UNIVERSITY OF SOUTH AFRICA

UNISA
university
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Vision

Towards ***the*** African university in the service of humanity

College of Economic and Management Sciences

Department of Finance & Risk Management & Banking

UNISA

college of economic and management sciences

General information

- Exam is 2 hours with 40 MCQ's
- Formula sheet will NOT be provided. *(Some of the formulas like Black Scholes and Merton, and FRAs formulas will be provided as part of the exam paper)*
- Formulate the LOS into questions to test your knowledge of the Subject Unit
- Examination includes both theory and calculations
- Mark composition:

	Questions	**Percentages**
Theory	14	35
Calculations	26	65
Total	**40**	**100%**

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INVESTMENTS: DERIVATIVES

CHAPTER 1

FORWARD MARKETS AND CONTRACTS

- Read
- Basic concepts and terminology

What is a:

- Forward contract
- Futures contract
- Swap
- Option
  - Call option
  - Put option

Forward commitment

Holder has a right but does not impose an obligation

Forward Contracts	Contingent Claim
No premium paid at inception	Premium Paid at inception

Question : *What is the advantage of a contingent claims over forward commitments ?*

Answer: Permit gain while protecting against losses……Why is it so?

If a risk free rate of interest is 7% and an investor enters into a transaction that has no risk, what would be the rate of return the investor should earn in the absence of the risk

A. 0%

B. between 0% and 7%

C. 7%

D. Less than 7%

The spot price of Gold is R930 per ounce and the risk free-rate of interest is 5% per annum. Calculate the equilibrium 6-month forward price per ounce of gold.

$$930 \times (1 + (0.05/2)) = R953.25$$

Why divide by 2… (6-months i.e. half a year)

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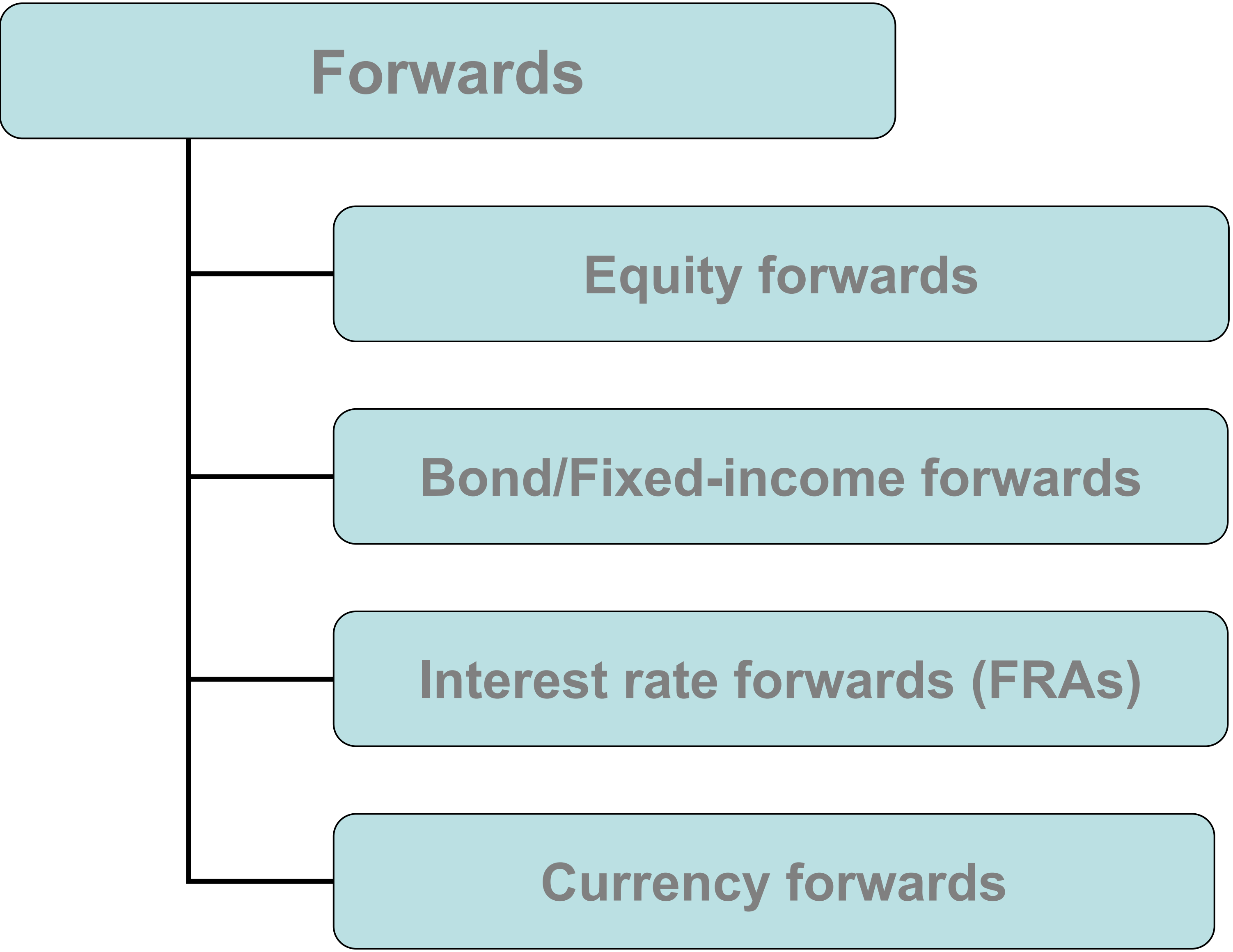
INVESTMENTS: DERIVATIVES

CHAPTER 2

FORWARD MARKETS AND CONTRACTS

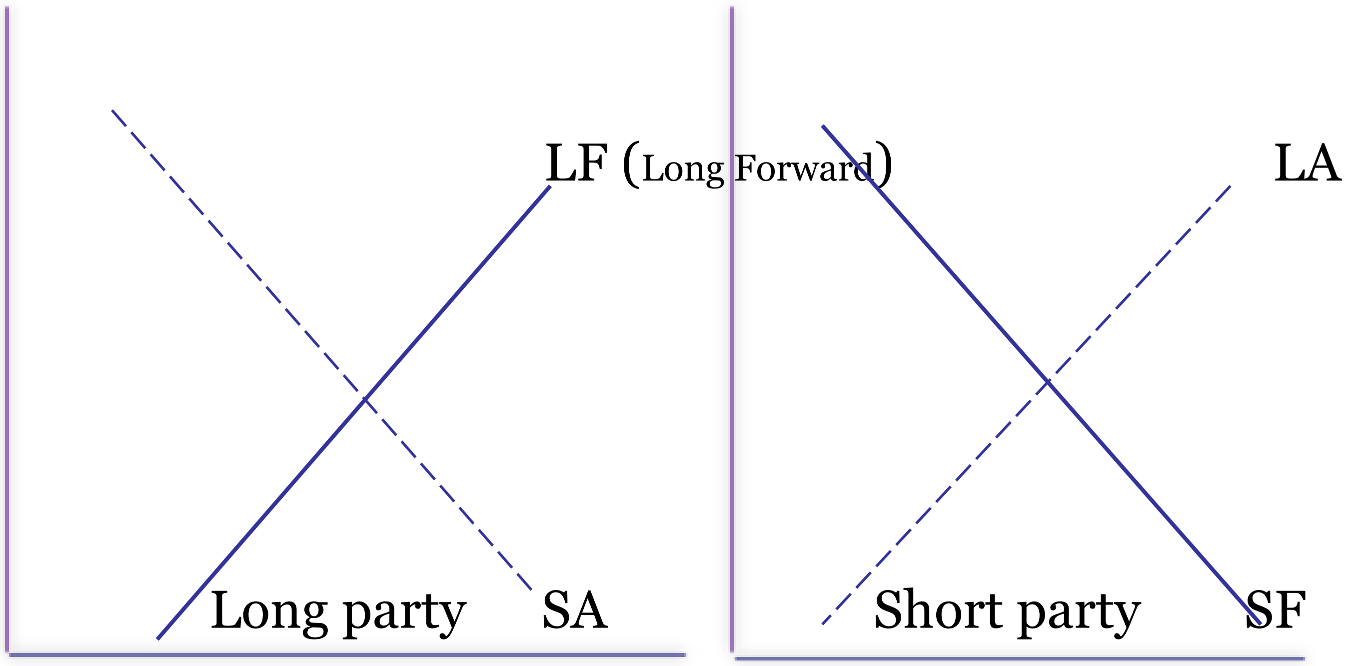
Definition

A **forward contract is an agreement between** two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established today. The contract is customised and each party is subject to the possibility that the other party will default.


Forwards
Equity forwards
Bond/Fixed-income forwards
Interest rate forwards (FRAs)
Currency forwards

Forwards	Futures
Over the counter	Futures exchange
Private	Public
Customized	Standardized
Default risk	Default free
Not marked to market	**Marked to market**
Held until expiration	Offset possible
Not liquid	Liquid
Unregulated	Regulated

Differentiate between the positions held by the long and short parties to a forward contract



- Party that agrees to buy the asset has a long forward position
- Party that agrees to sell the asset has a short forward position

Pricing and valuation of forward contracts

Are pricing and valuation not the same thing?

- The price is agreed on the initiation date (Forward price or forward rate) i.e. pricing means to determining the forward price or forward rate.

- Valuation, however, means to determine the amount of money that one would need to pay or would expect to receive to engage in the transaction

Pricing and valuation of forward contracts cont…

$F(0,T) = S0(1+r)^T$

The transaction is risk-free and should equivalent to investing **S0** *Rands* in risk free asset

Pricing and valuation of forward contracts cont…

$Vo(0,T) = S_0 - F(0,T)/(1+r)^T$

For forward contract Vo(0,T) should be ZERO (0)

If **Vo(0,T) ≠ 0** arbitrage would the prevail

The forward price that eliminates arbitrage:

$F(0,T) = S0(1+r)^T$

Pricing and valuation of forward contracts cont…

By definition an asset's value is the present value of future value thus,

$$V_t(0,T) = S_t - F(0,T)/(1+r)^{(T-t)}$$

(T-t) is the remaining time to maturity

Pricing and valuation of forward contracts cont…

$F(0,T) = (S0 - PV(D,0,T)) * (1+r)$^T

$PV(D,0,T) = \sum (Di/(1+r)^{(T-ti)})$

When dividends are paid continuously

$F(0,T) = S_0 e^{(-\partial c * t)} \cdot e^{(rc * t)}$

To convert discrete risk-free interest(r) to continuosly compounded equivalent(r_c):

$r_c = \text{Ln}(1+r)$

Pricing and valuation of forward contracts cont…

A portfolio manager expects to purchase a portfolio of stocks in 60 days. In order to hedge against a potential price increase over the next 60 days, she decides to take a long position on a 60-day forward contract on the S&P 500 stock index. The index is currently at 1150. The continuously compounded dividend yield is 1.85 percent. The discrete risk-free rate is 4.35 percent.

Calculate the no-arbitrage forward price on this contract, the value of the forward contract 28 days into the contract (index value 1225), and the value of the contract at expiration (index value 1235).

Decrease the spot index value by the dividend yield and thereafter calculate the future value (first convert the discrete rate to a continuously compounded rate).

$$F(0,T) = \left(1{,}150e^{-0.0185(60/365)}\right)e^{LN(1.0435)\times(60/365)} = \$1{,}154.56$$

The value of a contract is the difference between the discounted current spot price (at the dividend yield) and the discounted forward price (at the converted risk-free rate) for the remaining period.

$$V_t(0,T) = 1{,}225e^{-0.0185(32/365)} - 1{,}154.56e^{-LN(1.0435)\times(32/365)}$$

$$= 1{,}223.00 - 1{,}150.26$$

$$= \$72.76$$

At expiration, the value is simply the difference between the end-period spot index and the forward contract price, as calculated.

$$V_T(0,T) = 1{,}235 - 1{,}154.56 = \$80.44$$

Fixed-Income and interest rate forward contracts

Identify the characteristics of forward rate agreements

- Forward contract to borrow/lend money at a certain rate at some future date

Long position

- Borrows money (pays interest)
- Benefit when forward rate < market rate

Short position

– Lends money (receives interest)

– Benefit when forward rate > market rate

Calculate and interpret the payment at expiration of a FRA and identify each of the component terms

$$FRA_{payoff} = NP\left(\frac{(U_{rate} - FRA_{rate}) \times (\frac{U_{days}}{360})}{1 + U_{rate} \times (\frac{U_{days}}{360})}\right)$$

- ESKOM P/L is expecting to receive a cash inflow of R20, 000,000.00 in 90 days. Short term interest rates are expected to fall during the next 90 days. In order to hedge against this risk, the company decides to use an FRA that expires in 90 days and is based on 90day LIBOR. The FRA is quoted at 6%. At expiration LIBOR is 5%. Indicate whether the company should take a long or short position to hedge interest rate risk. Using the appropriate terminology, identify the type of FRA used here. Calculate the gain or loss to ESKOM P/L as a consequence of entering the FRA.

$$R20{,}000{,}000.00 \times \left(\frac{(0.05 - 0.06) \times (\frac{90}{360})}{1 + 0.05 \times (\frac{90}{360})} \right) = -49{,}382.72$$

- **Identify the characteristics of currency forwards**

- Exchange of currencies
- Exchange rate specified
- Manage foreign exchange risk

– Domestic risk-free rate
– Foreign risk free rate
– Interest rate parity (IRP)
– Covered interest arbitrage

Determine the price of a forward contract

- Initial or delivery price

$$F_T = S_0(1+r)^T = K$$

- Forward price during period

$$F_T = S_t(1+r)^{(T-t)}$$

Determine the value of a forward contract at initiation, during the life of the contract, and at expiration

$$Vo = So - \left(\frac{K}{(1+r)^T}\right) = 0$$

$$Vt = St - \left(\frac{K}{(1+r)^{T-t}}\right) = 0$$

alternatively

$$Vt = \frac{FT - K}{(1+r)^{T-t}} = 0$$

Calculate the price and value of a forward contract on a currency

- **Price – currency forward**

Discrete interest

$$f_0(T) = S_0\left(\frac{(1+r_d)^T}{(1+r_f)^T}\right)$$

Continuous interest

$$f_0(T) = S_0 e^{({r_d}^c - {r_f}^c)T}$$

Value – currency forward

Discrete interest

$$Vt = \frac{St}{(1+r_f)^{T-t}} - \frac{FT}{(1+r_d)^{T-t}}$$

$$V_t = St\left[e^{-r_f^c(T-t)}\right] - FT\left[e^{-r_d^c(T-t)}\right]$$

- **Covered interest arbitrage**

$$(1+rd)^T = (1+rf)^T\left(\frac{F}{S}\right)$$

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INVESTMENTS: DERIVATIVES

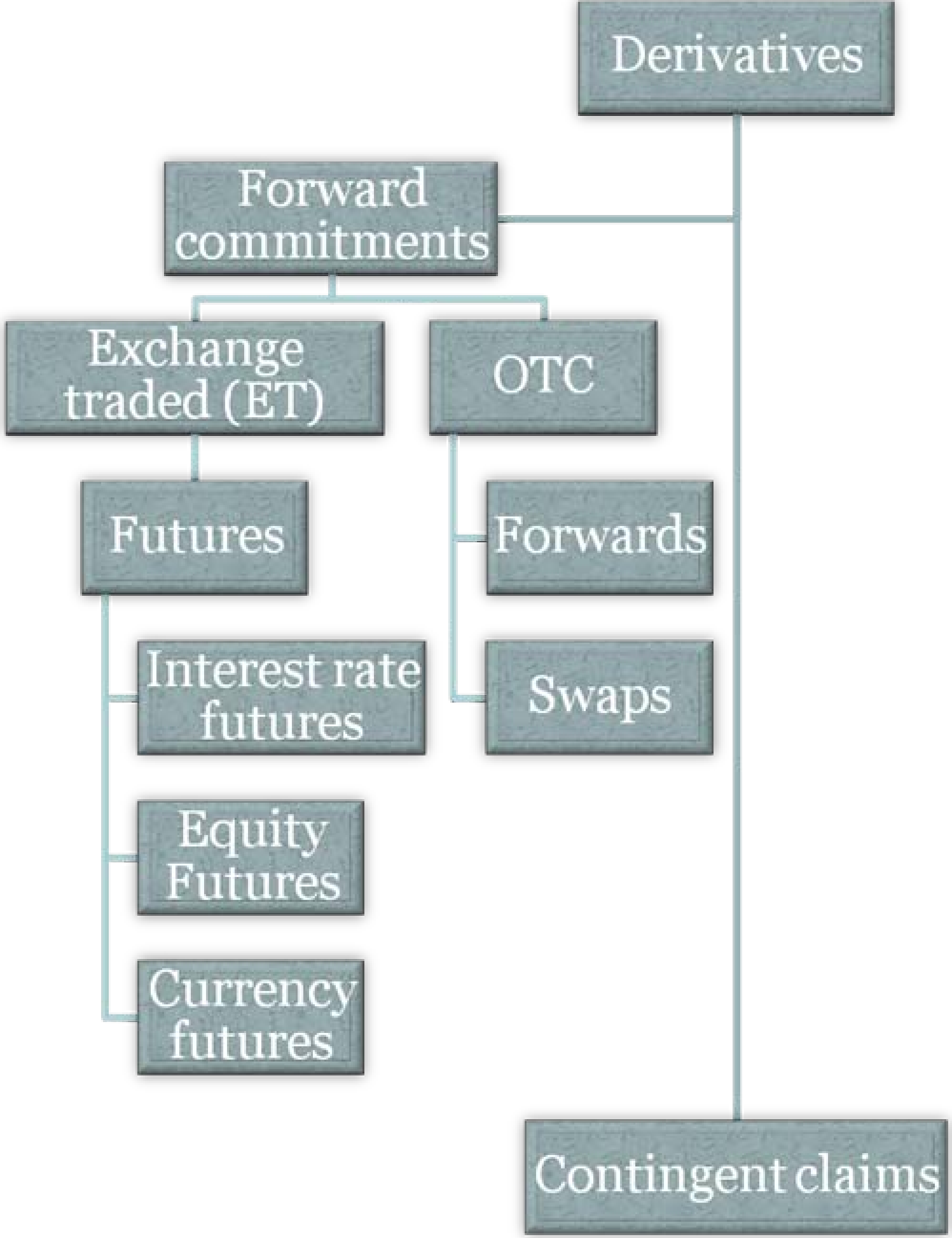
CHAPTER 3

FUTURES MARKETS AND CONTRACTS

Definition

A **forward contract** is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset at a future date at a price established today. The contract is customized and each party is subject to the possibility that the other party will default.

A **futures contract** is a variation of a forward contract that has essentially the same basic definition, but some clearly distinguishable additional features, the most important being that it is not a private and customized transaction. It is a public, standardized transaction that takes place on a futures exchange.


Derivatives
Forward commitments
Exchange traded (ET)
OTC
Futures
Forwards
Swaps
Interest rate futures
Equity Futures
Currency futures
Contingent claims

Identify the primary characteristics of futures contracts and distinguish between futures and forwards

Forwards	Futures
Over the counter	Futures exchange
Private	Public
Customized	Standardized
Default risk	Default free
Not marked to market	**Marked to market**
Held until expiration	Offset possible
Not liquid	Liquid
Unregulated	Regulated

Describe how a futures contract can be terminated at or prior to expiration by close out, delivery, equivalent cash settlement, or exchange for physicals

Close out (prior to expiration)

– Opposite (offsetting) transaction

Delivery

– Close out before expiration or take delivery

– Short delivers underlying to long (certain date and location)

Cash settlement

– No need to close out (leave position open)

– Marked to market (final gain/loss)

Exchange for physicals

– Counterparties arrange alternative delivery

Explain why the futures price must converge to the spot price at expiration

To prevent arbitrage: $f_t(T) = S_T$

Spot price (S) current price for immediate delivery

Futures price (f) current price for future delivery

At expiration f becomes the current price for immediate delivery (S)

If : $f_T(T) < S_T$

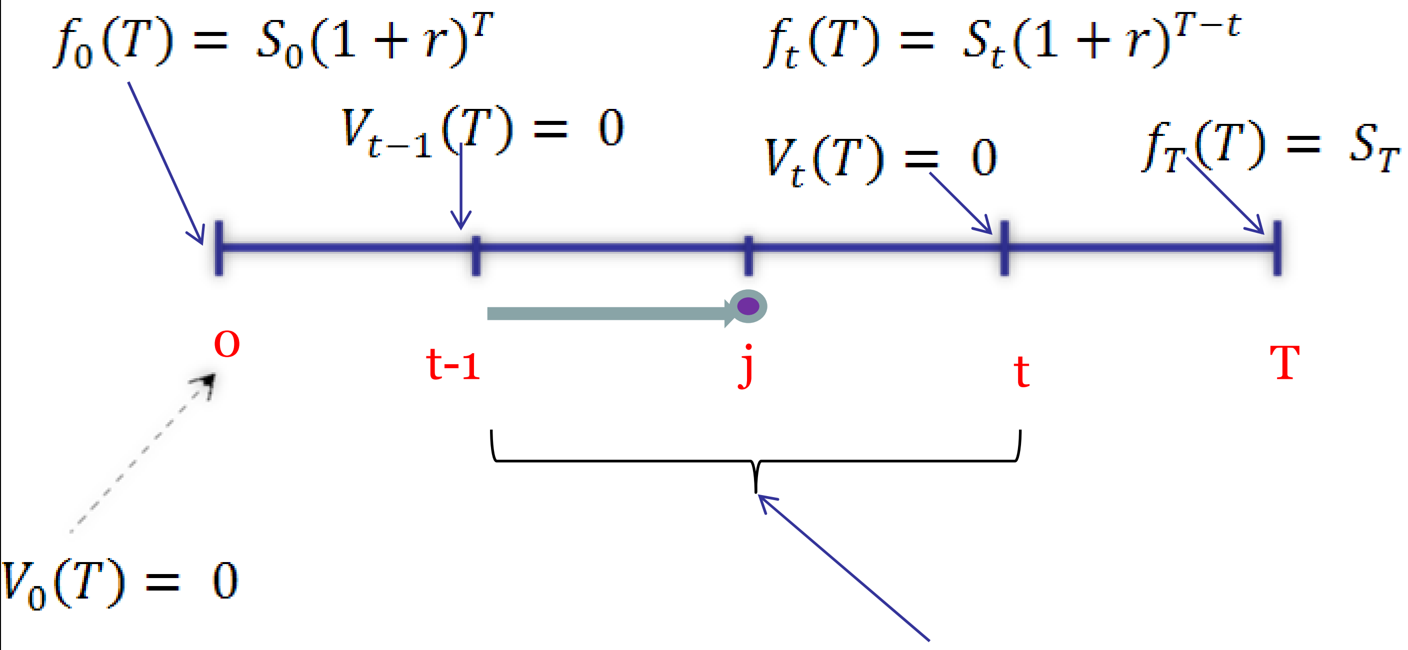
Buy contract; take delivery of underlying and pay lower futures price

If : $f_T(T) > S_T$

Sell contract; buy underlying, deliver and receive higher futures price

Determine the value of a futures contract

- Value before m-t-m = gain/loss accumulated since last m-t-m
- **Gain/loss captured through the m-t-m process**
- Contract re-priced at current market price and value = zero



$$V_{intraday}(T) = f_{intraday}(T) - f_{t-1}(T)$$

Summary: Value of a futures contract

- Like forwards, futures have no value at initiation
- Unlike forwards, futures do not accumulate any value
- Value always zero after adjusting for day's gain or loss (m-t-m)
- Value different from zero only during m-t-m intervals
- Futures value = current price – price at last m-t-m time
- Futures price increase -> value of long position increases
- Value set back to zero by the end-of-day mark to market

No-arbitrage futures prices

Cash-and-carry arbitrage: $f_0(T) = S_0(1+r)^T$

Today:
Sell futures contract
Borrow money
Buy underlying

At expiration:
Deliver asset and receive futures price
Repay loan plus interest

Reverse C&C arbitrage:

Today:
- Buy futures contract
- Sell/short underlying
- Invest proceeds

At expiration:
Collect loan plus interest
Pay futures price and take delivery

- **No cost or benefit to holding the asset**

$$f_0(T) = S_0(1+r)^T$$

- **Net cost or benefit to holding an asset**

$$f_0(T) = S_0(1+r)^T + \text{FV CB}$$

- Cost-of-carry model
- CB -> cost minus benefit (negative or positive value)

- Costs exceed benefits (net cost) – future value added
- Benefits exceed costs (net benefit) – FV subtracted

- Financial assets
- High(er) cash flows -> lower futures

- **Forward price**

$$F_T = [S_0 - PV(D)](1+r)^T$$

$$F_T = [S_0(1+r)^T] - FV(D)$$

- **Futures price**

$$f_0(T) = S_0(1+r)^T + FV(CB)$$

Contrast backwardation and contango

- **Backwardation**

– Futures price **below the spot price**

– Significant benefit to holding asset

– Net benefit (negative cost of carry)

- **Contango**

– Futures price **above spot price**

– Little/no benefit to holding asset

– Net cost (positive cost of carry)

Treasury bond futures

$$f_0(T) = \frac{[B_0 - PV(C)](1+r)^T}{CF}$$

$$f_0(T) = \frac{B_0(1+r)^T - FV(C)}{CF}$$

Example:

Calculate the no-arbitrage futures price of a 1.2 year futures contract on a 7% T-bond with exactly 10 years to maturity and a price of $1,040. The annual risk-free rate is 5%. Assume the cheapest to deliver bond has a conversion factor of 1.13.

Answer:

The semi-annual coupon is $35. A bondholder will receive two coupons during the contract term – i.e., a payment 0.5 years and 1 year from now.

$$FV(C) = 35(1.05)^{(1.2-0.5)} + 35(1.05)^{(1.2-1)} = \$71.56$$

$$PV(C) = \frac{35}{(1.05)^{0.5}} + \frac{35}{(1.05)^{1}} = \$67.49$$

$$f_0(T) = \frac{1040(1.05)^{1.2} - 71.56}{1.13} = \$912.52$$

$$f_0(T) = \frac{[1040 - 67.49](1.05)^{1.2}}{1.13} = \$912.52$$

Stock futures

$$F_T = [S_0(1+r)^T] - FV(D)$$

Example:

Calculate the no-arbitrage price for a 120-day future on a stock currently priced at $30 and expected to pay a $0.40 dividend in 15 days and in 105 days. The annual risk-free rate is 5%.

$$FV(D) = 0.40(1.05)^{\frac{105}{365}} + 0.40(1.05)^{\frac{15}{365}} = \$0.8065$$

$$f_0(T) = 30(1.05)^{\frac{120}{365}} - 0.8065 = \$29.68$$

Stock index futures

$$f_0(T) = S_0 e^{(r^c - \delta^c)T}$$

Example:

The current level of the Nasdaq Index is 1,780. The continuous dividend is 1.1% and the continuously compounded risk-free rate is 3.7%. Calculate the no-arbitrage futures price of an 87-day futures contract on this index.

$$f_0(T) = 1780e^{\left[(0.037-0.011)\left(\frac{87}{365}\right)\right]} = \$1{,}791.07$$

Currency futures

$$f_0(T) = S_0\left(\frac{(1+r_d)^T}{(1+r_f)^T}\right)$$

$$f_0(T) = S_0 e^{(r_d{}^c - r_f{}^c)T}$$

Example:

The risk-free rates are 5% in U.S. Dollars (\$) and 6.5% in British pounds (£). The current spot exchange rate is \$1.7301/£. Calculate the no-arbitrage \$ price of a 6-month futures contract.

Currency futures

Answer:

$r_d{}^c = \ln(1.05) = 0.0488 \quad r_f{}^c = \ln(1.065) = 0.0630$

$$f_0(T) = 1.7301\left(\frac{(1.05)^{0.5}}{(1.05)^{0.5}}\right) = \$1.7179$$

$$f_0(T) = 1.7301e^{(0.0488-0.0630)(0.5)} = \$1.7179$$

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INVESTMENTS: DERIVATIVES

CHAPTER 4

OPTION MARKETS AND CONTRACTS

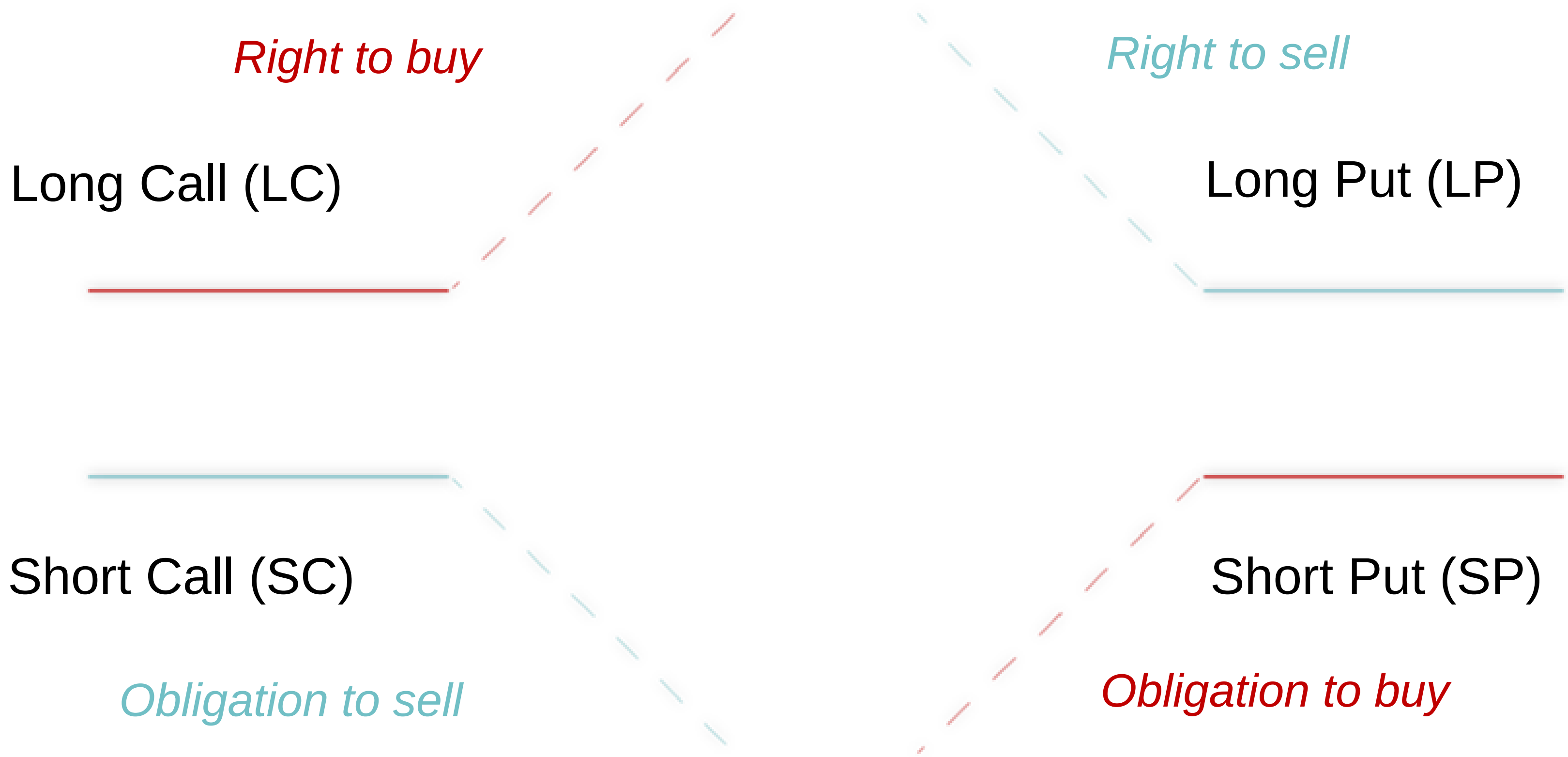
- Identify the basic elements and characteristics of opti on contracts

- Call options grant the holder (long position) the opportunity to buy the underlying security at a price below the current market price, provided that the market price exceeds the call strike before or at expiration (specified contingency).

- Put options grant the holder (long position) the opportunity to sell the underlying security at a price above the current market price, provided that the put strike exceeds the market price before or at expiration (specified contingency).

- The option seller (short position) in both instances receives a payment (premium) compelling performance at the discretion of the holder.

Option shapes



JSE Equity Options: Options - Terminology

American Style Options:

An option that can be exercised at any time prior to expiration is called an American option

European Style Options:

An option that can only be exercised at expiration is called an European option

JSE makes use of both American and European Style Options

In the money: $S>X$

•You will exercise the option

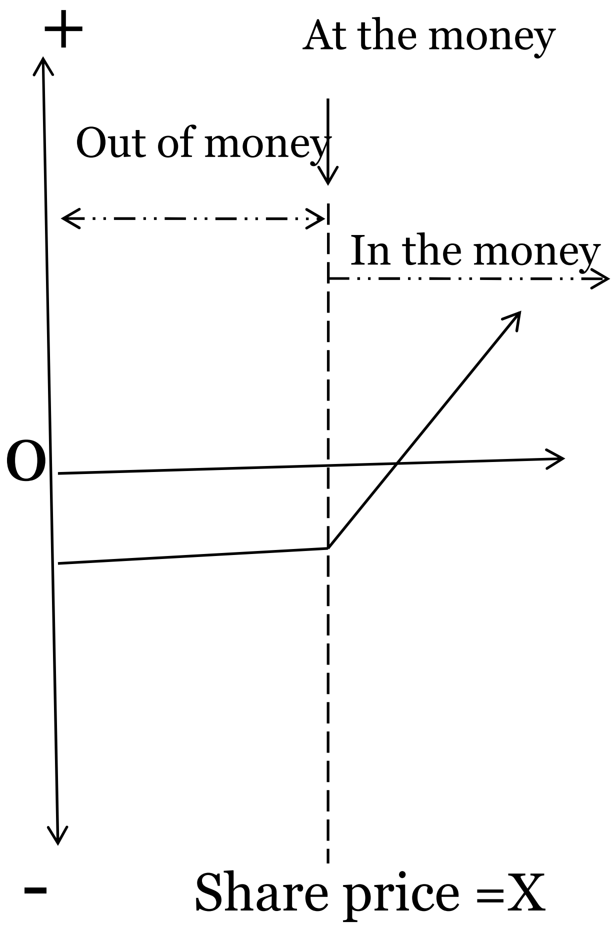
Out the money: $S<X$

•You will abandon the option

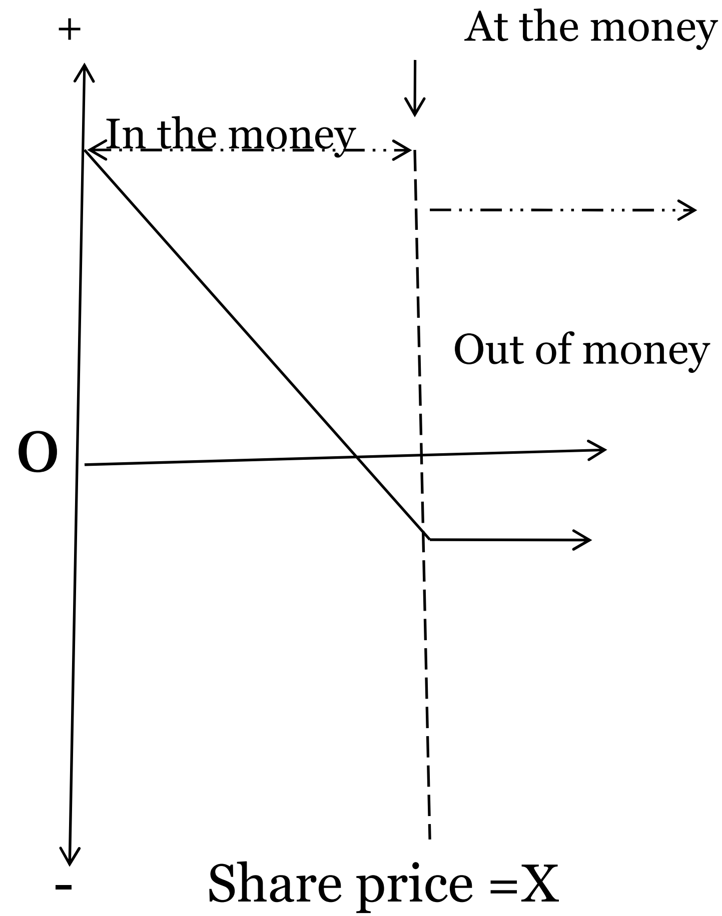
At the money: $S=X$

•Strike Price = Underlying Price

Call option



Put option



Identify the different varieties of options in terms of the types of instruments underlying them

- Financial options

  Equity options (individual or stock index), bond options, interest rate options, currency options

- Options on futures

  Call options – long position in futures upon exercise

  Put options – short position in futures upon exercise

- Commodity options

  Right to either buy or sell a fixed quantity of physical

  asset at a (fixed) strike price

JSE Equity Options: Call Option - Example

You don't have shares in a company but think this dynamic company is going to do well in the future (e.g. new CEO with great vision)

The company is currently trading at R100

- You buy a call option with a strike price the same as it's current price (R100) at a premium of R12.

- Scenario 1 – Company performed well!
- On future date the company is trading @ R120
- Exercise your option and buy the shares @ R100 even though it's trading at R120
- Profit = R8 (R120 – R100 - R12)

Scenario 2 – Company did NOT perform well!

- On future date the company is trading @ R90
- Why would you exercise your option and buy the share at R100 if you can buy it at its current trading price of R90?
- Loss = Premium of R12

- **JSE Equity Options: Put Option - Example**
- You've got shares in in a company and want to protect yourself as you're worried that this company is not going to do well in the future (e.g. new CEO with different vision)

  The company is currently trading at R200

  You buy a put option with a strike price of R200 at a premium of R20.

- Scenario 1 – Company did not perform well!
- On future date the company is trading @ R160
- Exercise your option and sell the shares @ R200 even though it's trading at R160
- Profit = R20 (R200 – R160 – R20)

- Scenario 2 – Company performed well!
- On future date the company is trading @ R240
- Why would you exercise your option and sell the share at R200 if you can sell it at its current trading price of R240?
- Loss = Premium of R20

- **JSE Equity Options: Options Compared To Common Stocks**

Similarities

- Both options and stocks are listed securities
- Like stocks, options trade with buyers making bids and sellers making offers.
- Option investors, like stock investors have the ability to follow price movements, trading volume etc. day by day or even minute by minute

Differences

- Price vs. Premium
- Unlike common stock, an option has a limited life. Common stock can be held indefinitely, while every option has an expiration date
- There is not a fixed number of options, as there is with common stock shares available
- Stock owners have certain voting rights and rights to dividends (if any), option owners participate only in the potential benefit of the stock's price movement

Notation and variables

Variable	Notation	State	Call option value	Put option value
Spot price	S	Increase	Increase	Decrease
Strike price	X	Higher	Lower	Higher
Volatility	σ	Higher	Higher	Higher
Time to maturity	t	Longer	Higher	Uncertain
Interest rates	r	Higher	Higher	Lower
Call option	C or c		max(0; S – X)	
Put option	P or p			max(0; X – S)

Explain putcall parity for European options and relate arbitrage and the construction of synthetic instruments

Fiduciary call – buying a call and investing PV(Payoff is X (otm) or X + (S-X) = S (itm)

Protective put – buying a put and holding asset

Payoff is S (otm) or (X-S) + S = X (itm)

Therefore:

When call is itm put is otm > payoff is S , -

When call is otm, put is itm -> payoff is X

S+ p = c+ PV(X)

S =c +PV(X) -p

P= c +PV(X)- S

C=S+ p -PV(X)

PV(X)= S+ p -c

Put call parity arbitrage

$S + p = c + PV(X)$

$c - p = S - PV(X)$

$c - p > S - PV(X)$

Sell call; buy put; buy spot; borrow PV(X)

$c - p < S - PV(X)$

Buy call; sell put; sell spot; invest PV(X)

Determine the minimum and maximum values of

European options

The lower bound for any option is zero (otm option)

	Upper bound	**Lower bound**
Call options	c ≤ S	c ≥ S – PV(X)
Put options	p ≤ PV(X)	p ≥ PV(X) - S

$$[S - X(1+r)^{-t}] \leq c \leq S$$

$$[X(1+r)^{-t} - S] \leq p \leq X(1+r)^{-t}$$

Chapter 5 - Swaps

- Determining the swap rate = **pricing** of swap
- As rates change over time, the PV of floating payments will either exceed or be less than the PV of fixed payments
  - Difference = **value** of swap
- Market value = difference between bonds
  - Fixed bond minus floating bond
  - Domestic bond minus foreign bond
- PV (receive) minus PV (pay)

Equity Swaps

Consider an equity swap in which the asset manager receives the return of the Russel 2000 Index in return for paying the return on the DJIA. At the inception of the equity swap, the Russel 2000 is at 520.12 and the DJIA is at 9867.33. Calculate the market value of the swap a few months later when the Russel 2000 is at 554.29 and the DJIA is at 9975.54. The notional principal of the swap is $15 million.

$$\text{Russel} = \frac{554.29}{520.12} = 1.0657 \qquad \text{DJIA} = \frac{9,975.54}{9,867.33} = 1.0110$$

$$V_{pay\_DJIA} = \$15,000,000(1.0657 - 1.0110) = \$820,500$$

Interest rate swaps

Consider a two-year interest rate swap with semi-annual payments. Assume a notional principal of $50 million.

Calculate the semi-annual fixed payment and the annualized fixed rate on the swap if the current term structure of LIBOR interest rates is as follows:

$L_0(180) = 0.0688$

$L_0(360) = 0.0700$

$L_0(540) = 0.0715$

$L_0(720) = 0.0723$

$$B_0(180) = \frac{1}{1 + 0.0688(180/360)} = 0.9667$$

$$B_0(360) = \frac{1}{1 + 0.0700(360/360)} = 0.9346$$

$$B_0(540) = \frac{1}{1 + 0.0715(540/360)} = 0.9031$$

$$B_0(720) = \frac{1}{1 + 0.0723(720/360)} = 0.8737$$

$$FS(0,4,180) = \frac{1 - 0.8737}{0.9667 + 0.9346 + 0.9031 + 0.8737} = 0.0343$$

$$\text{Fixed payment} = 0.0343 \times \$50{,}000{,}000 = \$1{,}715{,}000$$

$$\text{Annualized fixed rate} = 3.43\%(360/180) = 6.86\%$$

Calculate the market value of the swap 120 days later from the point of view of the party paying the floating rate and receiving the fixed rate, and from the point of view of the party paying the fixed rate and receiving the floating rate if the term structure 120 days later is as follows:

- $L_{120}(60) = 0.0620$
- $L_{120}(240) = 0.0631$
- $L_{120}(420) = 0.0649$
- $L_{120}(600) = 0.0687$

$$B_{120}(180) = \frac{1}{1 + 0.0620(60/360)} = 0.9898$$

$$B_{120}(360) = \frac{1}{1 + 0.0631(240/360)} = 0.9596$$

$$B_{120}(540) = \frac{1}{1 + 0.0649(420/360)} = 0.9296$$

$$B_{120}(720) = \frac{1}{1 + 0.0687(600/360)} = 0.8973$$

$$\text{Fixed} = 0.0343(0.9898 + 0.9596 + 0.9296 + 0.8973) + 1(0.8973)$$
$$= 1.0268$$

$$1^{st}\,\text{Floating payment} = \left[1 + (0.0688)(180/360)\right] = 1.0344$$

Ans cont…

Discounted with the 60 day present value factor of 0.9898:

$$\text{Float} = 1.0344 \times 0.9898 = 1.0239$$

$$V_{pay\_float} = \$50{,}000{,}000\left(1.0268 - 1.0239\right) = \$145{,}000$$

$$V_{pay\_fixed} = \$50{,}000{,}000\left(1.0239 - 1.0268\right) = -\$145{,}000$$

Equity and interest rate swap

Assume an asset manager enters into a one-year equity swap in which he will receive the return on the Nasdaq 100 Index in return for paying a floating interest rate. The swap calls for quarterly payments. The Nasdaq 100 is at 1651.72 at the beginning of the swap. Ninety days later, the rate $L_{90}(90)$ is 0.0665. Calculate the market value of the swap 100 days from the beginning of the swap if the Nasdaq 100 is at 1695.27, the notional principal of the swap is $50 million, and the term structure is:

$L_{100}(80) = 0.0654$

$L_{100}(170) = 0.0558$

$L_{100}(260) = 0.0507$

$$B_{100}(180) = \frac{1}{1 + 0.0654(80/360)} = 0.9857$$

$$\text{Next floating payment} = \left[1 + (0.0665)(90/360)\right] = 1.0168$$

Discounted with the 80 day present value factor of 0.9857:

$$\text{Float} = 1.0168 \times 0.9857 = 1.0023$$

$$\text{Equity} = \frac{1{,}695.27}{1{,}651.72} = 1.0264$$

$$V_{pay\_float} = \$50{,}000{,}000(1.0264 - 1.0023) = \$1{,}205{,}000$$

BEST OF LUCK IN YOUR EXAMS

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