## INV2601 DISCUSSION G G.A. A SEMESTER 1 <br> INVESTMENTS: AN INTRODUCTION DEPARTMENT OF FINANCE, RISK MANAGEMEN, <br> BANKING

## EXAMINATION

- Duration - 2 hours.
- 40 multiple choice questions.
- Total marks = 40 .
- Tested on study units 1 - 15 (Topic 5 study unit 16, excluded).
- Not provided: interest factor tables and formula sheet.
- I encourage you to create your own formula that you use in your revision for each chapter.
- Examination includes both theory and calculations.


## Mark Composition

|  | Questions | Percentages |
| :--- | :---: | :---: |
| Theory | 17 | 42 |
| Calculations | 23 | 58 |
| Total | $\mathbf{4 0}$ | $\mathbf{1 0 0 \%}$ |


|  |  | Questions |
| :--- | :--- | :---: |
| Topic 1 | The Investment Background | 15 |
| Topic 2 | Equity Analysis | 7 |
| Topic 3 | The Analysis of Bonds | 6 |
| Topic 4 | Portfolio Management | 12 |
| Total |  | $\mathbf{4 0}$ |

## CHAPTER 1

- An investment is:
- a current commitment of money, based on fundamental research
- to real and/or financial assets for a given period
- in order to accumulate wealth over the long term
- The goal of investment management:
- is to find investment returns that satisfy the investor's required rate of return
- Required rate of return - is the return that should compensate the investor for:
- Time value of money during the period of investment
- The expected rate of inflation during the period of investment
- The risk involved


## Required rate of return

- The real risk-free rate of return (RRFR) is the price charged for the exchange between current and future consumption.
- A risk free investment is one which provides the investor with certainty about the amount and timing of expected returns.
- Treasury bills are risk free because government has the unlimited ability to raise revenue from taxes which may be used to service its debt.
- To determine the required rate of return:
- The investor has to determine the nominal risk free rate of return (NRFR)
- Then add risk premium to compensate for the risk associated with the investment
- $\operatorname{NRFR}=[(1+\operatorname{RRFR})(1+\mathrm{EI})-1] \times 100$

Where: $\quad$ RRFR $=$ real rate of return (in decimal form)
$\mathrm{EI}=$ expected inflation (in decimal form)

- $\operatorname{RRFR}=(1+$ NRFR $)-1$
(1 + EI)


## Fundamentals of Investment

- Time value of money - an amount of money can increase in value because of the interest earned from an investment over time.
- Risk vs Return
- Risk is the uncertainty about whether an investment will earn its expected rate of return.
- Measure of risk of a single asset:
- Standard deviation
- Coefficient of variation (CV)
- Return is the sum of the cash dividends, interest and any capital appreciation or loss resulting from the investment.
- Historical return can be calculated using HPR and HPY.
- The risk and return principle:
- The greater the risk, the higher the investor's required rate of return.


## Example - HPY

- The annual holding period yield of an investment that was held for ten years is minus (-) $20 \%$. The beginning value of this investment was R220 500. The ending value is closest to:

$$
\begin{aligned}
\text { Annual } H P Y & =H P R^{1 / N} \\
-0.20 & =H P R^{1 / 10} \\
0.80 & =H P R^{1 / 10} \quad \text { where } 0.80=1-0.20 \\
H P R & =0.80^{10} \\
H P R & =0.1074 \\
H P R & =\frac{\text { Ending value }}{\text { Beginning value }} \\
0.1074 & =\frac{\text { Ending value }}{220500}
\end{aligned}
$$

Ending value $=0.1074 \times 220500$

$$
=R 23676.01
$$

## Example - Coefficient of Variation

- Calculate the Coefficient of Variation (CV) of Green Ltd given the following information.

$$
\begin{array}{|c|c|c|}
\hline \text { Possible outcomes } & \text { Probability(\%) } & \text { Return(\%) } \\
\hline \text { Pessimistic } & 20 & 5 \\
\hline \text { Most Likely } & 30 & 8 \\
\hline \text { Optimistic } & 50 & 10 \\
C V & =\frac{\delta}{E(r)} \\
\begin{aligned}
E(r) & =(0.20 \times 5)+(0.30 \times 8)+(0.50 \times 10) \\
& =8.40 \% \\
\delta & =\sqrt{\left[0.20(5-8.40)^{2}\right]+\left[0.30(8-8.40)^{2}\right]+\left[0.50(10-8.40)^{2}\right]} \\
& =\sqrt{3.64} \\
& =1.9079 \\
C V & =\frac{1.9079}{8.40} \\
& =0.23
\end{aligned}
\end{array}
$$

CHAPTER 2 - ORGANISATION AND FUNCTIONING OF SECURITIES MARKETS

- PRIMARY AND SECONDARY MARKETS
- Primary markets - sells newly issued securities of companies('new issues') and is also involved in initial public offerings(IPOs).
- Secondary market - supports the primary market by:
i) giving investors liquidity, price continuity and depth
ii) providing information about current prices and yields
- Third market - Over The Counter (OTC) trading of listed shares by a broker. This market may be used by investors to trade shares that are either suspended on the exchange or while the exchange is closed.
- Fourth market - direct trading of securities between two parties with no intermediary.


## Type of Transactions

- Market orders - orders to buy or sell securities at the best prevailing price. 'sell at best' or 'buy at best'. Provide liquidity.
- Limit orders - specify the buy or sell price.
- Short sales - the sale of shares the investor does not own with the intention of buying them back at a lower price at a later stage.
- He would have to borrow them from another investor, sell them in the market and subsequently replace them at (hopefully) a price lower than the price at which he sold them.
- Stop loss - conditional market order that directs the trade should the share price decline to a predetermined level.
- Stop buy order - used by short seller who want to minimise any loss if the share increases in value.


## CHAPTER 3-DEVELOPMENTS IN INVESTMENT THEORY

## - Capital Asset Pricing Model - CAPM

- CAPM indicates the return an investor should require from a risky asset assuming that he is exposed only to asset's systematic risk as measured by beta ( $\beta$ ).
- Rationale: For any level of risk, the SML indicates the return that could be earned by using the market portfolio and the risk-free asset.
- Required return: $(\mathrm{k})=\mathrm{rf}+\beta(\mathrm{rm}-\mathrm{rf})$
- Where: rf = risk free rate and $\mathrm{rm}=$ return of the market index
- An investor is not compensated for unsystematic risk because it is diversifiable.
- Systematic risk is measured by beta ( $\beta$ ). It is un-diversifiable because it is caused by factors that affect the entire market.
- Unsystematic risk is diversifiable because it is caused by factors that are unique to the company.
- Systematic risk + Unsystematic risk = Total risk.
- Total risk is measured by the standard deviation.


## Using CAPM to assess an asset

- An investment in an asset can be assessed by means of CAPM to determine whether an asset is over or undervalued.
- Estimated rate of return - is the actual holding period rate of return (HPR) that the investor anticipates.
- Estimated rate of return > required rate of return
- The share is undervalued.
- Estimated rate of return < required rate of return
- The share is overvalued.
- Highly efficient market - all assets should plot on the SML.
- Less efficient market - assets may at times be mispriced due to investors being unaware of all the relevant information.


## Example - Using CAPM to assess an asset

- You believe the share of Brown Stone Ltd is going to rise from R50 to R58 over one year and that you will received a dividend of R2 at end of the year. The beta of Brown Stone Ltd is 0.75 and its standard deviation is $13 \%$. The expected rate of return of the market is $12 \%$ and the risk-free rate of return is $8 \%$. Determine whether you will purchase the share.

$$
\begin{aligned}
& \text { Estimated rate of return }
\end{aligned}=\frac{\text { Ending value }(\text { including cash } \text { flows })}{\text { Beginning value }} \begin{aligned}
H P R & =\frac{58+2}{50} \\
& =(1.20-1) \times 100 \\
& =20 \%
\end{aligned}
$$

Required rate of return $=r f+\beta(r m-r f)$

$$
=8+0.8(12-8)
$$

$$
=11.20 \%
$$

Estimated rate of return $>$ Required rate of return

$$
20 \%>11.20 \%
$$

You would purchase the share as it is undervalued.

## Example - beta coefficient

- The beta coefficient of unit trusts $A$ and $B$ respectively, is:

| Unit trust | Average rate of <br> return (\%) | Variance <br> (\%) | Correlation <br> coefficient with <br> the market index |
| :--- | :--- | :--- | :--- |
| A | 27 | 6.00 | 0.85 |
| B | 15 | 2.00 | 0.55 |
| Market Index | 25 | 4.00 | - |

$\beta=\frac{\operatorname{corr}_{i, m} \times \delta_{i} \times \delta_{m}}{\delta_{m}{ }^{2}}$
$\beta_{A}=\frac{0.85 \times \sqrt{6} \times \sqrt{4}}{4}=1.04$
$\beta_{B}=\frac{0.55 \times \sqrt{2} \times \sqrt{4}}{4}=0.39$

CHAPTER 5: VALUATION PRINCIPLES AND PRACTICES

- Valuation concepts
- Par value - the value at the which a financial asset is originally issued in the primary market. Also known as face value.
- Market value - is determined by the price that is determined in the secondary market.
- Book value:
- Fixed assets = value of fixed assets indicated in the firm's balance sheet.
- Ordinary shares $=($ par value $\times$ no. of shares issued $)+$ cumulative retained earnings + capital contributed in excess of par.
- Intrinsic (fair) value - is determined by calculating the present value of the cash flows expected from an asset.
- Required input variables
- Cash flows (returns) - the value of an asset depends on the cash flows that it is expected to generate during the period it is owned.
- Timing - earlier cash flows are preferred to later cash flows.
- Discount rate - should reflect the risk-return relationship of the asset concerned.


## Two-stage dividend model

- An investor in Imperial Ltd's ordinary share expects it to pay annual cash dividends of R0.50 in year one, R0.90 per share in year two. The dividend is expected to grow at a constant rate of 5\% in future. Imperial Ltd's required rate of return is $10 \%$. Calculate the intrinsic value of the share using the two stage dividend model.

$$
\begin{aligned}
V_{0} & =\frac{D_{1}}{(1+k)^{1}}+\frac{D_{2}}{(1+k)^{2}}+\frac{P_{2}}{(1+k)^{2}} \\
V_{0} & =\frac{0.50}{(1.10)^{1}}+\frac{0.90}{(1.10)^{2}}+\frac{\frac{0.945}{(0.10-0.05)}}{(1.10)^{2}} \\
& =0.4545+0.7438+15.6198
\end{aligned}
$$

$$
=R 16.82
$$

$$
\text { NOTE: } P_{2}=\frac{D_{3}}{(k-g)}
$$

$$
D_{3}=D_{2}(1+g)
$$

$$
=0.90(1.05)
$$

$$
=0.945
$$

## Two-stage dividend model (Alternative calculation)



|  | INPUTS |
| :--- | :--- |
| CF0 | 0 |
| CF1 | 0.50 |
| CF2 | $19.80(18.90+0.90)$ |
| I/YR | $10 \%$ |
| COMP NPV | R16.82 |

## Three-stage dividend model

- Global Corporation has just paid dividends of R1.00 per share. Assume that over the next three years, dividends will grow as follows: 5\% next year, 10\% in year two and $15 \%$ in year 3. After that growth is expected to level off to a constant growth rate of $8 \%$ per year. The required rate of return is $12 \%$. Calculate the intrinsic value using the multistage model.

$$
\begin{aligned}
& D_{0}=1.00 \\
& D_{1}=1(1.05)=1.05 \\
& D_{2}=1.05(1.10)=1.1550 \\
& D_{3}=1.1550(1.15)=1.3283 \\
& D_{4}=1.3283(1.08)=1.4345
\end{aligned}
$$

$$
\begin{aligned}
V_{0} & =\frac{D_{1}}{(1+k)^{1}}+\frac{D_{2}}{(1+k)^{2}}+\frac{D_{3}}{(1+k)^{3}}+\frac{P_{3}}{(1+k)^{3}} \\
V_{0} & =\frac{1.05}{(1.12)^{1}}+\frac{1.1550}{(1.12)^{2}}+\frac{1.3283}{(1.12)^{3}}+\frac{\left[\frac{1.4345}{(0.12-0.08)}\right]}{(1.12)^{3}} \\
& =\frac{1.05}{(1.12)^{1}}+\frac{1.1550}{(1.12)^{2}}+\frac{1.3238}{(1.12)^{3}}+\frac{35.8625}{(1.12)^{3}}
\end{aligned}
$$

Where $\quad P_{3}=\frac{D_{4}}{(k-g)}$

$$
=0.9375+0.9208+0.9455+25.5262
$$

$$
=R 28.33
$$

## Three-stage dividend model (Alternative calculation)

| 0 | 1.05 | 1.1550 | $\begin{array}{r} 35.8625 \\ 1.3283 \end{array}$ | Terminal value (P3) Dividends |
| :---: | :---: | :---: | :---: | :---: |
|  | 1.05 | 1.1550 |  |  |
|  |  |  |  |  |
|  | - |  |  |  |
| 0 | 1 | 2 | 3 | Years |


|  | INPUTS |  |
| :--- | :--- | :--- |
| CF0 | 0 |  |
| CF1 | 1.05 |  |
| CF2 | 1.1550 |  |
| CF3 | 37.1908 | $(35.8625+1.3283)$ |
| I/YR | $12 \%$ |  |
| COMP NPV | R28.33 |  |

## CHAPTER 6: FUNDAMENTAL ANALYSIS



## CHAPTER 9 - COMPANY VALUATION

- Grey Stone Ltd currently retains $40 \%$ of its earnings which are R5.50 a share this year. It earns a ROE of $25 \%$. Assuming a required rate of return of $16 \%$, how much would you pay for Grey Stone Ltd on the basis of the earning multiplier model?

| Required rate of return (k) | 16\% |
| :---: | :---: |
| ROE | 25\% |
| Retention rate (RR) | 40\% |
| Earnings per share (EPS) | R5.50 |
| Growth rate (g) $=$ ROE $\times \mathrm{RR}$ |  |
| $=25 \% \times 0.40$ |  |
| = 10\% |  |

## Example - Company Valuation

$$
\begin{aligned}
P_{0} / E_{1} & =\frac{D / E}{(k-g)} \\
& =\frac{(1-0.40)}{(0.16-0.10} \\
& =\frac{0.60}{0.06} \\
& =10.00 \times \\
E_{1} & =E_{0}(1+g) \\
& =5.50(1.10) \\
& =R 6.05 \\
P_{0} & =P_{0} / E_{1} \times E_{1} \\
& =10.00 \times 6.05 \\
& =R 60.50
\end{aligned}
$$

Note: Dividend payout $(D / E)=1-\operatorname{Retention} \operatorname{Rate}(R R)$

## CHAPTER 11: BOND FUNDAMENTALS

- Bonds are issued in the capital market (financial market for long term debt obligations and equity securities).
- Bonds provide an alternative to direct lending as a source of funding.
- Basics of bonds:
- Principal value/Face value/Par value (FV)
- Coupon rate (PMT)
- Term to maturity (N)
- Market value (PV)
- Yield to maturity (I/YR)


## Bond Fundamentals

- Interest rate risk - effect of changes in the prevailing market rate on the return on a bond (price risk and reinvestment risk).
- Price risk - arises when a bond is sold before maturity.
- Reinvestment risk - arises from the market rate being different from the yield to maturity.
- Credit risk - risk that creditworthiness of a bond issuer will deteriorate. It is sub-divided into the following:
- Default risk - possibility that issuer will fail to meet its obligations regarding timely payment of coupons and principal
- Credit spread risk - risk that the credit spread will increase
- Downgrade risk - risk that a rating agency assigns a lower rating to a bond causing a rise in yield and drop in price
- Yield curve risk - arises from a non parallel shift in the yield curve.
- Liquidity risk - risk of having to sell a bond at a price below fair value due to lack of liquidity.


## Bond Fundamentals

- Call risk:
- Applies to callable bonds
- It is the risk that the bond is eventually called from the holder by the issuer when the market rate falls
- Call protection reduces call risk
- Non-callable bonds have no call risk


## ALTERNATIVE BOND STRUCTURES

- Coupon bonds
- Zero-coupon bonds
- Bonds with embedded options:
- Call provision
- Put provision
- Sinking fund provision - principal amount is repaid periodically
- Floating rate notes


## CHAPTER 12: VALUATION OF BONDS

| Relation | Effect | Issue |
| :--- | :--- | :--- |
| Coupon rate < Discount <br> rate | Bond price < Principal <br> value | Discount bond |
| Coupon rate $>$ <br> rate | Bond price > Principal <br> value | Premium bond |
| Coupon rate $=$ Discount <br> rate | Bond price $=$ Principal <br> value | Par value bond |

## Yield measures

- Nominal yield - coupon rate of bond.
- Current yield - only considers a bond's annual interest income ignoring any capital gains/losses, or reinvestment income.
- Yield to maturity - annualised rate of return based on bond's price, coupon payments and par value.
- Yield to call:
- A provision that gives the bond issuer the right to call the bond at a predetermined price that is at/above par
- Has a higher return than an identical non-callable bond
- Advantageous to the issuer
- Bond is called when interest rates have dropped significantly
- Yield to put:
- Advantageous to the holder forcing the issuer to repurchase the bond prior to maturity at a predetermined price
- Arises when prevailing interest rate have risen significantly
- Holder reinvest (new issue) at a higher rate (lower price)
- Realised yield - takes into account of the expected rate of return during the investment.
- Spot and forward rates - the appropriate discount rates for cash flows at different points in time.


## Example - Realised yield

- Assume that you purchase a 3-year R1 000 par value bond, with an 15\% coupon, and a yield to maturity of 12\%. After you purchase the bond, one year interest rates are as follows (these are the reinvestment rates)

| Year 1 | $9 \%$ |
| :--- | :--- |
| Year 2 | $7 \%$ |
| Year 3 | $4 \%$ |

- Calculate the realised compound or horizon yield, if you hold the bond to maturity. Interest is paid annually.
- Determine the market price of the bond (If it is not given to you in the question).

| HP 10BII |  |
| :--- | :--- |
| Input | Function |
| 1000 | FV |
| 150 | PMT |
| 3 | N |
| 12 | I/YR |
|  | PV (Market price) |
|  | R1 072.05 |

## Example - Realised yield



Step 1: Calculate the future value of the coupon payments reinvested.

$$
\begin{aligned}
& =150(1.07)(1.04)+150(1.04)+150 \\
& =166.92+156+150 \\
& =R 472.92
\end{aligned}
$$

Step 2: Add the face value of the bond to the future value of the coupon payments.
= R1 000 + R472.92
= R1 472.92

## Example - Realised yield

Step 3: Calculate the actual yield received.

| HP 10BII |  |
| :--- | :--- |
| Input | Function |
| R1 472.92 | FV |
| -R1 072.05 | PV |
| 3 | N |
|  | I/YR |
|  | 11.17\% |

## Example - Spot Rates

- Calculate the equivalent 6-month spot rate, 12-month spot rate and 18month spot rate using the bootstrapping method. All bonds have a face value of R100 and semi-annual coupon payments.

| Bonds | Maturity (months) | Annual coupon | Price | Yield to maturity |
| :--- | :---: | :---: | :--- | :--- |
| M | 6 | $5 \%$ | R100.49 | $4 \%$ |
| N | 12 | $10 \%$ | R101.89 | $8 \%$ |
| O | 18 | $15 \%$ | R104.01 | $12 \%$ |

- 6-month spot rate:

$$
\begin{aligned}
\frac{102.50}{(1+x)^{1}} & =100.49 \\
\frac{102.50}{100.49} & =1+x \\
1.02 & =1+x \\
x & =(1.02-1) \times 100 \times 2 \\
& =4.00 \%
\end{aligned}
$$

- NB: If the annual coupon = yield to maturity, the 6-month spot rate will be equal to the yield to maturity.


## Example - Spot Rates

- 12-month spot rate:

$$
\begin{aligned}
\frac{5}{(1.02)^{1}}+\frac{105}{(1+x)^{2}} & =101.89 \\
4.9020+\frac{105}{(1+x)^{2}} & =101.89 \\
\frac{105}{(1+x)^{2}} & =101.89-4.9020 \\
\frac{105}{(1+x)^{2}} & =96.9880 \\
\frac{105}{96.9880} & =(1+x)^{2} \\
1.0826 & =(1+x)^{2} \\
x & =\left(1.0826^{1 / 2}-1\right) \times 100 \times 2 \\
& =(1.0405-1) \times 100 \times 2 \\
& =8.10 \%
\end{aligned}
$$

## Example - Spot Rates

- 18-month spot rate:

$$
\begin{aligned}
\frac{7.50}{(1.02)^{1}}+\frac{7.50}{(1.0405)^{2}}+\frac{107.50}{(1+x)^{3}} & =104.01 \\
7.3529+6.9275+\frac{107.50}{(1+x)^{3}} & =104.01 \\
\frac{107.50}{(1+x)^{3}} & =104.01-7.3529-6.9275 \\
\frac{107.50}{(1+x)^{3}} & =89.7296 \\
\frac{107.50}{89.7296} & =(1+x)^{3} \\
1.1980 & =(1+x)^{3} \\
x & =\left(1.1980^{1 / 3}-1\right) \times 100 \times 2 \\
& =(1.0621-1) \times 100 \times 2 \\
& =12.42 \%
\end{aligned}
$$

## Measurement of Interest Rate Risk

- Interest rate risk is the risk that changing market rates will impact negatively on the return of a bond.
- Duration-convexity approach to measuring interest rate risk or price sensitivity provides an approximation of the actual interest rate sensitivity.
- Duration allows for managing the price sensitivity of a bond portfolio:
- Declining interest rate environment - lengthen duration to take full advantage of the increase in the value through an increased interest rate sensitivity.
- Increasing interest rate environment - shorten duration so as to limit the decline in bond value.


## Duration

- Properties of duration:
- Duration of a zero coupon bond will equal its term to maturity
- Duration of a coupon bond will always be less than its term to maturity
- Positive relationship between term to maturity and duration
- Inverse relationship between coupon and duration
- Inverse relationship between yield to maturity and duration
- Calculation of duration
- Macaulay duration - sums the weighted discounted cash flows to arrive at a basic duration value.
- Modified duration - discount the Macaulay duration at the yield to maturity.
- Effective duration - straight forward way to calculate duration. It is equal to modified duration for an option free bond.


## Example - Effective duration

- A 15 year, 8\% coupon bond pays interest semi-annually and sells for R846.28. Calculate the effective duration of this bond if the yield to maturity changes by 150 basis points.

|  | V- | Vo |  | V+ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| FV | 1000 | 1000 |  | 1000 |  |
| PMT | 40 | 40 | $(80 \div 2)$ | 40 |  |
| N | 30 |  | 30 | $(15 \times 2)$ | 30 |
| I/YR | 4.25 | $[(10-1.50) \div 2]$ | 5 | $(10 \div 2)$ | 5.75 |
| PV | 958.05 |  | $\mathbf{8 4 6 . 2 8}$ | $\mathbf{7 5 2 . 5 3}$ |  |

$$
\begin{aligned}
\text { Effective duration } & =\frac{V_{-}-V_{+}}{2 V_{0}(\Delta y / 100)} \\
& =\frac{958.05-752.53}{2 \times 846.28 \times(1.50 / 100)} \\
& =\frac{205.52}{25.3884} \\
& =8.10
\end{aligned}
$$

## Example - Duration

- A $6 \%$ coupon bond pays interest semi-annually, has a modified duration of 10 , sells for R800, and is priced at a yield to maturity (YTM) of $8 \%$. If the market rate increases to $9 \%$, the estimated change in price, using the duration concept, is:
Modified duration = effective duration ( D ) $=10$
Change in yield $(\Delta y)=9-8=1 \%=0.01$
Duration effect: $\quad \% \Delta \mathrm{PD}=-\mathrm{D}(\Delta \mathrm{y})$

$$
=-10(0.01)=-0.10
$$

Estimating prices with duration: $\mathrm{PD}(+1)=\mathrm{V} 0 \times(1-\% \Delta \mathrm{PD})$ = R800 ( 1 - 0.10)
= R720
Estimated change in price $=$ R720 - R800
= -R80

## Convexity

- Duration ignores the curvature of the price-yield relationship:
- It is a poor approximation of price sensitivity to larger yield changes
- Increases in price are underestimated
- Decreases in price are overestimated
- Convexity adjustment accounting for the convex shape of the price-yield curve improves the accuracy of the duration measure.
- If you have two bonds which equal duration but bond A had a higher convexity than bond $B$. You will prefer bond $A$ because :
- It has a better price performance when yields fall (greater price increase) and also when yields rise (smaller decrease in price).


## Example - Convexity

- A 20-year, 14\% semi-annual coupon bond (R1 000 par value) is priced at a yield to maturity (YTM) of $12 \%$. Determine :

1. The convexity adjustment with a 75 basis point decrease in yield
2. The change in price due to convexity

Effective convexity $=(\mathrm{V}-)+(\mathrm{V}+)-2 \mathrm{~V}_{\mathbf{0}}$
$2 \mathrm{~V}_{0}(\mathrm{y} / 100)^{2}$

|  | V- | Vo | V+ |  |
| :--- | :--- | :--- | :--- | :--- |
| FV | 1000 | 1000 | 1000 |  |
| PMT | 70 | 70 | $(140 \div 2)$ | 70 |
| N | 40 | 40 | $(20 \times 2)$ | 40 |
| I/YR | $5.625[(12-0.75) \div 2)$ | 6 | $(12 \div 2)$ | $6.375[(12+0.75) \div 2]$ |
| PV | $\mathbf{1 2 1 7 . 0 6}$ | 1150.46 | $\mathbf{1 0 8 9 . 7 6}$ |  |

## Example - Convexity

$$
\text { Effective convexity } \begin{aligned}
& =\frac{V_{-}+V_{+}-2 V_{0}}{2 V_{0}(\Delta y / 100)^{2}} \\
& =\frac{1217.06+1089.76-(2 \times 1150.46)}{2 \times 1150.46 \times(0.75 / 100)^{2}} \\
& =\frac{5.90}{0.1294} \\
& =45.60 \\
\Delta P & =V_{0} \times \text { convexity } \times(\Delta y / 100)^{2} \\
& =1150.46 \times 45.60 \times(0.75 / 100)^{2} \\
& =R 2.95
\end{aligned}
$$

## CHAPTER 13: DERIVATIVE INSTRUMENTS

- Major categories of derivatives
- Forwards:
- Agreement between two parties in which one party the buyer agrees to buy from the other party, the seller, an underlying asset at a future date at a price established today
- The contract is customized - (privately traded on an over the counter (OTC) market
- Risk of default by either party is high
- Futures:
- Agreement between two parties in which the buyer agrees to buy from the seller, an underlying asset at a future date at a price established today
- Public traded on a futures stock exchange
- Standardized transaction


## Derivative Instruments

- Options:
- Call option: the right to buy a specific amount of a given share at a specified price (strike price) during a specified period of time.
- Provided the market price (S) exceeds the call strike $(X)$ before or at expiration. NB: S > X
- Put option: the right to sell a specific amount of a given share at a specified price (strike price) during a specified period of time.
- Provided the put strike price $(X)$ exceeds the market price $(S)$ before or at expiration. NB: $\mathrm{X}>\mathrm{S}$
- Swaps:
- An agreement between two parties to exchange a series of future cash flows.
- A variation of a forward contract; equivalent to a series of forward contracts.


## Arbitrage Opportunity

- Any deviation from the theoretical or fair value as calculated may lead to a specific arbitrage strategy to exploit and profit from this discrepancy.
- Principle: Buy low and sell high
- Cash and carry arbitrage ( $\mathrm{P}>\mathrm{F}$ ):
- Applicable when the market price $(P)$ is greater than the theoretical price (F)
- Sell the futures contract at the quoted market price
- Borrow money at the risk-free rate for the period until expiry
- Buy the underlying at the spot price
- Reverse cash and carry arbitrage ( $\mathbf{F}>\mathrm{P}$ ):
- Applicable when the theoretical price ( $F$ ) is greater than the market price (P)
- Buy the futures contract at the quoted market price
- Sell the underlying at the spot price
- Invest or lend the money at the risk-free rate for the period until expiny


## Buying or selling a call option

- Call holder(buyer) can exercise his right to purchase the underlying should the spot price exceed the strike price ( $\mathrm{S}>\mathrm{X}$ ).
- When $S>X$, the call option has an intrinsic value (in-the-money).
- [c = max(0; S - X)]
- Profit potential:
- Call holder is unlimited
- Call writer is limited to the premium received (p)
- Potential loss:
- Call holder is the premium paid (p)
- Call writer is unlimited


## Buying or selling a put option

- The put holder can exercise his right to sell the underlying should the strike price exceed the spot price ( $\mathrm{X}>\mathrm{S}$ ).
- When $X>S$, the put option has an intrinsic value(in-the-money).
- [ $\mathrm{p}=\max (0 ; \mathrm{X}-\mathrm{S})$ ]
- Potential profit:
- Put holder is limited to the breakeven value (X-p)
- Put writer is limited to the premium received ( $p$ )
- Potential loss:
- Put holder is premium paid (p)
- Put writer is the breakeven value ( $\mathrm{X}-\mathrm{p}$ )


## CHAPTER 14

- General portfolio construction

| Probability of <br> occurrence | Rate of Return <br> Security $\mathbf{P}$ | Rate of Return - <br> Security Q |
| :---: | :---: | :---: |
| $20 \%$ | $20 \%$ | $14 \%$ |
| $35 \%$ | $15 \%$ | $10 \%$ |
| $45 \%$ | $10 \%$ | $6 \%$ |

- Calculate the following:

1. The standard deviation of both securities.
2. The correlation coefficient between the two assets.
3. The portfolio risk, if $40 \%$ of the portfolio is invested in $P$ and $60 \%$ in $Q$.

## General Portfolio Construction

- Standard of deviation for both assets:

$$
\begin{aligned}
E_{P} & =(0.20 \times 20)+(0.35 \times 15)+(0.45 \times 10) \\
& =13.75 \% \\
E_{Q} & =(0.20 \times 14)+(0.35 \times 10)+(0.45 \times 6) \\
& =9.00 \% \\
\delta_{P} & =\sqrt{0.20(20-13.75)^{2}+0.35(15-13.75)^{2}+0.45(10-13.75)^{2}} \\
& =\sqrt{7.8125+0.5469+6.3281} \\
& =\sqrt{14.6875} \\
& =3.83 \% \\
\delta_{Q} & =\sqrt{0.20(14-9)^{2}+0.35(10-9)^{2}+0.45(6-9)^{2}} \\
& =\sqrt{5+0.35+4.05} \\
& =\sqrt{9.40} \\
& =3.07 \%
\end{aligned}
$$

## General portfolio construction

- Correlation between both assets:

$$
\begin{aligned}
\operatorname{Correlation}\left(r_{P . Q}\right)= & \frac{\text { Covariance }_{P . Q}}{\delta_{P} \times \delta_{Q}} \\
\text { Covariance }_{P . Q}= & {[0.20(20-13.75)(14-9)]+[(0.35(15-13.75)(10-9)]} \\
& +[0.45(10-13.75)(6-9) \\
= & 6.25+0.4375+5.0625 \\
= & 11.75
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{Correlation}\left(r_{P, Q}\right) & =\frac{11.75}{3.83 \times 3.07} \\
& =0.99
\end{aligned}
$$

## General portfolio construction

- Portfolio standard deviation $\left(\delta_{\mathrm{p}}\right)$ :

$$
\delta_{P}=\sqrt{\left[W_{P}^{2} \times \delta_{P}^{2}\right]+\left[W_{Q}^{2}+\delta_{Q}{ }^{2}\right]+\left[2 \times W_{P} \times W_{Q} \times r_{P, Q} \times \delta_{P} \times \delta_{Q}\right.}
$$

Where: $W_{P}=0.40 \quad W_{Q}=0.60 \quad r_{P, Q}=0.99 \quad \delta_{P}=3.83 \% \quad \delta_{Q}=3.07 \%$

$$
\begin{aligned}
\delta_{P} & =\sqrt{\left[0.40^{2} \times 3.83^{2}\right]+\left[0.60^{2} \times 3.07^{2}\right]+[2 \times 0.40 \times 0.60 \times 0.99 \times 3.83 \times 3.07]} \\
& =\sqrt{2.347+3.393+5.5874} \\
& =\sqrt{11.3274} \\
& =3.37 \%
\end{aligned}
$$

## CHAPTER 15: EVALUATION OF PORTFOLIO MANAGEMENT

| Unit trust | Average rate of <br> return | Variance | Beta |
| :--- | :--- | :--- | :--- |
| New Mutual | 14 | 1.80 | 0.40 |
| Invest | 25 | 3.90 | 0.80 |
| Grand Merchant | 32 | 5.26 | 1.20 |
| Total Market Index | 12 | 1.50 |  |

Assume the risk free rate of return is $8 \%$.

1. Evaluate the performance of New Mutual unit trust according to the method of Treynor
2. Evaluate the performance of Invest unit trust according to the method of Sharpe
3. The performance of Grand Merchant unit trust according to the method of Jensen

## Performance measurement

$$
\begin{aligned}
& \text { Treynor }_{\text {New Mutual }}=\frac{r_{p}-r_{f}}{\beta} \\
&=\frac{14-8}{0.40} \\
&=15.00 \\
& \text { Sharpe }_{\text {Invest }}=\frac{r_{p}-r_{f}}{\delta_{p}} \\
&=\frac{25-8}{\sqrt{3.90}} \\
&=8.61 \\
&{\text { Iensen's alpha }(\propto)_{\text {Grand Merchant }}}=r_{p}-\left[r_{f}+\beta\left(r_{m}-r_{f}\right)\right] \\
&=32-[8+1.20(12-8)] \\
&=32-12.80 \\
&=19.20 \%
\end{aligned}
$$

## BEST OF WISHES IN YOUR EXAMS!

