CALCULATIONS.

SHARP EL-738 users: Press 2ndF M-CLR 0 0 and 2ndF CA if you just want to clear the financial keys.

HP10BII users: Press  C ALL

5.1 Assignment 01

1. This is a compound interest calculation as the term *compound interest* is found in the question and only one principle value is mentioned in the question. The formula for a compound interest calculation is:

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm}$$

Now given is the principle or present value, P , of R5 000 that she needs now. You are asked to calculate the amount that she will have to pay back in the future thus we need to calculate the future value or S . The time period of the loan is specified as 15 months. The time period t must always be expressed as years. Thus we need to change the 15 months to a fraction of a year by dividing the 15 months by the number of months in a year namely 12. Thus $t = \frac{15}{12}$ or $t = 1\frac{3}{12}$ or $t = 1,25$. As the interest is compounded monthly there are 12 compounding periods in one year thus $m = 12$. Now

$$\begin{aligned} S &= P \left(1 + \frac{j_m}{m} \right)^{tm} \\ &= 5\,000 \left(1 + \frac{0,28}{12} \right)^{\frac{15}{12} \times \frac{12}{1}} \\ &= 7\,066,87 \end{aligned}$$

Sarah will pay R7 066,87.

EL-738	HP10BII
<p>2ndF CA <i>Use financial mode</i> 2ndF P/Y 12 ENT ON/C ± 5 000 PV 28 I/Y <i>Remember you enter the interest rate as a percentage and you do not enter the divide by 12.</i> <i>The calculator does that automatically</i> $15 \div 12 \times 12 = N$ or use $15 \div 12 = 2ndF$ $\times P/Y N$ COMP FV 7066.86534... <i>is displayed.</i></p>	<p>C ALL 12 P/YR 5 000± PV 28 I/YR <i>Remember you enter the interest rate as a percentage and you do not enter the divide by 12.</i> <i>The calculator does that automatically</i> $15 \div 12 = \times 12 = N$ or $15 \div 12$ $\times P/YR$ FV 7066.86534 <i>is displayed.</i></p>

Or alternatively

EL-738	HP10BII
<p><i>Use normal mode</i> $5\,000(1 + 0.28 \div 12)$ 2ndF y^x [on the DEPR key] $(15 \div 12 \times 12) =$ 7066.87 <i>is displayed.</i></p>	<p><i>Use normal mode</i> $0.28 \div 12 = +1 = \text{y}^x$ [on the \times key] $(15 \div 12 \times 12)$ $= \times 5\,000 =$ 7066.87 <i>is displayed.</i></p>

[Option 4]

2. Firstly this is a compound interest problem as the term *compound interest* is found in the question. Secondly it is a conversion between two types of compound interest rates as the question ask for the equivalent daily compound rate of a given monthly compound interest rate.

To change the compounding period and thus the interest rate we can use the conversion formula in your guide – chapter 3 – see example 3.5 namely





$$j_n = n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right)$$

Now the given interest rate is 13,75% (j_m) and is compounded monthly thus $m = 12$. The

new compounded interest rate is daily thus $n = 365$.

$$\begin{aligned} j_n &= n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right) \\ &= 365 \left(\left(1 + \frac{0,1375}{12} \right)^{12 \div 365} - 1 \right) \\ &= 0,13674 \end{aligned}$$

The daily compounded interest rate is 13,674%.

EL-738	HP10BII
<p>2ndF CA <i>Use normal mode</i> $365((1 + 0.1375 \div 12) 2\text{ndF } y^x(12 \div 365) - 1) =$ 0.13674... <i>is displayed.</i> <i>Multiply by 100 to calculate %</i> $\times 100 =$ 13.674... <i>is displayed.</i></p>	<p> C ALL <i>Use normal mode</i> $0.1375 \div 12 = +1 =$  y^x  $(12 \div 365$ ) $= -1 = \times 365 =$ 0.13674... <i>is displayed.</i> <i>Multiply by 100 to calculate %</i> $\times 100 =$ 13.674... <i>is displayed.</i></p>

[Option 1]

3. This is a continuous compounding calculation as the term *continuous compounding rate* is found in the question. The formula for continuous compounding is:

$$S = Pe^{ct}.$$

Note: The mathematical constant e is a unique real number. The number e is of considerable importance in mathematics. The numerical value of e truncated to 20 decimal places is 2,71828182845904523536. Some calculators have a specific key (for example e^x) to calculate the value of the power of e . You should be able to call up the specified value of e by entering e^1 into your calculator.

Now given the future value or accumulated sum S that she needs in five years' time of R36 000, the continuous interest rate (c) of 7,5% and the number of years of the loan (t) of 5 years. We need to determine the present value of the amount of money she needs to invest now, P . Thus

$$\begin{aligned} S &= Pe^{ct} \\ 36\,000 &= Pe^{(0,075 \times 5)} \\ P &= \frac{36\,000}{e^{(0,075 \times 5)}} \\ &= 24\,742,41 \end{aligned}$$

Jeremy must invest R24 742,41 now.

EL-738	HP10BII
2ndF CA <i>Use normal mode</i> $36\,000 \div 2\text{ndF } e^x$ [on the \pm key] $(0.075 \times 5) =$ 24742.41 is displayed.	C ALL <i>Use normal mode</i> $0.075 \times 5 = \text{C}$ e^x [on the 1 key] $\rightarrow \text{M}$ [store in memory] $36\,000 \div \text{RM} =$ 24742.41 is displayed.

[Option 1]

4. This is a conversion between two types of interest rates as asked is to write the *effective interest rate* in terms of the *nominal interest rate*. Now we have a formula of the effective interest rate if the nominal interest rate j_m is given namely $j_{eff} = 100 \left(\left(1 + \frac{j_m}{m} \right)^m - 1 \right)$. Now given is $j_m = 16,5\%$ and $m = 6$ (compounded every second month). Thus

$$\begin{aligned}
 j_{eff} &= 100 \left(\left(1 + \frac{j_m}{m} \right)^m - 1 \right) \\
 &= 100 \left(\left(1 + \frac{0,165}{6} \right)^6 - 1 \right) \\
 &= 17,676\%.
 \end{aligned}$$

The effective rate equals 17,676%.

EL-738	HP10BII
2ndF CA $6(x,y)$ [on the key next to ENT] 16.5 2ndF \rightarrow EFF [on the PV key] 17.677 is displayed if your calculator is set to 3 decimal places.	C ALL 6 P/YR [on PMT key] 16.5 NOM\% [on PV key] EFF\% [on I/YR key] 17,677 is displayed if your calculator is set to 3 decimal places.

[Option 4]



5. In this problem we have *equal deposits in equal time periods* plus the interest rate that is specified is *compounded*. Thus we are working with *annuities*. As the deposits are not specified as being paid at the beginning of the period we take it as being paid at the end of each time period thus we have an ordinary annuity. As the future value is given we make use of the future value formula of an annuity namely $S = Rs_{\overline{n}|i}$ to calculate n . Now given is $S = 35000$; $i = 0,1132/12$ and the deposits (R) are R500 each.

$$S = Rs_{\overline{n}|i}$$

$$35\,000 = 500s_{\overline{n}|0,1132\div 12}$$

$$n = 54$$

It will take Eva 54 months.

EL-738	HP10BII
2ndF CA	 C ALL
<i>Use financial mode</i>	<i>Use financial mode</i>
2ndF P/Y 12 ENT ON/C	12  P/YR
11.32 I/Y	11.32 I/YR
35 000 FV	35 000 FV
500± PMT	500± PMT
COMP N	N
54 <i>is displayed.</i>	54 <i>is displayed.</i>

[Option 2]

6. This is a compound interest calculation as the term *compound interest* is found in the question and only one principle value is mentioned in the question. The formula for compound interest calculations is:

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm}$$

Now given is the principle or present value (P) of R6 000 that accumulates to the future value (S) of R9 000. The interest rate is given as 8%, compounded every three months thus $j_m = 0,08$ and $m = 4$, as there are 4 three months periods in one year. We need to determine the number of years thus t .

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm}$$

$$9\,000 = 6\,000 \left(1 + \frac{0,08}{4} \right)^{t4}$$

Using our calculator's financial mode we determine

$$n = 4t = 20,4753$$

You must remember that when using your calculator it determines the number of compounding periods n which is $t \times m$. We must therefore divide it by four to obtain the number of years (t).

$$\begin{aligned} t &= 20,4753 \div 4 \\ &= 5,1188 \\ &\approx 5,12 \end{aligned}$$

OR alternatively,

solving it using normal mode of the calculator we first have to simplify the calculation

$$\begin{aligned} S &= P \left(1 + \frac{j_m}{m} \right)^{tm} \\ 9\,000 &= 6\,000 \left(1 + \frac{0,08}{4} \right)^{t4} \\ \frac{9\,000}{6\,000} &= \left(1 + \frac{0,08}{4} \right)^{t4} \end{aligned}$$

Take \ln on both sides of the equation

$$\ln \left(\frac{9\,000}{6\,000} \right) = \ln \left(1 + \frac{0,08}{4} \right)^{t4}$$

But $\ln a^x = x \ln a$, thus

$$\begin{aligned} \ln \left(\frac{9\,000}{6\,000} \right) &= 4t \ln \left(1 + \frac{0,08}{4} \right) \\ \frac{\ln \left(\frac{9\,000}{6\,000} \right)}{\ln \left(1 + \frac{0,08}{4} \right)} &= 4t \\ t &= \frac{\frac{\ln \left(\frac{9\,000}{6\,000} \right)}{\ln \left(1 + \frac{0,08}{4} \right)}}{4} \\ t &= 5,1188 \\ t &\approx 5,12 \end{aligned}$$

It will take 5,12 years.

EL-738	HP10BII
<p>2ndF CA <i>Use financial mode</i> 2ndF P/Y 4 ENT ON/C 8 I/Y 6 000± PV 9 000 FV COMP N 20.4753 is displayed. To calculate the years t we divide N by 4. $\div 4 =$ 5.12 to two decimals is displayed.</p> <p><i>Use normal mode</i> 2ndF $\ln(9\,000 \div 6\,000) \div$ 2ndF $\ln(1 + 0.08 \div 4) =$ $\div 4 =$ 5.12 is displayed.</p>	<p> C ALL <i>Use financial mode</i> 4 P/YR 8 I/Y 6 000± PV 9 000 FV N 20.4753 is displayed. To calculate the years t we divide N by 4. $\div 4 =$ 5.12 to two decimals is displayed.</p> <p><i>Use normal mode</i> 9 000 \div 6 000 = LN <i>Store for later use</i> $\rightarrow M$ 0.08 \div 4 = +1 = LN $\div 4 =$ 5.12 is displayed.</p>

[Option 2]

7. According to definitions $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$ and $a_{\overline{n}|i} = \frac{(1+i)^n - 1}{i(1+i)^n}$.

Now if we multiply both sides of the $a_{\overline{n}|i}$ formula with $(1+i)^n$ we get:

$$(1+i)^n a_{\overline{n}|i} = \frac{(1+i)^n - 1}{i(1+i)^n} \times (1+i)^n$$

$$(1+i)^n a_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

But

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

$$(1+i)^n a_{\overline{n}|i} = s_{\overline{n}|i}$$

Thus $x = a_{\overline{n}|i}$ per definition.

[Option 1]

8. This is a simple discount calculation as the term *discount rate* is found in the question. The formula for simple discount is $P = S(1 - dt)$.

Given is the future value of the loan (S) which is R48 000, the time period (t) which equal 7 months and the discount rate (d) of 11,9%. The time period that we use must always be in years. As the given time is in months we change it to a fraction of a year by dividing the months by the number of months in a year which is 12. Thus $t = \frac{7}{12}$. We need to determine P . Thus

$$\begin{aligned} P &= S(1 - dt) \\ &= 48\,000 \left(1 - 0,119 \times \frac{7}{12} \right) \\ &= 44\,668,00 \end{aligned}$$

You will receive R44 668 now.

EL-738
2ndF CA
<i>Use normal mode</i>
$48\,000(1 - 0,119 \times 7 \div 12) =$
$44\,668.00$ is displayed.



HP10BII
C ALL
<i>Use normal mode</i>
$0,119 \times 7 \div 12 = \pm + 1 \Rightarrow M$
$48\,000 \times \text{RM} =$
$44\,668.00$ to two decimal places is displayed.
<i>Or alternatively</i>
$48\,000 \times (1 -$
$(0,119 \times 7 \div 12$
$) =$
$44\,668.00$ is displayed.

[Option 1]

9. In this question *compound interest* as well as *continuous compounding rate* are mentioned. This is called a conversion problem as we need to write the one in terms of the other. Now the nominal rate j_m is given as 19,4%; m is given as compounded monthly thus equal to 12 and asked is to determine the continuous rate c . Now

$$\begin{aligned} c &= m \ln \left(1 + \frac{j_m}{m} \right) \\ &= 12 \ln \left(1 + \frac{0,194}{12} \right) \\ &= 0,19245 \end{aligned}$$

The continuous compounding rate is 19,245%.

EL-738	HP10BII
2ndF CA <i>Use normal mode</i> 12 2ndF ln [on the 2 key] $(1 + 0.194 \div 12) =$ 0.19245 <i>is displayed.</i> <i>Multiply with 100 to calculate %</i> $\times 100 =$ 19.245... <i>is displayed.</i>	 C ALL <i>Use normal mode</i> $0.194 \div 12 = +1$ $=$  LN [on the 2 key] $\times 12 =$ 0.19245 <i>is displayed.</i> <i>Multiply with 100 to calculate %</i> $\times 100 =$ 19.245... <i>is displayed.</i>

[Option 5]

10. This is a simple interest rate calculation as the term *simple interest* is found in the question. The formula for a simple interest calculation is $S = P(1 + rt)$ or $I = Prt$. Given is the value invested or present value (P) of R25 000; the simple interest rate (r) of 8,75% and the time period (t) of the period between 5 March and 15 November.

Now to calculate the number of days between 5 March and 15 November we make use of the number of days table at the back of your study guide Appendix C.


Now the rows in the table represent the day, the columns the month and where the two intersect we read off the date value.

Thus 5 March is day number 64 and 15 November day number 319. Thus the total number of days between 5 March and 15 November is $319 - 64 = 255$ days.

The time period t must always be expressed as years. Thus we need to change the 255 days to a fraction of a year by dividing the 255 days by the number of days in a year namely 365. Thus $t = \frac{255}{365}$. Now we need to determine the interest earned over the period of time namely I . Thus

$$\begin{aligned}
 I &= Prt \\
 &= 25\,000 \times 0,0876 \times \frac{(319 - 64)}{365} \\
 &= 1\,530
 \end{aligned}$$

The accumulated interest is R1 530.

EL-738	HP10BII
2ndF CA <i>Use normal mode</i> $25\,000 \times 0.0876 \times (319 - 64)$ $\div 365 =$ 1 530 <i>is displayed.</i>	 C ALL <i>Use normal mode</i> $319 - 64 = \div 365 =$ $\times 0.0876 \times 25\,000 =$ 1 530 <i>is displayed.</i>

[Option 3]

11. The terms *simple interest* and *simple discount* are found in the wording of the problem. Thus if it was just a simple interest rate calculation the formula used would have been $S = P(1 + rt)$. If it was only a simple discount calculation the formula used would have been $P = S(1 - dt)$. Now as both of them are mentioned we need a formula which expresses the relationship between them. In the solution of Exercise 2.3.2 in the guide we derived a formula for the relationship between the simple interest rate and the simple discount rate as being: $r = \frac{d}{1 - dt}$. Now given is the simple interest rate and simple discount rate and asked is the time period t . Thus we need to change the formula to have t as the subject of the formula. Now substituting the given values of r as 24% and d as 20,5% we determine t in years as:

$$\begin{aligned}
 r &= \frac{d}{1 - dt} \\
 1 - dt &= \frac{d}{r} \\
 -dt &= \frac{d}{r} - 1 \\
 dt &= 1 - \frac{d}{r} \\
 t &= \frac{1 - \frac{d}{r}}{d} \\
 t &= \frac{1 - \frac{0,205}{0,24}}{0,205} \\
 t &= 0,71138 \text{ years}
 \end{aligned}$$

Now as all the answers to the problem are given as days we need to change the years to days. Now there are 365 days in a year thus:

$$\begin{aligned}
 t &= 0,71138 \times 365 \text{ days} \\
 &= 259,65 \text{ days} \\
 &\approx 260 \text{ days}
 \end{aligned}$$

The time under consideration is approximately 260 days.

EL-738	HP10BII
2ndF CA <i>Use normal keys</i> <i>Enter as</i> $(1 - 0.205 \div 0.24) \div 0.205 =$ 0.71138 <i>years is displayed</i> <i>To get days \times by 365</i> $\times 365 =$ 259.65 <i>is displayed.</i>	C ALL <i>Use normal keys</i> <i>Enter as</i> $1 - \text{C ALL} (0.205 \div 0.24 \text{C ALL}) = \div 0.205 =$ 0.71138 <i>years is displayed</i> <i>To get days \times by 365</i> $\times 365 =$ 259.65 <i>is displayed.</i>

[Option 1]

12. This is an *odd period calculation* using the *method of simple interest for odd periods and compound interest for full period*. First we draw a time line of the situation:



To determine the odd periods it is very important to note when the interest is being paid. In this situation interest is paid monthly starting at the beginning of the year. Interest is thus paid on the 1 January, 1 February and 1 March etc. Thus there are two odd periods. One from 8 January to 1 February and one from 1 November to 21 November. If we use the days table in the study guide page 201 then the number of days between 8 January and 1 February is $32 - 8 = 24$ days and the number of days between 1 November to 21 November is $325 - 305 = 20$ days. The time between 1 February and 1 November consists of full periods. Be careful when calculating the full periods. If you count the interest dates you get 10 but remember the value of t is the number of time periods and a time period is between two dates, thus the value of t is 9. Now we use simple interest for odd periods and compound interest for the full periods. Thus the accumulated value at 21 November is:

$$\begin{aligned}
 S &= P(1 + rt) \left(1 + \frac{j_m}{m}\right)^{tm} (1 + rt) \\
 &= 360\,000 \left(1 + 0,0791 \times \frac{24}{365}\right) \left(1 + \frac{0,0791}{12}\right)^{\frac{9}{12} \times \frac{12}{1}} \left(1 + 0,0791 \times \frac{20}{365}\right) \\
 &= 361\,872,39 \left(1 + \frac{0,0791}{12}\right)^{\frac{9}{12} \times \frac{12}{1}} \left(1 + 0,0791 \times \frac{20}{365}\right) \\
 &= 383\,915,31 \left(1 + 0,0791 \times \frac{20}{365}\right) \\
 &= 385\,579,29
 \end{aligned}$$

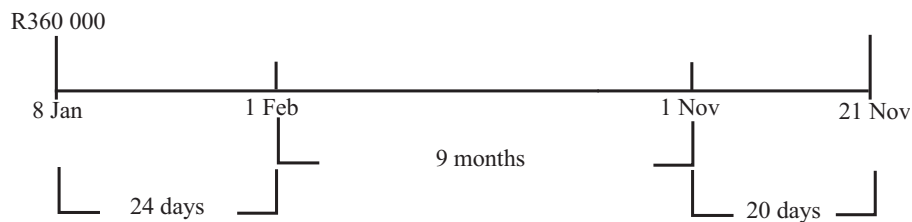
Jabu will have R385 579,29 available.

EL-738	HP10BII
<p>2ndF CA <i>First use normal mode</i> $360\,000(1 + 0.0791 \times 24 \div 365) =$ $361\,872.39 \dots$ is displayed. <i>Use as PV in next calculation in financial mode. Change answer to negative</i> $\times \pm 1 = PV$ 2ndF P/Y 12 ENT ON/C $7.91 I/Y 9 \div 12 \times 12 = N$ or $9 \div$ 2ndF $\times P/Y N$ COMP FV $383\,915.3087$ is displayed. <i>Use as PV in normal mode for last calculation.</i> $\times (1 + 0.0791 \times 20 \div 365) =$ $385\,579.29$ is displayed.</p>	<p>C ALL <i>First use normal mode</i> $360\,000 \times \text{C ALL} (1 + \text{C ALL} (0.0791 \times 24 \div$ $365 \text{C ALL}) \text{C ALL}) =$ $361\,872.39 \dots$ is displayed. <i>Use as PV in next calculation in financial mode</i> $\pm PV$ 12 $\text{C ALL} P/YR$ 7.91 I/YR $9 \div 12 \times 12 = N$ or $9 \times P/YR N$ FV $383\,915.3087$ is displayed. <i>Use as PV in next calculation in financial mode</i> $\times \text{C ALL} (1 + \text{C ALL} (0.0791 \times 20 \div$ $365 \text{C ALL}) \text{C ALL}) =$ $385\,579.29$ is displayed.</p>

[Option 5]

13. This is an odd period calculation using the method of *fractional compounding*. We make use of ordinary compound interest $S = P \left(1 + \frac{im}{m}\right)^{tm}$ but we express the odd periods as a fraction of a year because t must always be in years.

First we draw a time line:



Next we express the odd periods and periods as a fraction of a year. Thus $t = \frac{20}{365} + \frac{24}{365} + \frac{9}{12}$. We divide the days by 365 as there are 365 days in a year and divide the 9 months by 12. Now given is the present value of R360 000, the interest of 7,91% compounded monthly and asked is the future value on 21 November.

Thus

$$\begin{aligned}
 S &= P \left(1 + \frac{j_m}{m} \right)^{tm} \\
 &= 360\,000 \left(1 + \frac{0,0791}{12} \right)^{\left(\frac{20+24}{365} + \frac{9}{12} \right) \times \frac{12}{1}} \\
 &= 385\,576,04
 \end{aligned}$$

Jabu will have R385 576,04 available.

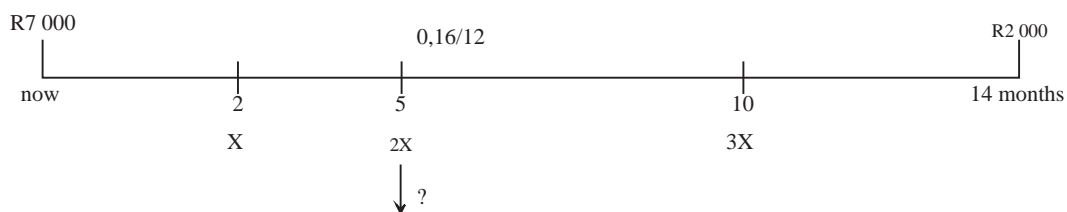
EL-738	HP10BII
2ndF CA	<input type="checkbox"/> C ALL
2ndF P/Y 12 ENT ON/C	12 <input type="checkbox"/> P/YR 360 000± PV
±360 000 PV	7.91 I/YR 20 + 24 = ÷365+ <input type="checkbox"/> (9÷
7.91 I/Y	12 <input type="checkbox"/>) = ×12 = N
(24 + 20) ÷ 365 + (9 ÷ 12) =	FV
×12 = N	385 576.04 is displayed.
COMP FV	
385 576.04 is displayed.	

[Option 4]

14. This is a compound interest calculation as the term *compound interest* is found in the question. The formula for compound interest calculations is:

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm}$$

First we draw a time line:



In this question the person is *rescheduling* her payments towards her loans thus we make use of the *equation of value* principle namely what you owe you must pay back thus

the total of all the loans = total of all the payments.

But you cannot add money values at different time periods together. You must first move them to the same date. We use the date that the question specify namely month five as comparison date.

First we calculate the value of each loan at month five. We move the R7 000 from now to year five, thus five months forward in time. We thus calculate a future value. Now $S = P(1 + j_m/m)^{tm}$ where t is the number of years and m the number of compounding periods. Thus

$$S = 7\,000(1 + 0,16/12)^{(5/12 \times 12)}$$

The R2 000 must be moved backwards in time from month 14 to month five thus nine months in total back. We thus calculate a present value:

$$P = 2\,000(1 + 0,16/12)^{-(9/12 \times 12)}$$

Remember that $2\,000(1 + 0,16/12)^{-(9/12 \times 12)}$ is the same as $2\,000/(1 + 0,16/12)^{(9/12 \times 12)}$

Now the total owing at month five is:

$$\begin{aligned} & 7\,000 (1 + 0,16/12)^{(5/12 \times 12)} + 2\,000(1 + 0,16/12)^{-(9/12 \times 12)} \\ & = 9\,254,52616. \end{aligned}$$

Secondly we calculate the value of the total payments at month five:

Now you are paying X at month two and $2X$ at month five and $3X$ at month ten. Thus the first payment X must be moved forward from month two to month five thus three months in total forward: $X(1 + 0,16/12)^{3/12 \times 12}$. Then $2X$ is already at month five thus the value of $2X$ stays $2X$. The $3X$ must move back from month ten to month five thus five months back in total: $3X(1 + 0,16/12)^{-(5/12 \times 12)}$.

Thus the total of the payments at month five is:

$$X(1 + 0,16/12)^{(3/12 \times 12)} + 2X + 3X(1 + 0,16/12)^{-(5/12 \times 12)}$$

Taking X out as a common factor:

$$\begin{aligned} & = X[(1 + 0,16/12)^{(3/12 \times 12)} + 2 + 3/(1 + 0,16/12)^{(5/12 \times 12)}] \\ & = 5,84829X \end{aligned}$$

Now we equate what you owe to what you pay and solve for X .

$$\begin{aligned}
7\,000 \left(1 + \frac{0,16}{12}\right)^5 + 2\,000 \left(1 + \frac{0,16}{12}\right)^{-9} &= X \left(1 + \frac{0,16}{12}\right)^3 + 2X + 3X \left(1 + \frac{0,16}{12}\right)^{-5} \\
9\,254,52616 &= X \left[\left(1 + \frac{0,16}{12}\right)^3 + 2 + 3 \left(1 + \frac{0,16}{12}\right)^{-5} \right] \\
9\,254,52616 &= 5,84829X \\
\frac{9\,254,52616}{5,84829} &= X \\
X &= 1\,582,43
\end{aligned}$$

Remember they ask the payment at month five which is $2X$. Thus we still need to multiply the answer by 2. Thus the value of the payment at month five is:

$$\begin{aligned}
X &= 1\,582,43 \\
2X &= R3\,164,86
\end{aligned}$$

The size of the payment at month five is R3 164,86.

EL-738

2ndF M-CLR 00

Use financial keys

Calculate the total of the payments and store in memory for later use:

Calculate the PV of the 3X

2ndF P/Y 12 ENT ON/C

16 I/Y

5 N

±3 FV

COMP PV

2.80776... *is displayed.*

Add the two of 2X

+2 =

4.80776... *is displayed.* *Store it in memory for later use*

M+

Now the P/Y and I/Y are still the same and entered already thus just enter values that have changed from step 1 to calculate the PV of X

3 N

± 1 PV

COMP FV

1.04054... *is displayed.*

Add it in memory for later use

M+

Calculate the total of the loans and store in ALPHA keys memory. I/Y and P/Y already entered.

Calculate the FV of 7000

HP10BII

 C ALL

Use financial keys

Calculate the total of the payments and store in memory for later use:

Calculate the PV of the 3X

12  P/YR

16 I/YR

5 N

3± FV

PV

2.80776... *is displayed.*

Add the two of 2X

+2 =

4.80776... *is displayed.*

Store it in memory for later use

→ M

Now the P/YR and I/YR are still the same and entered already thus just enter values that have changed from step 1 to calculate the PV of X

3 N

1± PV

FV

1.04054... *is displayed.*

Add it in memory for later use

M+

Calculate the total of the loans

Calculate the FV of 7000






EL-738	HP10BII
<p>± 7000 PV 5 N COMP FV 7479.278... <i>is displayed.</i> <i>Store it in ALPHA memory keys for later use</i> STO A <i>Calculate the PV of 2000</i> ± 2000 FV 9 N COMP PV 1775.248... <i>is displayed.</i> <i>Store it in ALPHA memory keys for later use</i> STO B <i>Add two ALPHA memories together to calculate total of loans</i> ALPHA A + ALPHA B = 9254.5261... <i>is displayed.</i> <i>Divide total loans by payments value to calculate value of X</i> \div RCL M+ = 1582.43... <i>is displayed.</i> <i>Multiply by 2 to calculate 2X</i> $\times 2 =$ 3164.86... <i>is displayed.</i></p>	<p>$7000 \pm$ PV 5 N FV 7479.278... <i>is displayed.</i> <i>Write it down for later use</i> <i>Calculate the PV of 2000</i> $2000 \pm$ FV 9 N PV 1775.248... <i>is displayed.</i> <i>Write it down for later use</i> <i>Add two values of loans together to calculate total of loans</i> $7479.278 \dots + 1775.248 \dots =$ 9254.5261... <i>is displayed.</i> <i>Divide total loans by payments value to calculate value of X</i> \div RM = 1582.43... <i>is displayed.</i> <i>Multiply by 2 to calculate 2X</i> $\times 2 =$ 3164.86... <i>is displayed.</i></p>

[Option 3]

15. In this problem we have *equal payments in equal time periods* (monthly) plus the interest rate is specified as *compounded*, thus we are working with annuities. If the payments were at the end of the time we would have use the formula $S = Rs_{\overline{n}|i}$. But the word **immediately** indicates that the payments are being made at the beginning of the time intervals thus an annuity due instead of an ordinary annuity. We need to determine the future value of an annuity due. Thus the formula to use is $S = (1 + i)Rs_{\overline{n}|i}$.

$$\begin{aligned}
 S &= (1+i)Rs_{\overline{\infty}|i} \\
 &= (1+i)6\,000s_{\overline{6 \times 12}|0,0894 \div 12} \\
 &= 573\,187
 \end{aligned}$$

Romeo will have R573 187 available.

EL-738	HP10BII
<p>2ndF M-CLR 0 0 <i>Switch the BGN/END mode on:</i> 2ndF BGN/END [on FV key] <i>Calculate FV for R6 000's</i> 2ndF P/Y 12 ENT ON/C 6 000± PMT 6 × 12 =N or 2ndF ×P/Y N 8.94 I/Y COMP FV 573 187 is displayed. <i>Note: Switch the BGN/END mode off</i> [press 2ndF BGN/END:]</p>	<p> C ALL <i>Switch the BGN/END mode on:</i>  BGN/END <i>Calculate FV for R6 000's</i> 12  P/YR 6 000± PMT 6 × 12 =N or 6  ×P/YR 8.94 I/YR FV 573 187 is displayed. <i>Note: Switch the BGN/END mode off</i> [press  BGN/END:]</p>


[Option 5]

5.2 Assignment 02

1. This is annuity calculation as there are *equal payments in equal time periods* as well as a given *compound interest* rate. But the payments are being made for an indefinite period thus we have a perpetuity. The present value of a perpetuity can be calculated as $P = \frac{R}{i}$. Given is the interest rate of 10,75% compounded monthly, thus $i = \frac{0,1075}{12}$. The present value of the price money is R2 500 000, thus $P = R2\,500\,000$. We need to calculate R . Thus

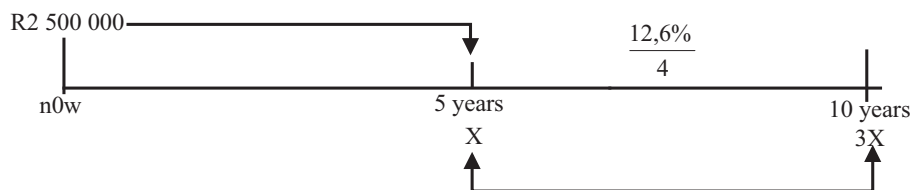
$$\begin{aligned}
 P &= \frac{R}{i} \\
 2\,500\,000 &= \frac{R}{0,1075 \div 12} \\
 2\,500\,000 \times (0,1075 \div 12) &= R \\
 R &= 22\,395,83
 \end{aligned}$$

Mandla will receive R22 395,83 every month.

EL-738	HP10BII
2ndF CA <i>Use normal mode</i> $2\,500\,000 \times (0.1075 \div 12) =$ 22 395.83 is displayed.	<div style="text-align: center;">  C ALL </div> <i>Use normal mode</i> $0.1075 \div 12 = \times 2\,500\,000 =$ 22 395.83 is displayed.

[Option 2]

2. This is a compound interest calculation as the term *compound interest* is found in the question. Secondly we found the words payments specified in the question as well, thus it is an annuity? NO! An annuity is equal payments in equal time intervals. Here we have two payments that are different in size and at different time periods. Thus we do not have annuities. In this example Mandla is *rescheduling* the way his prize money must be paid out to him thus we have an *equations of value* problem. Now drawing the time line:




We need to determine the payment size at year five. Now let it be X . Now there are two payments: RX five years from now and one three times X due in 10 years' time thus $R3X$. Now the amount owed to him must be equal to the sum of the two payments. We cannot just add the two payments together because of the time value of money. Now as all the moneys are at different dates thus to compare them we need to move them all to the same date namely year five as it is asked.

First we move R2 500 000 from now to year five. Thus five years forward. If we move a value forward in time we are calculating a future value thus because the formula for compound interest calculations is $S = P \left(1 + \frac{j}{m}\right)^{tm}$, we multiply by the interest component to calculate the future value of the R2 500 000.

Secondly the $R3X$ payment must be moved back from year 10 to year five thus five years back. If we move a value back in time we divide by the interest component or multiply by the negative of the exponent. As the payment of X is already at year five we leave the value as it is. Now we can equate them as follows: The winnings = the sum of the two payments. Thus

$$\begin{aligned}
 2\,500\,000 \left(1 + \frac{0,126}{4}\right)^{5 \times 4} &= X + 3X \left(1 + \frac{0,126}{4}\right)^{-5 \times 4} \\
 4\,648\,626,51 &= X \left[1 + 3 \left(1 + \frac{0,126}{4}\right)^{-5 \times 4}\right] \\
 4\,648\,626,51 &= X [1 + 1,61338] \\
 X &= \frac{4\,648\,626,51}{2,61338} \\
 &= 1\,778\,779,55
 \end{aligned}$$

Mandla will receive R1 778 780.

EL-738	HP10BII
<p>2ndF M-CLR 00 <i>Calculate the PV of the 3X</i> 2ndF P/Y 4 ENT ON/C 12.6 I/Y $5 \times 4 = N$ ± 3 FV COMP PV 1.61338... <i>is displayed.</i> <i>Add the one of X</i> +1 = 2.61338... <i>is displayed.</i> <i>Store it in memory for later use</i> M+ <i>Now the N, P/Y and I/Y is still the same and entered already thus just enter values that have changed from step 1</i> <i>Calculate the FV of 2 500 000</i> $\pm 2\,500\,000$ PV COMP FV 4 648 626.51... <i>is displayed.</i> <i>Divide FV with previous answer</i> \divRCL M+ = 1778 779.55 <i>is displayed.</i></p>	<p><i>Calculate the PV of the 3X</i> 4  P/YR 12.6 I/YR $5 \times 4 = N$ ± 3 FV PV 1.61338... <i>is displayed.</i> <i>Add the one of X</i> +1 = 2.61338... <i>is displayed.</i> <i>Store it in memory for later use</i> \rightarrowM <i>Now the N, P/Y and I/Y is still the same and entered already thus just enter values that have changed from step 1</i> <i>Calculate the FV of 2 500 000</i> $\pm 2\,500\,000$ PV FV 4 648 626.51... <i>is displayed.</i> <i>Divide FV with previous answer</i> \divRM = 1778 779.55 <i>is displayed.</i></p>

3. In this problem we have *equal payments in equal time periods* plus the interest rate that is specified is *compounded*. Thus we are working with annuities. As the payments are not specified as being paid at the beginning of the period we take it as being paid at the end of each time period thus we have to calculate the present value of an ordinary annuity or $P = Ra_{\overline{n}|i}$.

But the compounding is calculated quarterly and the deposits are monthly. Now very important is that when working with annuities your compounding period and payment periods must be the same! If not you have to change the compounding period and thus the interest rate to the same period as the payment periods using the conversion formula in your guide – chapter 3 - see example 3.5 namely $j_n = n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right)$.

In this case we firstly need to change the quarterly compounded interest rate to monthly using the above formula. After that you can make use of the normal present value calculation of an annuity to calculate the amount P . Now firstly we convert the 12,9% compounded quarterly to a compounded monthly rate:

$$\begin{aligned} j_n &= n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right) \\ &= 12 \left(\left(1 + \frac{0,129}{4} \right)^{4 \div 12} - 1 \right) \\ &= 0,12764 \dots \end{aligned}$$

Secondly we use the new monthly compounded interest rate to calculate the present value of the annuity. **PLEASE DO NOT ROUND OFF THE NEW INTEREST RATE USE ALL THE DIGITS!!!!**

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= 1\,100 a_{\overline{8 \times 12}|0,12764 \div 12} \\ &= 65\,965,49 \end{aligned}$$

The present value is R65 965.

EL-738	HP10BII
<p>2ndF CA Calculate the new interest rate $12((1 + 0.129 \div 4) 2\text{ndF } y^x(4 \div 12) - 1) =$ 0.12764 ... is displayed. Store this as I/Y after making it a percentage and calculate the PV: $\times 100 = I/Y \pm 100 \text{ PMT}$ 2ndF P/Y 12 ENT ON/C $8 \times 12 = N$ or 8 2ndF $\times P/Y N$ COMP PV 65 965.49 is displayed.</p>	<p>C ALL Calculate the new interest rate $0.129 \div 4 = +1 =$ $\text{y}^x (4 \div 12)$ $= -1 = \times 12 =$ 0.12764... is displayed. Store it as I/YR after making it a percentage and calculate the PV: $\times 100 = I/YR 100 \pm \text{PMT}$ 12 P/YR $8 \times 12 = N$ or 8 $\times P/YR$ PV 65 965.49 is displayed.</p>

[Option 4]

4. This is an *average rate of return* problem as it is mentioned in the question. The income or cash inflows are given. First we calculate the average of all the inflows. That means add them all together and divide by the number of inflows and secondly you express the average calculated in the first step as a ratio to the outflow and multiply by 100 to get %.


$$\text{ARR} = \frac{\text{Average after-tax income}}{\text{Investment level}}$$

$$0,08421 = \frac{(2 + 5 + 3 + 4 + 7 + 3) \text{ in hundred thousand} \div 6}{\text{Investment level}}$$

$$\text{Investment level} = \frac{400\,000}{0,08421}$$

$$= 4\,750\,029,69$$

The original investment rounded to the nearest thousand of rands is R4 750 000.




EL-738	HP10BII
2ndF M-CLR 00 <i>Using normal mode</i> Calculate the average 200 000 + 500 000 + 300 000 +400 000 + 700 000 + 300 000 = ÷6 = 400 000 is displayed. Now divide by ARR ÷0.08421 = 4 750 029.69 is displayed.	 C ALL <i>Using normal mode</i> Calculate the average 200 000 + 500 000 + 300 000 +400 000 + 700 000 + 300 000 = ÷6 = 400 000 is displayed. Now divide by ARR ÷0.08421 = 4 750 029.69 is displayed.

[Option 4]

5. This is an amortization problem as a *home loan* is mentioned together with *payments*. Asked is the *real cost* of the loan if inflation is taken into account. The real cost of the loan is the present value of all the payments calculated using the inflation rate minus the present value of the home loan. Thus first we calculate the present value of all the payments calculated using the inflation rate. Thus

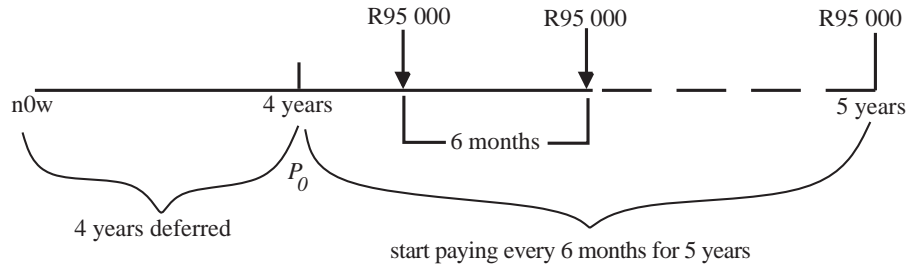
$$\begin{aligned}
 T_v &= Ra_{\overline{n}|r} \\
 &= 9\,680,70a_{\overline{20 \times 12}|0,092 \div 12} \\
 &= 1\,060\,749,00
 \end{aligned}$$

The total value of the loan is R1 060 749. The real cost of the loan is R270 749 (1 060 749 – 790 000).

EL-738	HP10BII
2ndF CA <i>Use financial mode</i> 2ndF P/Y 12 ENT ON/C 9.2 I/Y 20 × 12 = N or 20 2ndF ×P/Y N 9680.70± PMT COMP PV 1060749.00 is displayed. To calculate real cost –790 000 = 270 749 is displayed.	 C ALL <i>Use financial mode</i> 12  P/YR 9.2 I/YR 20 × 12 = N or 20  ×P/YR 9680.70± PMT PV 1060749.00 is displayed. To calculate real cost –790 000 = 270 749 is displayed.

[Option 3]

6. This is annuity calculation as there are *equal payments in equal time periods* as well as a given *compound interest rate*. But Lilian will only start her payments after four years thus the *payments are deferred* for four years. Thus this is a deferred annuity problem. Time line:



Now we need to determine the value of the loan when Lilian starts paying aunt Josephine back. Thus the present value of an annuity P_0 with payments of $R = R95\,000$, interest rate of 18,6% and compounding periods of every six months or $m = 2$. Thus

$$\begin{aligned}
 P &= Ra_{\overline{n}|i} \\
 &= 95\,000a_{\overline{5 \times 2}|0,186 \div 2} \\
 &= 601\,708,66
 \end{aligned}$$

Lilian owes aunt Josephine R601 708,66, when she starts paying her back.

EL-738	HP10BII
2ndF CA	<input type="checkbox"/> C ALL
<i>Use financial mode</i>	<i>Use financial mode</i>
2ndF P/Y 2 ENT ON/C	2 <input type="checkbox"/> P/YR
18.6 I/Y	18.6 I/YR
$\pm 95\,000$ PMT	95 000 \pm PMT
$5 \times 2 = N$ or 5 2ndF \times P/Y N	$5 \times 2 = N$ or 5 <input type="checkbox"/> \times P/YR
COMP PV	PV
601 708.66 <i>is displayed.</i>	601 708.66 <i>is displayed.</i>

[Option 3]

7. Now in question 6 we calculated the present value of the debt when Lilian started to pay back the debt. But asked is the value of the debt now. We must thus move this amount back in time to now for a period of four years. To move money back in time we divide by

the compound interest component. Thus

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm}$$

$$601\,708,66 = P \left(1 + \frac{0,186}{2} \right)^{4 \times 2}$$

$$\frac{601\,708,66}{\left(1 + \frac{0,186}{2} \right)^{4 \times 2}} = P$$

$$P = 295\,410$$

Aunt Josephine gave Lilian R295 410.

EL-738	HP10BII
<p>2ndF CA <i>Use financial mode</i> 2ndF P/Y 2 ENT ON/C 18.6 I/Y $4 \times 2 = N$ or 4 2ndF \timesP/Y N 601 708.66\pm FV COMP PV 295 410 rounded to an integer is displayed.</p> <p><i>Use normal mode</i> $601\,708.66 \div ((1 + 0.186 \div 2))$ 2ndF $y^x(4 \times 2) =$ 295 410 rounded to an integer is displayed.</p>	<p> C ALL <i>Use financial mode</i> 2 P/YR 18.6 I/YR $4 \times 2 = N$ or 4 \times P/YR 601 708.66\pm FV PV 295 410 rounded to an integer is displayed.</p> <p><i>Use normal mode</i> $0.186 \div 2 = +1 =$ y^x $(4 \times 2$ $) =$ <i>Store in memory</i> \rightarrow M 601 708.66\div RM = 295 410 is displayed rounded to an integer .</p>

[Option 4]

8. We need to determine the *slope of the regression line* that represents the data. The equation of the regression line is equal to $y = a + bx$, where a denote the y -intercept and b the slope. Using the statistical keys of our calculator and entering the data points we determine the slope. The slope of the regression line is $-0,65747$.

EL-738	HP10BII
2ndF M-CLR 0 0 MODE 1 1 3.7 (x,y) 16 DATA [on the ENT key] 5.6 (x,y) 25 DATA 7.5 (x,y) 80 DATA 18.9 (x,y) 45 DATA 28.4 (x,y) 12 DATA To determine the slope or b-value ALPHA b = b = -0.65747 is displayed. Cancel STA mode: MODE 0	$\boxed{\text{CL}} \sum$ [on $\rightarrow M$ key] 3.7 INPUT 16 $\sum +$ 5.6 INPUT 25 $\sum +$ 7.5 INPUT 80 $\sum +$ 18.9 INPUT 45 $\sum +$ 28.4 INPUT 12 $\sum +$ 0 $\boxed{\text{y}}$, m [on 5 key] (that is the value of a) $\boxed{\text{SWAP}}$ [on K key] value of b is displayed. -0.65747

[Option 1]

9. We need to determine the *correlation coefficient* r of the fitted line that represents the data. The correlation coefficient tells you how strong the linear relationship between the two sets of data is.

Using the statistical keys of our calculator and entering the data points we determine that the correlation coefficient is -0.24810 .

EL-738	HP10BII
2ndF M-CLR 0 0 MODE 1 1 3.7 (x,y) 16 DATA [on the ENT key] 5.6 (x,y) 25 DATA 7.5 (x,y) 80 DATA 18.9 (x,y) 45 DATA 28.4 (x,y) 12 DATA ALPHA r = -0.24810 is displayed. Cancel STA mode: MODE 0	$\boxed{\text{CL}} \sum$ [on $\rightarrow M$ key] 3.7 INPUT 16 $\sum +$ 5.6 INPUT 25 $\sum +$ 7.5 INPUT 80 $\sum +$ 18.9 INPUT 45 $\sum +$ 28.4 INPUT 12 $\sum +$ To get the value of r press $\boxed{\text{r}}$ $\boxed{\text{SWAP}}$ -0.24810 is displayed.

[Option 2]

10. We need to determine the *settlement date* of a bond. Now given is the accrued interest, the half year coupon rate and the next coupon date after the settlement date. Drawing a time line:



Now given the accrued interest. We can use the formula to determine R :

$$\text{Accrued interest} = \frac{H - R}{365} \times C$$

Now the given coupon date is 28 October each year. Thus the second coupon date is six months before or after (coupon payable every six months) the given date, thus 28 April each year. H is the number of days in the six months where the settlement date falls in thus between 28/10/12 and 28/04/12. H equals day number 301 (28 October) minus day number 118 (28 April) and that is 183 and we know that the half year coupon is 7,375 thus the yearly coupon C is $7,375 \times 2$. Thus

$$\begin{aligned} 5,49589 &= \frac{183 - R}{365} \times (7,375 \times 2) \\ \left(\frac{5,49589 \times 365}{7,375 \times 2} \right) &= 183 - R \\ \left(\frac{5,49589 \times 365}{7,375 \times 2} \right) - 183 &= -R \\ R &= 183 - \left(\frac{5,49589 \times 365}{7,375 \times 2} \right) \\ R &= 47 \end{aligned}$$

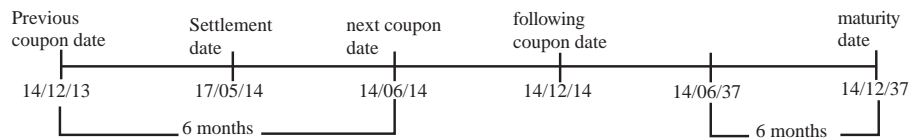
Now R is the number of days between the settlement date and the next coupon date. We must now move 47 days back from 28 October 2012 to obtain the settlement date. Now 28 October is date number 301. Thus $301 - 47$ will give us day number 254. Using the day table in the guide we determine 254 to be 11 September 2012.

EL-738
2ndF CA
<i>Use normal mode</i>
$183 - (5.49589 \times 365) \div$
$(7.375 \times 2) =$
<i>47 is displayed.</i>

HP10BII
C ALL
<i>Use normal mode</i>
$5.49589 \times 365 =$
$\div \text{ (7.375 } \times 2 \text{)} =$
$\rightarrow \text{M}$
$183 - \text{RM} =$
<i>47 is displayed.</i>

[Option 4]

11. We need to determine the *all-in price* of the Bond 525. First we draw the time line of the Bond 525:



Now the price on 14/06/2014 can be determined using

$$P = da_{\overline{n}|z} + 100(1 + z)^{-n}$$

Now given is $z = 0,104 \div 2$; and $d = 9,6 \div 2$. All we need is n .

TIPS TO CALCULATE n IN GENERAL:

Now n is the number of half years from the coupon date after the settlement date, until the maturity date. As a start we determine the first coupon date after the settlement date. Secondly we determine the number of half years until the maturity date. Now there are two situations that can exist when calculating n :

- (a) **If the month of the next coupon date is the same as the month of the maturity date** then subtract the year of the next coupon date from the year of the maturity date - that gives you the number of years until maturity. But you need the number of half years until maturity thus multiply the years by 2 to calculate n .

For example:

Settlement date is 14/9/2013

Next coupon is 4/10/2013

Maturity date is 4/10/2034

Now because the months of the next coupon date and the maturity date are the same namely month 10 we subtract the years namely $2034 - 2013 = 21$ years thus $21 \times 2 = 42$ half years. Thus $n = 42$.

- (b) **If the month of the next coupon date is the different from the month of the maturity date** then ignore the next coupon date and move to the second coupon date from the settlement date - thus you try to get the months the same.

Subtract the year of the second coupon date from the year of the maturity date as in method 1 - that gives you the number of years. But you need the number of half years thus multiply the years by 2 and then add 1 for the period you have ignored.

For example:

Settlement date is 14/9/2013

Next coupon is 4/10/2013

Maturity date is 4/4/2034

Now the month of the next coupon and maturity is different. Thus ignore the first coupon date 4/10/2013 and look at the next coupon date which is 4/4/2014. Now the month of the coupon date 2 and the maturity date is the same namely month 4. We subtract $2034 - 2014 = 20$ years thus $20 \times 2 = 40$ half years but we have ignored one period thus $n = 40 + 1 = 41$.

Now to calculate n we must first determine the number of years from the next coupon date until the maturity date:

As the months of the next coupon (06) and maturity date (12) are not the same we ignore the coupon of 14/06/14 and look at the next coupon 14/12/14. Now the month of the coupon date (12) and the maturity date (12) is the same. We subtract the years $2037 - 2014 = 23$ years.

This 23 must be multiplied by two and then add one (because we have ignored the 14/06/14).

$$\begin{aligned} n &= 23 \times 2 + 1 \\ &= 47 \end{aligned}$$

Now the price on 14/06/14 is:

$$\begin{aligned} P(14/06/14) &= \frac{9,6}{2} a_{\overline{47}|0,104 \div 2} + 100 \left(1 + \frac{0,104}{2} \right)^{-47} \\ &= 83,78654 + 9,23124 \\ &= 93,01779 \end{aligned}$$

As the settlement date is more than ten days from the next coupon date we must add the coupon that was due on 14/06/2014. It is a cum interest case.

$$\begin{aligned} P(14/06/14) &= 93,01779 + 4,8 \\ &= 97,81779 \end{aligned}$$

This R97,81779% must now be discounted back to the settlement date by using the discount factor of $f = \frac{R}{H}$.

R is the number of days between the settlement date and the next coupon date, R equals day number 165 (14 June) minus day number 137 (17 May). $R = 28$.

H is the number of days in the six months in which the settlement date falls. We must first complete the number of days in 2012 thus 365 minus 348 (14 December) plus day number 165 (14 June) $H = 182$

$$\begin{aligned} \text{All-in price} &= 97,81779 \left(1 + \frac{0,104}{2} \right)^{-\frac{28}{182}} \\ &= 97,05788 \end{aligned}$$

The all-in price is R97,05788%.

EL-738

2ndF M-CLR 0 0

*First we calculate the PV
of the 100*

2ndF P/Y 2 ENT ON/C

±100 FV

10.4 I/Y

47 N

COMP PV

9.23124 *is displayed.*

Store in memory for later use

M+

*Secondly we calculate the
PV of the coupon flow.*

2ndF CA

9.6 ÷ 2 = × ± 1 = PMT

2ndF P/Y 2 ENT ON/C

10.4 I/Y

47 N

COMP PV

83.78654 *is displayed.*

Add to memory.

M+

The value at 14/06/14 is

RCL M+

93.01779 *is displayed.*

Add a coupon.

+4.8 =

97.81779 *is displayed.*

*Now use it to get the
all in price*

2ndF CA

2ndF P/Y 2 ENT ON/C

97.81779 ± FV

10.4 I/Y

28 ÷ 182 = N

COMP PV

97.05788 *is displayed.*

HP10BII

 C ALL

*First we calculate the PV
of the 100*

2  P/YR

100 ± FV

10.4 I/YR

47 N

PV

9.23124 *is displayed.*

C

*Secondly we calculate the
PV of the coupon flow.*

9.6 ÷ 2 = ± PMT

10.4 I/YR

47 N

PV

The value at 14/06/14

93.01779 *is displayed.*

Add a coupon.

+4.8 =

97.81779 *is displayed.*

*Now use it to get the
all-in price*

 C ALL

2  P/YR

97.81779 ± FV

10.4 I/YR

28 ÷ 182 = N

PV

97.05788 *is displayed.*







12. As the *payments increase by a fixed amount in fixed time intervals* we can conclude that this is an *increasing annuity*. The formula for the future value of an increasing annuity is

$$S = \left[R + \frac{Q}{i} \right] s_{\overline{n}|i} - \frac{nQ}{i}$$

Now given is the yearly interest rate (i) of 10%; the increase (Q) of R360; the initial payment (R) of R3 600 and the total time (n) of 20 years. We need to determine the future value:

$$\begin{aligned} &= \left[3\,600 + \frac{360}{0,10} \right] s_{\overline{20}|0,10} - \frac{20(360)}{0,10} \\ &= 412379,9964 - 432000 \\ &= 340\,379,99 \\ &\approx 340\,380 \end{aligned}$$

The amount available after 20 years is R340 380

EL-738	HP10BII
<p>2ndF M-CLR 0 0 <i>Calculate the $s_{\overline{n} i}$ part of the formula using Financial mode</i> 2ndF P/Y 1 ENT ON/C 10 I/Y 20 N <i>Calculate the PMT</i> 3 600 + 360 ÷ 0.10 = <i>Make the answer negative and store as PMT</i> × ± 1 = PMT COMP FV 412 379,99...<i>is displayed.</i> <i>Subtract second part of formula</i> -(20 × 360 ÷ 0.10) = 340 379,99... <i>is displayed.</i> <i>Rounded 340 380.00</i></p>	<p> C ALL <i>Calculate the $s_{\overline{n} i}$ part of the formula using Financial mode</i> 1  P/YR 10 I/YR 20 N <i>Calculate the PMT</i> 3 600 +  (360 ÷ 0.10 ) = <i>Make the answer negative and store as PMT</i> ± PMT FV 412 380.00...<i>is displayed.</i> <i>Subtract second part of formula</i> -  (20 × 360 ÷ 0.10 ) = 340 380.00... <i>is displayed.</i></p>

[Option 2]

13. Given the NPV, PI and IRR we need to determine if we want to invest in Investment A or B. Now

Criteria	Investment A	Investment B
NPV	44 000 > 0 Accept	-22 000 < 0 Decline
PI	1,945 > 1 Accept	0,071 < 1 Decline
IRR	16,00 > 12,00 Accept	8,04 < 12,00 Decline


Accept Investment A as the NPV is positive, the PI is greater than 1 and the IRR is greater than the cost of capital. **[Option 1]**

14. The value of A which is the interest due at month end 147 can be calculated as:

Monthly payment at month 147 – principal repaid at month 147

$$\begin{aligned}\text{Value of } A &= 2\,080,54 - 2\,014,27 \\ &= 66,27\end{aligned}$$

The value of A is R66,27.

EL-738	HP10BII
2ndF M-CLR 00 <i>Use normal keys</i> $2\,080.54 - 2\,014.27 =$ <i>66.27... is displayed.</i>	 C ALL <i>Use normal keys</i> $2\,080.54 - 2\,014.27 =$ <i>66.27 ... is displayed.</i>

[Option 3]


15. The value of B which is the outstanding principal at the beginning of month 149 can be calculated as:

outstanding principal at the beginning of month 148 - principal repaid 148

$$\begin{aligned}\text{Value of } B &= 6\,141,56 - 2\,030,64 \\ &= 4\,110,92\end{aligned}$$

The outstanding principal is R4 110,92.

The value of B is R4 110,92.

EL-738	HP10BII
2ndF M-CLR 00 <i>Use normal keys</i> $6\,141.56 - 2\,030.64 =$ <i>4\,110.92 is displayed.</i>	 C ALL <i>Use normal keys</i> $6\,141.56 - 2\,030.64 =$ <i>4\,110.92 is displayed.</i>

[Option 4]

6 WHEN TO USE WHICH FORMULA

Below are guidelines with regard to the identification and application of formulae (i.e. when to use which formula).

6.1 Formulæ

6.1.1 Simple Interest

You go to the bank once and the calculation is done at the end of the investment time.

The words “**simple interest**” must be mentioned in the question.

$$I = Prt$$

Use this formula when you want to determine the interest received/paid.

$$S = P(1 + rt)$$

Use this formula when the accumulated amount is mentioned in the question.

6.1.2 Simple Discount

The words “**simple discount**” or “**discount rate**” must be mentioned in the question.

$$P = S(1 - dt)$$

The **relationship** between **simple interest** and **simple discount** is given as:

$$r = \frac{d}{1 - dt} \quad \text{or} \quad d = \frac{r}{1 + rt}$$

6.1.3 Compound Interest

You go to the bank once (one principle) and the calculations are done according to the number of compounding periods. The word “**compounded**” must be mentioned in the question.

$$S = P \left(1 + \frac{j_m}{m} \right)^{tm} \quad \text{or} \quad S = P(1 + i)^n$$

$\frac{j_m}{m}$ or i is the nominal interest rate per year divided by the number of compounding periods per year.

$tm = n$ where t is the total time expressed as years or a fraction of a year and m the number of compounding periods in one year.

6.1.4 Effective Rate

Refers to the rate that you will, in effect, receive/pay in one year. **Effective interest** rate will be mentioned in the question.

$$J_{eff} = 100 \left(\left(1 + \frac{j_m}{m} \right)^m - 1 \right)$$

6.1.5 Continuous Compounding

When the rate at which compounding takes place tends to be infinity, the calculations of the interest are continuous.

$$c = m \ln \left(1 + \frac{j_m}{m} \right)$$

This formula is used when a nominal interest rate with m compounded periods per year is converted to a continuous compounding rate.

$$S = Pe^{ct}$$

This formula is used when either the principal (present value) and/or the accumulated amount (future value) are given and the interest rate is continuous.

$$J_\infty = 100(e^c - 1)$$

This formula is used when the continuous rate is converted to an effective rate.

NB! Remember when a c is in the formula **continuous compounding** is applicable.

6.1.6 Convert Interest Rates

$$j_n = n \left(\left(1 + \frac{j_m}{m} \right)^{m \div n} - 1 \right)$$

This formula is used when you want to convert a given nominal interest rate j_m compounded m periods per year to n compounding periods per year and still get the same return.

6.1.7 Annuities

You go to the bank with **equal amounts** (payments or deposits) of money at the **same time intervals**. The applicable interest rate is **compounded**. Remember the compound interest rate periods must be the same as the payment periods.

6.1.8 Present Value

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \end{aligned}$$

You use this formula when the balance in the account, at the end of the time, is zero or you are asked to calculate the present value or it is given.

6.1.9 Future Value

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R \left[\frac{(1+i)^n - 1}{i} \right] \end{aligned}$$

You use this formula when, at the end of the time, there is a balance in the account or you are asked to calculate the future value or it is given.

6.1.10 Annuity Due

Payments are made at the beginning of the periods.

$$\begin{aligned} P &= (1+i) Ra_{\overline{n}|i} \\ S &= (1+i) Rs_{\overline{n}|i} \end{aligned}$$

Use this formula when the words “**begin, start immediately or in advance**” are mentioned in the sentence. Multiply the formulae with $(1+i)$ and use n in the formulae as the number of payments made.

6.1.11 Increasing Annuity

$$S = \left(R + \frac{Q}{i} \right) s_{\overline{n}|i} - \frac{nQ}{i}$$

The original payment R is **increased yearly with the same amount Q** .

$$A = nR + Q \frac{n(n-1)}{2}$$

The formula represents the actual amount paid for an increasing annuity where n is the number of payments made.

6.1.12 Perpetuity

$$P = \frac{R}{i}$$

You will receive the payment R **indefinitely**.

6.1.13 Deferred Annuity

With a deferred annuity, you are unable to start to repay your debt immediately. **Your first payment is a number of payment intervals after the end of the first interest period.** First calculate the present value of the payments made and then discount this amount back to now by using compound interest. Do not confuse the word “discount” with “simple discount rate”. “Discount back” is just another term for “moving back”. A deferred annuity can also apply in a future value annuity situation.

6.1.14 Amortisation

Use the present value formula for annuities and determine the payment.

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \\ R &= \frac{P}{\left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]} \end{aligned}$$

6.1.15 Sinking Fund

Use the future value formula for annuities and determine the payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= R \left[\frac{(1+i)^n - 1}{i} \right] \\ R &= \frac{S}{\left[\frac{(1+i)^n - 1}{i} \right]} \end{aligned}$$

6.1.16 Stocks

$$P = da_{\overline{n}|z} + 100(1+z)^{-n}$$

d is the half yearly coupon payments, z the half yearly interest rate and n the number of outstanding coupons.

6.1.17 Net Present Value

$$NPV = PV_{in} - I_{out}$$

6.1.18 Profitability Index

$$PI = \frac{\text{Present value of cash inflows}}{\text{Present value of cash outflows}}$$

OR

$$PI = \frac{NPV + \text{Outlays (initial investment)}}{\text{Outlays (initial investment)}}$$

6.1.19 Handling Of The Data

Use your calculator to do the calculations.

NB: Please read your questions in the examination attentively before answering them.

I hope these guidelines will help you to better understand the application of the formulae (i.e. when to use which formula).

7 LIST OF FORMULÆ

$I = Prt$	$r = \frac{d}{1 - dt}$
$S = P(1 + rt)$	$S = (1 + i)Rs_{\overline{n} i}$
$P = S(1 - dt)$	$P = (1 + i)Ra_{\overline{n} i}$
$S = P \left(1 + \frac{j_m}{m}\right)^{tm}$	$P = da_{\overline{n} z} + 100(1 + z)^{-n}$
$j_{eff} = 100 \left(\left(1 + \frac{j_m}{m}\right)^m - 1 \right)$	$\frac{H - R}{365} \times c$
$S = Pe^{ct}$	$\frac{-R}{365} \times c$
$j_{\infty} = 100(e^c - 1)$	$MIRR = \left(\frac{C}{PV_{out}} \right)^{\frac{1}{n}} - 1$
$c = m \ln \left(1 + \frac{j_m}{m}\right)$	$-P = \frac{R}{i}$
$j_m = m \left(e^{\frac{c}{m}} - 1\right)$	$S = \left[R + \frac{Q}{i}\right] s_{\overline{n} i} - \frac{nQ}{i}$
$j_n = n \left(\left(1 + \frac{j_m}{m}\right)^{\frac{m}{n}} - 1 \right)$	$T_r = Ra_{\overline{n} r}$
$S = R \left(\frac{(1 + i)^n - 1}{i} \right)$	$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
$S = Rs_{\overline{n} i}$	$\bar{x}_w = \frac{\sum_{i=1}^n x_i w_i}{\sum_{i=1}^n w_i}$
$P = Ra_{\overline{n} i}$	$\sum_{i=1}^n i = \frac{n(n+1)}{2}$
$P = R \left(\frac{(1 + i)^n - 1}{i(1 + i)^n} \right)$	$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$
$A = nR + Q \left[\frac{n(n-1)}{2} \right]$	