

**INV3703**

(495361)

May/June 2014

INVESTMENTS. DERIVATIVES

Duration 2 Hours

70 Marks

EXAMINERS
 FIRST
 SECOND
 EXTERNAL

 MR CF ERASMUS
 MR G MAROZVA
 MS M REYERS

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

A non-programmable financial and/or a scientific calculator is permissible

This paper consists of 22 pages, including two pages for rough work (pp 17 – 18), the standard normal distribution table (p 19), a formula sheet (pp 20 – 22), and instructions for completing a mark-reading sheet

INSTRUCTIONS

Complete section A on the mark-reading sheet provided Write your student number and the unique number on the mark-reading sheet

UNIQUE NUMBER: 495361Complete sections B, C, D and E on the examination paper

Assume all risk-free rates as discrete unless stated otherwise

Round all answers to at least 4 decimals and your final answer to 2 decimals

Mark allocation for the paper

Section A	10 multiple-choice questions	[20 marks]
Section B	Forwards and Futures	[20 marks]
Section C	Options	[15 marks]
Section D.	Swaps	[10 marks]
Section E	Theory and application	[5 marks]
Total		[70 marks]

SECTION A: MULTIPLE-CHOICE QUESTIONS

[10 x 2 marks = 20 marks]

- 1 Which one of the following is a characteristic of an exchange traded derivative?
 - 1 Bears credit risk
 - 2 Standardized
 - 3 Unregulated
 - 4 Illiquid

- 2 Mark took a long position in a call option, what would *most likely* be the advantage of engaging in this contingent claim compared to a forward commitment?
 - 1 Contingent claims permit gains while protecting against losses
 - 2 Contingent claims are easier to offset than forward commitments
 - 3 Contingent claims have lower default risk than forward commitments
 - 4 Contingent claims are typically cheaper to initiate than forward commitments

- 3 A call for delivery of a 90-day T-bill with a \$1 notional par value in 60 days sells for \$0.9859, the implied discount rate for the T-bill would be closest to
 - 1 1.41%
 - 2 1.88%
 - 3 2.82%
 - 4 5.64%

- 4 A 3 × 6 FRA contract
 - 1 expires in 3 months based on a 60-day LIBOR rate
 - 2 expires in 3 months based on a 90-day LIBOR rate
 - 3 expires in 6 months based on a 90-day LIBOR rate
 - 4 expires in 6 months based on a 180-day LIBOR rate

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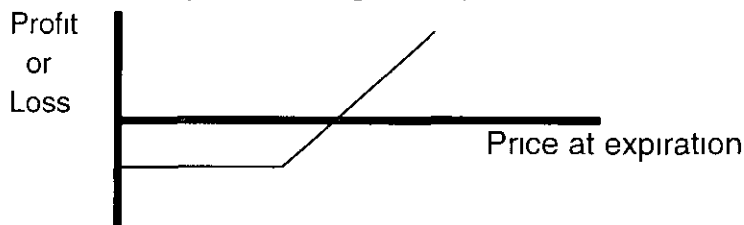
5 A Eurodollar futures price changes from 99.02 to 99.09. The gain or loss to an investor who is long four contracts is closest to

- 1 -\$700
- 2 -\$175
- 3 \$175
- 4 \$700

6 A five-year Treasury bond with a par value of \$1000, 7% annual yield and a 10% semi-annual coupon is priced at \$1124.75. A futures contract calling for delivery of the bond expires one year from now. The one-year risk-free rate is 6%. The futures price would be closest to

- 1 \$ 983.69
- 2 \$ 985.44
- 3 \$1000.00
- 4 \$1090.48

7 The following option diagram represents a



- 1 Long put
- 2 Long call
- 3 Short put
- 4 Short call

8 An analyst is currently pricing an option using the Black-Scholes option pricing model. The calculated value for $d_1 = -0.53$. The value of $N(d_1)$ would be closest to

1. 0.2981
2. 0.3192
3. 0.6808
4. 0.7019

- 9 A US based company exchanges \$14 000 for €10 000 with an European company. If you were to value the swap at initiation of this contract, the value of the transaction would be
- 1 \$0
 - 2 \$14 000 paid by the US company and €10 000 paid by the European company
 - 3 \$14 000 paid by the US company and €10 000 received by the European company
 - 4 unable to be determined as no exchange rate is provided
- 10 A British company enters into a currency swap in which it pays a fixed rate of 6 percent in dollars and the counterparty pays a floating rate of LIBOR+2.5 percent in pounds. The notional principals are £20 million and \$30 million. Payments are made quarterly and on the basis of 30 days per month and 360 days per year. Calculate the quarterly payments if the LIBOR rates are as follow.

90-day LIBOR rate 2.50%
360-day LIBOR rate 5.5%

	Dollar payment	Pounds payment
1	\$300 000	£375 000
2	\$375 000	£300 000
3	\$1 200 000	£1 500 000
4	\$1 500 000	£1 200 000

SECTION B: FORWARDS AND FUTURES

(20 marks)

- 1 Fikile, the head of treasury at Sanral, wishes to hedge against an increase in borrowing costs due to a possible downgrade in Sanrals' credit rating. Fikile proposes to the management of Sanral to hedge the risk by entering into a long 6 × 12 FRA. The current structure for LIBOR is as follows

Term	Interest rate
30 day	6.15%
90 day	6.30%
180 day	6.60%
360 day	6.95%

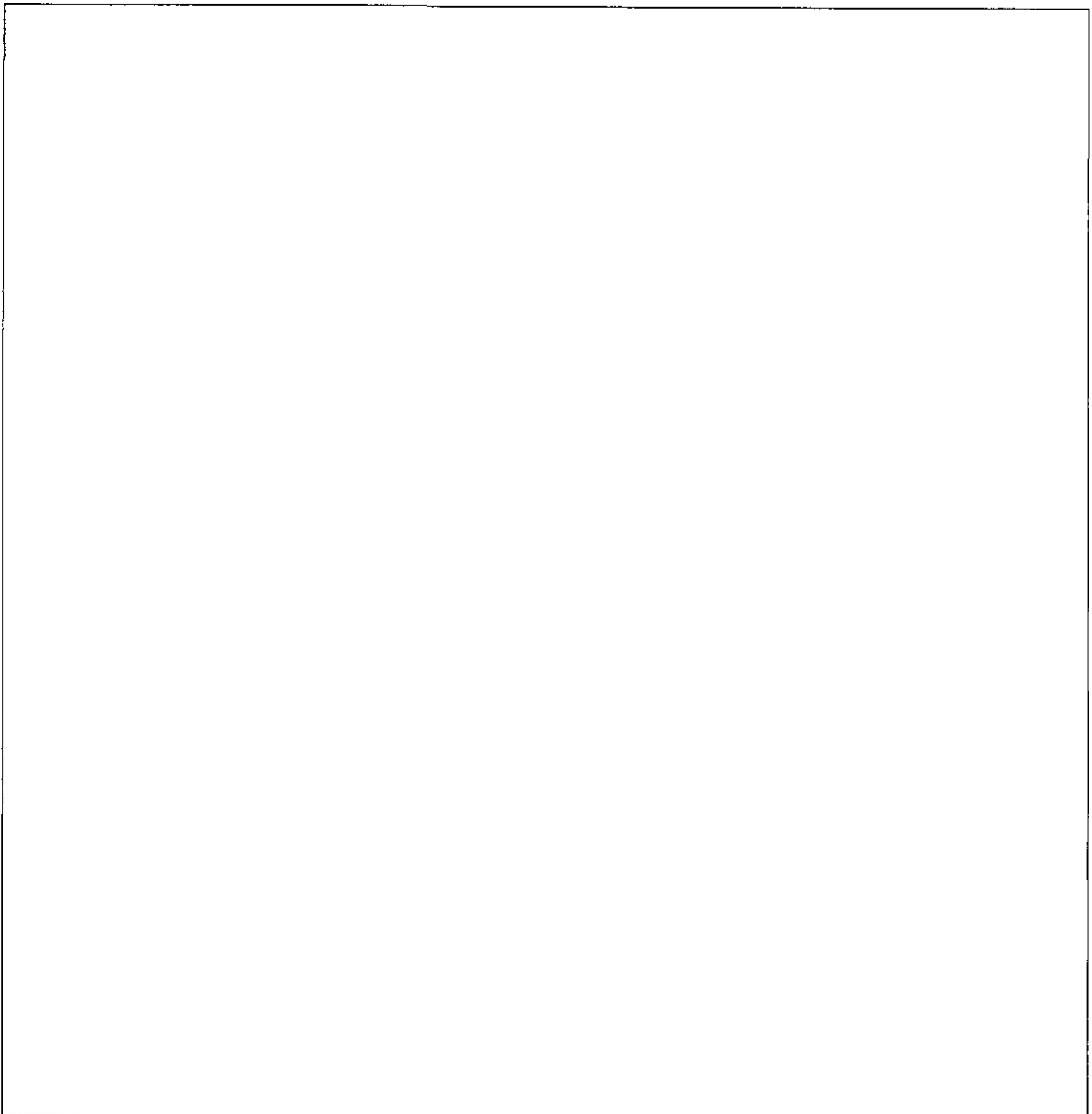
- 1.1 Calculate the rate that Fikile would receive on a 6 × 12 FRA (3)
(Show all calculations. Only the calculations will be marked in this question.)

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1.2 Suppose Fikile went long in the FRA and the FRA rate is 0.0707. Now 55 days later the interest rates have risen and the LIBOR term structure is as follows

Term	Interest rate
125 day	6.65%
305 day	7.24%

Calculate the market value of this FRA based on a notional principal of R50 000 000. (4)



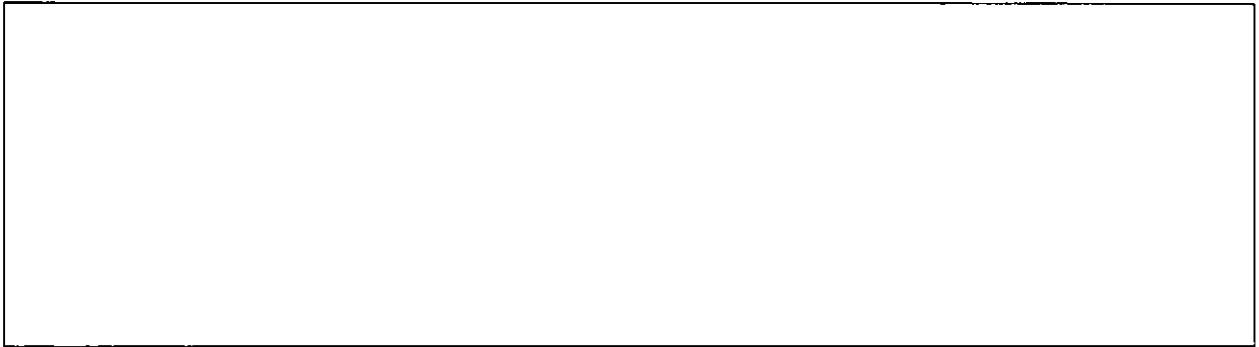
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- 2 Eskom expects to receive a cash inflow of £200 000 000 in 90 days to fund an alternative energy project. The economist in the treasury department expects short-term interest rates to decrease during the next 90 days. The economist advises the treasurer of Eskom to enter into a FRA that expires in 90 days and that is based on the 90-day LIBOR rate. A dealer quoted this FRA at 6 per cent. At expiration the LIBOR rate is 5 per cent. Assume a notional principal on the contract of £200 000 000.
- 2.1 Thandi, the head of treasury, is uncertain of the position she should take on the FRA to hedge against the interest rate risk. Would you recommend a short or a long position and why would you recommend this position? (2)

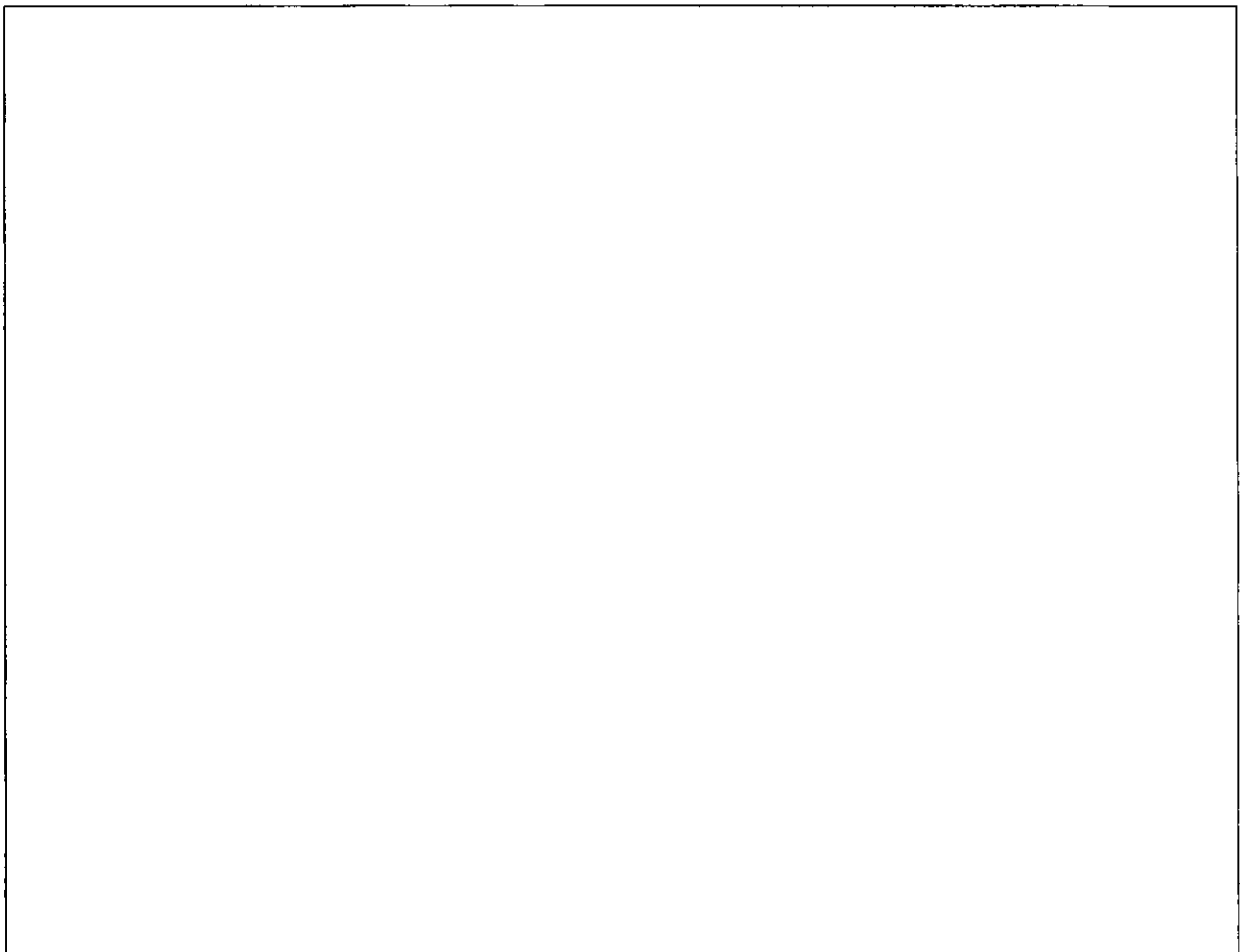
- 2.2 Calculate whether Eskom made a gain or a loss as a consequence of entering the FRA. (3)

3 Platinum currently trades at \$1224.46 per ounce in the spot market. The risk-free interest rate is 7 per cent. Assume the net cost of carry for platinum futures is zero.

3.1 Calculate the price of a platinum futures contract that expires in 135 days. (1)



3.2 Use your answer calculated in 3.1 and illustrate how an arbitrage transaction could be executed if the platinum futures contract is priced at \$1261.13 per ounce. (5)



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- 4 A copper futures contract requires the long trader to buy 100 tonnes of copper. The initial margin is \$2 000 and the maintenance margin requirement is \$1 500.
- 4.1 Alicia sells one August copper futures contract at a futures price of \$323 per tonne. When would Alicia receive a maintenance margin call? (2)

SECTION C: OPTIONS

(15 marks)

- 1 Ciska, an analyst at Anna List Investments collects the following information on call and put prices of a share

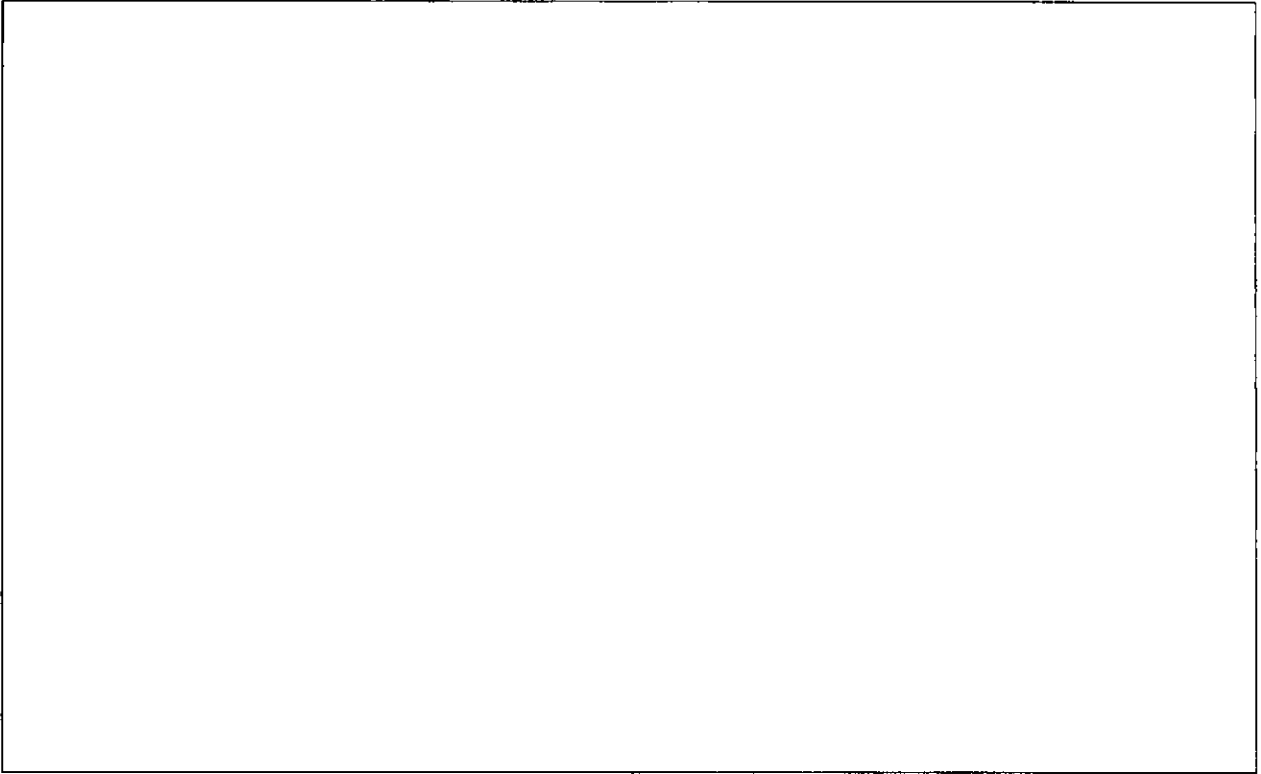
Call price	\$ 5 60
Put price	\$ 6 60
Share price	\$85 00
Exercise price	\$88 00
Risk-free rate	6 per cent
Days to expiration	239
Assume a 365 day year	

- 1 1 Use put-call parity to calculate the price of the synthetic put option (1)

- 1 2 Use put-call parity to calculate the price of the synthetic underlying share (1)

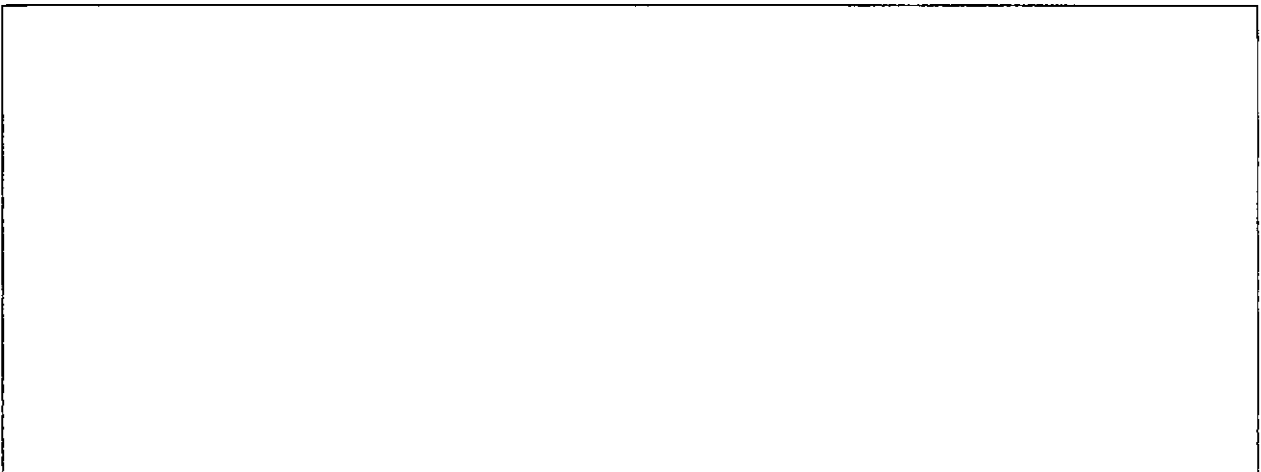
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- 1.3 If the value of the share is \$90 at expiration, indicate how you will derive an arbitrage profit by using a synthetic call (4)

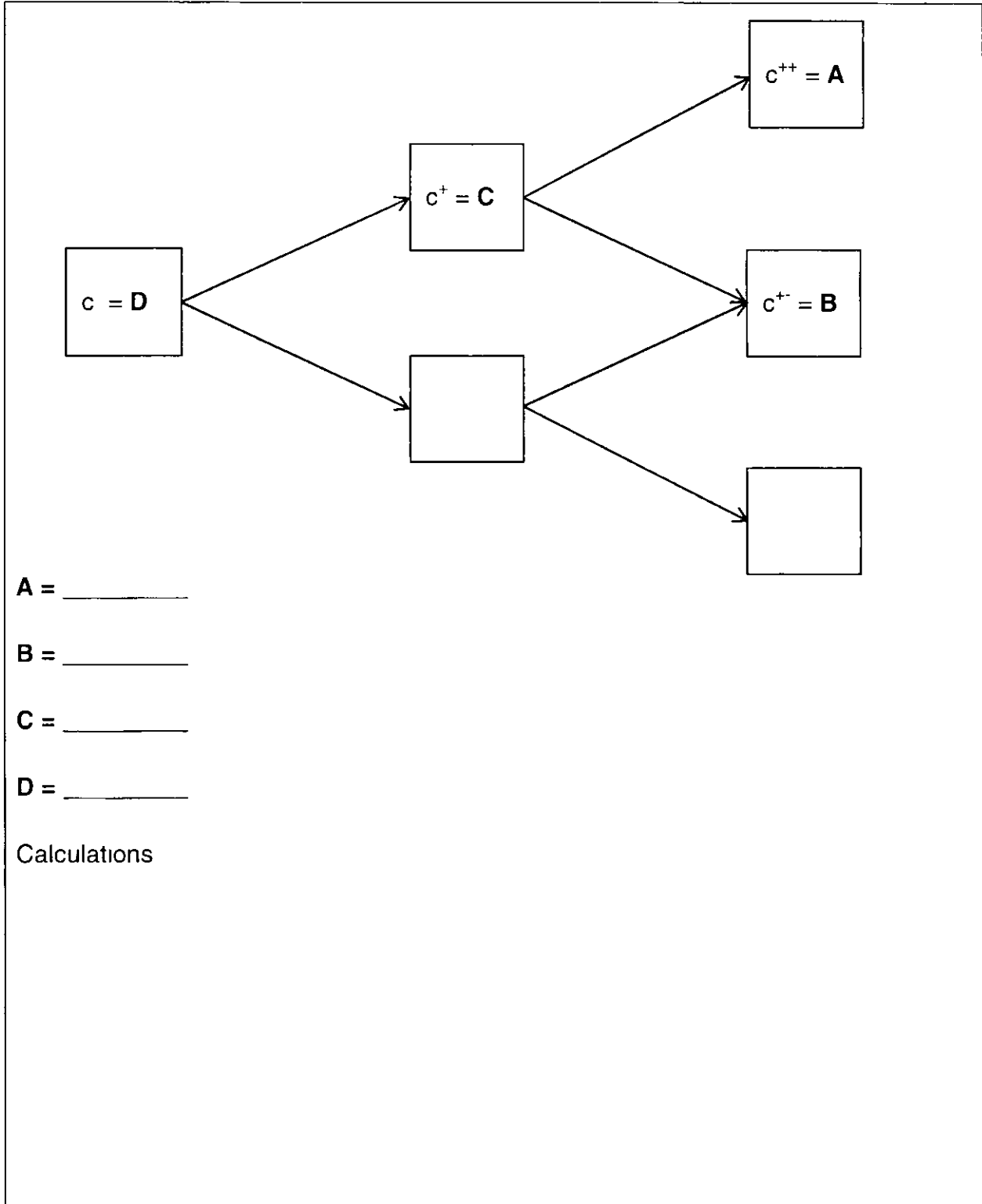


2. A two-period binomial model with a share that currently trades at \$50 in the spot market can either increase in value by 20 per cent or decrease by 20 per cent in the highly volatile market. The annual risk-free rate is 8 per cent and each period of the binomial model is equal to one year.

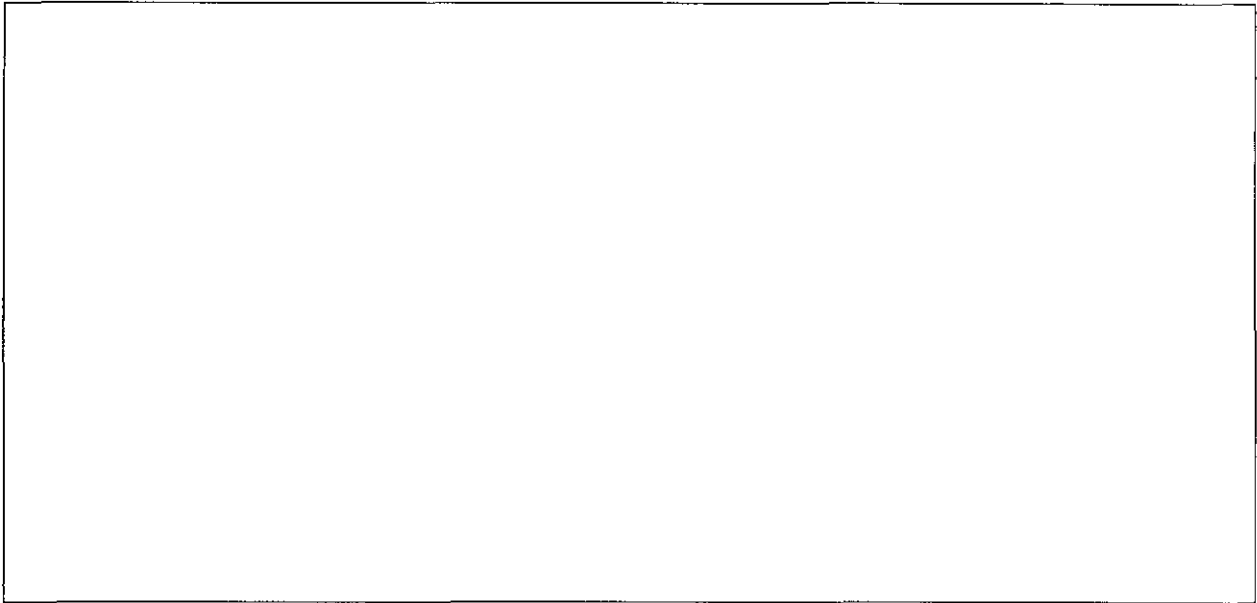
- 2.1 Calculate the risk-neutral probability of the binomial model (2)



2 2 Use the diagram below to find the European call option values of A, B, C, and D if the strike price for this option is \$57 (4)



- 3 A Barclays Africa (BGA) share is currently trading at R133.15, call and put options associated with this share have an exercise price of R145.00. These options expire in 129 days and the volatility is 0.48. The continuously compounded risk-free interest rate is 4 per cent. Before the analyst could finish the option pricing he had to leave his desk but left you with the correct values for $d_1 = -0.11$ and $d_2 = -0.39$.
- 3.1 Calculate the value of the European call option on a BGA share. (3)



SECTION D: SWAPS

(10 marks)

- 1 Vodacom (JSE VOD), has issued its first 2 year ZAR denominated bond with a coupon rate set at 6%, which means that Vodacom pays a fixed rate of interest in ZAR for the next two years. However Vodacom would prefer to have a USD liability as it believes the USD will weaken against the ZAR over the next two years.

AT&T (NYSE T), has issued a 2 year USD denominated bond with a coupon rate of 4%, which means that AT&T pays a fixed rate of interest in USD for the next two years. However AT&T would prefer to have a ZAR liability as it believes the USD will strengthen against the ZAR over the next two years.

The companies decide to enter into a swap, the terms of which are contained in table below.

Contract date	1 July 2014
Contract type	Currency swap (fixed for fixed)
ZAR Fixed rate payer	Vodacom
USD Fixed rate payer	AT&T
Start date	1 July 2014
Maturity date	1 July 2016
Notional principal	ZAR 10 000 000
ZAR Fixed interest rate per annum	6%
USD Fixed interest rate per annum	4%
Payment date	1 July of each year

- 1.1 The exchange rate was ZAR10.00/USD at the initiation of the swap. Indicate the cash flows between the companies from the perspective of AT&T mobile. (4)

Cash flows paid and received by AT&T mobile

Date	USD cash flow	ZAR cash flow
1 July 2014		
1 July 2015		
1 July 2016		

[TURN OVER]

- 1.2 Identify the type of risk either company, Vodacom or AT&T, may incur as a result of the swap contract. Briefly explain the potential effect of this risk. (2)

2. An asset manager wishes to reduce his exposure to large-cap stocks and increase his exposure to small cap stocks. He seeks to do so using an equity swap. He agrees to pay a dealer the return on a large-cap index, and the dealer agrees to pay the manager the return on a small-cap index. The value of the small-cap index starts off at 712.70, and the large-cap index starts at 1230.10. In six months, the small-cap index is at 653.30 and the large-cap index is at 1270.92. Assume that payments are made semi-annually.

- 2.1 If the notional principal is R10 million, calculate the first overall payment and indicate which party makes the payment. (4)

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SECTION E: THEORY AND APPLICATION

(5 marks)

- 1 If a trader would enter into a derivative transaction where s/he pays a premium to buy an underlying asset at a predetermined price sometime in the future, which derivative would the trader hold and what is the position that the trader took? (2)

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- 2 If a futures price is lower than the expected spot price the situation is called normal (1)

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- 3 Swaptions are options on a swap contract. There are two types of swaptions, a payer swaption and receiver swaptions. Briefly explain which rate is paid and which rate is received by a trader who entered a **receiver swaption**. (2)

TOTAL**[70 MARKS]**

ROUGH WORK

(WILL NOT BE MARKED)

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ROUGH WORK

(WILL NOT BE MARKED)

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Cumulative probabilities for a standard normal distribution

$$P(X \leq x) = N(x) \text{ for } x \geq 0 \text{ or } 1 - N(-x) \text{ for } x < 0$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0 5000	0 5040	0 5080	0 5120	0 5160	0 5199	0 5239	0 5279	0 5319	0 5359
0.10	0 5398	0 5438	0 5478	0 5517	0 5557	0 5596	0 5636	0 5675	0 5714	0 5753
0.20	0 5793	0 5832	0 5871	0 5910	0 5948	0 5987	0 6026	0 6064	0 6103	0 6141
0.30	0 6179	0 6217	0 6255	0 6293	0 6331	0 6368	0 6406	0 6443	0 6480	0 6517
0.40	0 6554	0 6591	0 6628	0 6664	0 6700	0 6736	0 6772	0 6808	0 6844	0 6879
0.50	0 6915	0 6950	0 6985	0 7019	0 7054	0 7088	0 7123	0 7157	0 7190	0 7224
0.60	0 7257	0 7291	0 7324	0 7357	0 7389	0 7422	0.7454	0 7486	0 7517	0 7549
0.70	0 7580	0 7611	0 7642	0 7673	0 7704	0 7734	0 7764	0 7794	0 7823	0 7852
0.80	0 7881	0 7910	0 7939	0 7967	0 7995	0 8023	0 8051	0 8078	0 8106	0 8133
0.90	0 8159	0 8186	0 8212	0 8238	0 8264	0 8289	0 8315	0 8340	0 8365	0 8389
1.00	0 8413	0 8438	0 8461	0 8485	0 8508	0 8531	0 8554	0 8577	0 8599	0 8621
1.10	0 8643	0 8665	0 8686	0 8708	0 8729	0 8749	0 8770	0 8790	0 8810	0 8830
1.20	0 8849	0 8869	0 8888	0 8907	0 8925	0 8944	0 8962	0 8980	0 8997	0 9015
1.30	0 9032	0 9049	0 9066	0 9082	0 9099	0 9115	0 9131	0 9147	0 9162	0 9177
1.40	0 9192	0 9207	0 9222	0 9236	0 9251	0 9265	0 9279	0 9292	0 9306	0 9319
1.50	0 9332	0 9345	0 9357	0.9370	0 9382	0 9394	0 9406	0 9418	0 9429	0 9441
1.60	0 9452	0 9463	0 9474	0 9484	0 9495	0 9505	0 9515	0 9525	0 9535	0 9545
1.70	0 9554	0 9564	0 9573	0 9582	0 9591	0 9599	0 9608	0 9616	0 9625	0 9633
1.80	0 9641	0 9649	0 9656	0 9664	0 9671	0 9678	0 9686	0 9693	0 9699	0 9706
1.90	0 9713	0 9719	0 9726	0 9732	0 9738	0 9744	0 9750	0 9756	0 9761	0 9767
2.00	0 9772	0 9778	0 9783	0 9788	0 9793	0 9798	0 9803	0 9808	0 9812	0 9817
2.10	0 9821	0 9826	0 9830	0 9834	0 9838	0 9842	0 9846	0 9850	0 9854	0 9857
2.20	0 9861	0 9864	0 9868	0 9871	0 9875	0 9878	0 9881	0.9884	0 9887	0 9890
2.30	0 9893	0 9896	0 9898	0 9901	0 9904	0 9906	0 9909	0 9911	0 9913	0 9916
2.40	0 9918	0 9920	0 9922	0 9925	0 9927	0 9929	0 9931	0 9932	0 9934	0 9936
2.50	0 9938	0 9940	0 9941	0 9943	0 9945	0 9946	0 9948	0 9949	0 9951	0 9952
2.60	0 9953	0 9955	0 9956	0 9957	0 9959	0 9960	0 9961	0 9962	0 9963	0 9964
2.70	0 9965	0 9966	0 9967	0 9968	0 9969	0 9970	0 9971	0 9972	0 9973	0 9974
2.80	0 9974	0 9975	0 9976	0 9977	0 9977	0 9978	0 9979	0 9979	0 9980	0 9981
2.90	0 9981	0 9982	0 9982	0 9983	0 9984	0 9984	0 9985	0 9985	0 9986	0 9986
3.00	0 9987	0 9987	0 9987	0 9988	0 9988	0 9989	0 9989	0 9989	0 9990	0 9990

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PRICING AND VALUATION OF FORWARD

Discrete interest (r)

$$(1+r)^T$$

Continuous interest (r^c)

$$e^{r^c T}$$

Conversion ($r \leftrightarrow r^c$)

$$r^c = \ln(1+r)$$

$$r = e^{r^c} - 1$$

Discount interest

$$\text{Amount} \left[1 - r \left(\frac{d}{360} \right) \right]$$

Add-on interest

$$\text{Amount} \left[1 + r \left(\frac{d}{360} \right) \right]$$

Forward rate agreement (FRA)**Price/rate**

$$\text{FRA}_{\text{rate}} = \left[\frac{1 + L_0 \left(\frac{h+m}{360} \right)}{1 + L_0 \left(\frac{h}{360} \right)} - 1 \right] \left(\frac{360}{m} \right)$$

Value

$$V_g = \left[\frac{1}{1 + L_g \left(\frac{h-g}{360} \right)} \right] - \left[\frac{1 + \text{FRA}_{\text{rate}} \left(\frac{m}{360} \right)}{1 + L_g \left(\frac{h+m-g}{360} \right)} \right]$$

Payoff

$$\text{FRA}_{\text{payoff}} = \text{NP} \left[\frac{(U_{\text{rate}} - \text{FRA}_{\text{rate}}) \left(\frac{U_{\text{days}}}{360} \right)}{1 + U_{\text{rate}} \left(\frac{U_{\text{days}}}{360} \right)} \right]$$

Forward contract – no cash flows**Price**

$$F_T = S_0 (1+r)^T$$

Value

$$V_0 = S_0 - \frac{F_T}{(1+r)^T} \quad V_t = S_t - \frac{F_T}{(1+r)^{T-t}}$$

Forward contract – equity**Price and value – discrete compounding**

$$F_T = [S_0 - \text{PV}(D)](1+r)^T$$

$$V_t = S_t - \text{PV}(D) - \left[\frac{F_T}{(1+r)^{T-t}} \right]$$

Price and value – continuous compounding

$$F_T = S_0 e^{(r^c - \delta^c)T}$$

$$V_t = S_t e^{-\delta^c(T-t)} - F_T e^{-r^c(T-t)}$$

Forward contract – fixed income**Price and value**

$$F_T = [B_0 - \text{PV}(C)](1+r)^T$$

$$V_t = B_t - \text{PV}(C) - \left[\frac{F_T}{(1+r)^{T-t}} \right]$$

Forward contract – currency**Price and value – discrete compounding**

$$F_T = S_0 \left[\frac{(1+r_d)^{d/365}}{(1+r_f)^{d/365}} \right]$$

$$V_t = \left[\frac{S_t}{(1+r_f)^{T-t}} \right] - \left[\frac{F_T}{(1+r_d)^{T-t}} \right]$$

Price and value – continuous compounding

$$F_T = S_0 e^{(r_d^c - r_f^c)(d/365)}$$

$$V_t = S_t e^{-r_f^c(T-t)} - F_T e^{-r_d^c(T-t)}$$

Interest rate parity (IRP)

$$(1+r_d)^{d/365} = (1+r_f)^{d/365} \left(\frac{F}{S} \right)$$

PRICING OF FUTURES**Futures price – no cost or benefit**

$$f_0(T) = S_0(1+r)^T$$

Futures price – net cost or benefit

$$f_0(T) = S_0(1+r)^T + \text{FV}(\text{CB})$$

Futures price – stock

$$f_0(T) = S_0(1+r)^T - \text{FV}(D)$$

Futures price – stock index

$$f_0(T) = S_0 e^{(r-\delta^s)T}$$

Futures price – Treasury bill

$$f_0(T) = \frac{B_0(1+r)^T - \text{FV}(C)}{\text{Conversion Factor}}$$

Futures price – currency*Discrete compounding*

$$f_0(T) = S_0 \left[\frac{(1+r_d)^{d/365}}{(1+r_f)^{d/365}} \right]$$

Continuous compounding

$$f_0(T) = S_0 e^{(r_d^c - r_f^c)(d/365)}$$

PRICING AND VALUATION OF SWAPS**Net fixed payment**

$$\text{NFP} = (\text{swap rate} - \text{LIBOR}) \left(\frac{\text{days}}{360} \right) \text{NP}$$

Swap fixed rate

$$C = \left(\frac{1 - Z_4}{Z_1 + Z_2 + Z_3 + Z_4} \right)$$

Discount rate

$$Z_{\text{day}} = \frac{1}{1 + \left(R_{\text{day}} \times \frac{\text{days}}{360} \right)}$$

Market value of interest rate swap

$$\text{MV}_{\text{IRS}} = V_{\text{floating-rate bond}} - V_{\text{fixed-rate bond}}$$

Market value of currency swap

$$\text{MV}_{\text{CS}} = V_{\text{domestic bond}} - V_{\text{foreign bond}}$$

Return on equity

$$\text{Return} = \left(\frac{\text{Ending value}}{\text{Beginning value}} \right)$$

Yield on equity

$$\text{Yield} = \left(\frac{\text{Ending value}}{\text{Beginning value}} \right) - 1$$

Payment on equity position

$$\text{PMT} = \text{Yield} \times \text{NP}$$

Market value of equity swap

$$\text{MV}_{\text{ES}} = \text{NP} (\text{Return}_X - \text{Return}_Y)$$

Swaption payoffs

$$\text{Payoff}_{\text{Swaption-payer}} = (\text{SFR} - X) \left(\frac{\text{days}}{360} \right) \text{NP}$$

$$\text{Payoff}_{\text{Swaption-receiver}} = (X - \text{SFR}) \left(\frac{\text{days}}{360} \right) \text{NP}$$

PRICING OF OPTION CONTRACTS**Intrinsic values**

$$c = \max[0, (S - X)]$$

$$p = \max[0, (X - S)]$$

Bounds – European options*No cash flows – upper and lower*

$$[S - X(1+r)^{-1}] \leq c \leq S$$

$$[X(1+r)^{-1} - S] \leq p \leq X(1+r)^{-1}$$

Cash flows – lower bounds

$$c \geq [S - PV(CF)] - PV(X)$$

$$p \geq PV(X) - [S - PV(X)]$$

Bounds – American options

$$[S - X(1+r)^{-1}] \leq C \leq S$$

$$(X - S) \leq P \leq X$$

Put-call parity – European options**No cash flows**

$$S + p = c + X(1+r)^{-1}$$

Cash flows

$$[S - PV(CF)] + p = c + PV(X)$$

Futures contracts

$$F_T(1+r)^{-1} + p = c + X(1+r)^{-1}$$

Put-call parity – American options

$$S - X \leq C - P \leq S - X(1+r)^{-1}$$

Binomial model

$$p = \frac{(1+r) - d}{u - d}$$

$$f = \frac{[(p)(f^+) + (1-p)(f^-)]}{(1+r)}$$

$$f = \frac{[(p)^2(f^{++}) + 2(p)(1-p)(f^{+-}) + (1-p)^2(f^{--})]}{(1+r)^2}$$

Black-Scholes Merton model**Black-Scholes model**

$$d_1 = \frac{\ln(S/X) + [r^c + (\sigma^2/2)]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = SN(d_1) - Xe^{-r^c T}N(d_2)$$

$$p = Xe^{-r^c T}N(-d_2) - SN(-d_1)$$

Merton's model

$$d_1 = \frac{\ln(S/X) + [(r^c - \delta) + (\sigma^2/2)]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = Se^{-\delta T}N(d_1) - Xe^{-r^c T}N(d_2)$$

$$p = Xe^{-r^c T}N(-d_2) - Se^{-\delta T}N(-d_1)$$

[$r^c = r_d^c$ and $\delta = r_f^c$ when pricing currency options]

Black's model

$$d_1 = \frac{\ln(F/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = e^{-r^c T} [FN(d_1) - XN(d_2)]$$

$$p = e^{-r^c T} [XN(-d_2) - FN(-d_1)]$$

Delta

$$\text{Delta} = \frac{f_1 - f_0}{S_1 - S_0} = N(d_1)$$

Interest rate options

$$IR_{\text{call}} = NP(U_{\text{rate}} - X_{\text{rate}}) \left(\frac{d}{360} \right)$$

$$IR_{\text{put}} = NP(X_{\text{rate}} - U_{\text{rate}}) \left(\frac{d}{360} \right)$$

