**INV3703**

(473681) October/November 2014

INVESTMENTS: DERIVATIVES

Duration 2 Hours

70 Marks

EXAMINERS

FIRST

MR CF ERASMUS

SECOND

MR G MAROZVA

EXTERNAL

MS M REYERS

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

A non-programmable financial and/or a scientific calculator is permissible.

This paper consists of 27 pages, including three pages for rough work (pp 21–23), the standard normal distribution table (p 24), a formula sheet (pp 25–27), and instructions for completing a mark-reading sheet

INSTRUCTIONS

Complete section A on the mark-reading sheet provided Write your student number and the unique number on the mark-reading sheet

UNIQUE NUMBER: 473681Complete sections B, C, D and E on the examination paper

Assume all risk-free rates as discrete unless stated otherwise

Round all answers to at least 4 decimals and your final answer to 2 decimals

Mark allocation for the paper

Section A	10 multiple-choice questions	[20 marks]
Section B	Forwards and Futures	[20 marks]
Section C	Options	[15 marks]
Section D	Swaps	[10 marks]
Section E	Theory and application	[5 marks]
Total		[70 marks]

SECTION A: MULTIPLE-CHOICE QUESTIONS

[10 x 2 marks = 20 marks]

- 1 For all parties involved, which of the following financial instruments is an example of a contingent claim?
- 1 Swaps
 - 2 Put options
 - 3 Futures contracts
 - 4 Forward contracts
- 2 Renault entered into a forward contract with a platinum supplier, which one of the following statements is *least accurate* regarding a forward contract?
- 1 Futures contracts are easier to offset than forward contracts
 - 2 Forward contracts are generally less liquid than futures contracts
 - 3 Forward contracts are easier to tailor to specific needs than futures contracts
 - 4 Forward contracts are characterised by having a clearinghouse as an intermediary Futures contracts have a clearinghouse.
- 3 A 90-day T-bill is selling at a discount of 8%, the price per \$1 par for the T-bill would be closest to
- 1 \$0 92
 - 2 \$0 98
 - 3 \$1 02
 - 4 \$1 08
- Discount yield = $0.08 = (1 - \text{Price}/1)(360/90) = 1 * 0.08 * 90 / 360 = 0.02$
 $1 - 0.02 = 0.98$
 Similar to page 67 Q1 a.
- 4 A five-year Treasury bond with a par value of \$1000, 7% annual yield and a 10% semi-annual coupon is priced at \$1214 75 A futures contract calling for delivery of the bond expires one year from now The one-year risk-free rate is 6% Assume that coupons are reinvested at the 7% annual yield of the Treasury bond The futures price would be closest to
- 1 \$1 000 00
 - 2 \$1 090 48
 - 3 \$1 101 75
 - 4 \$1 185 89
- $1000 * 0.05 = \$50$ (10%/2=5%)
 $50(1 + 0.035) + 50 = 101.75$ (7% reinvestment/2= 0.035)
 $1214.75(1.06) - 101.75 = \1185.89

[TURN OVER]

- 5 Ngomandi, the head of treasury at Eskom, wishes to hedge against an increase in borrowing costs due to a possible downgrade in Eskom's credit rating. Ngomandi proposes to the management of Eskom to hedge the risk by entering into a long 3 × 6 FRA. The current structure for LIBOR is as follows

Term	Interest rate
90 day	6.15%
180 day	6.30%
270 day	6.60%

The rate that Ngomandi would receive on a 6 × 12 FRA is closest to

- 1 3.18% ERROR
2 4.88%
3 6.65%
4 6.72%

- 6 Suppose Retha, a trader, went long in a 6 × 12 FRA and the FRA rate is 0.0606. Suppose it is now 55 days after the initiation of the contract and the interest rates have risen with the LIBOR term structure is as follows

Term	Interest rate
125 day	6.65%
305 day	7.24%

The market value of this FRA based on a notional principal of R50 000 000 would be closest to

- 1 R95 000 $h=180$ $=1/1+0.0665(125/360) - (1$
 2 R335 000 $m=180$ $+0.0606(180/360)/(1+0.0724(305/360))$
 3 R370 000 $g=55$ $=0.00668 * 50\,000\,000$
 4 R2 616 000 $h-g=125$ 6.65% $=R334\,000$
 $h+m-g=180+180-55=305$ 7.24%

- 7 A share index option that expires in 75 days is currently at 1240.89 and makes no cash payments during the life of the option. Assume a share multiplier of 1 and a risk-free rate of 3%. The lowest possible price for a European-style call option on this index with an exercise price of 1225 would be closest to. Similar to Q6 a P245. Also example of Put option.

- 1 0
2 15.89 $1240.89-1225/(1.03)^{75/365}$
 3 23.31 $=23.31\$$
4 1240.89

[TURN OVER]

- 8 A two-period binomial model with a share that currently trades at \$50 in the spot market can either increase in value by 18 per cent or decrease by 15 per cent in the highly volatile market. The annual risk-free rate is 9 per cent and each period of the binomial model is equal to one year. The risk-neutral probability for the binomial model would be closest to

1	0.27	$(1+r)-d / u-d$
2	0.30	$(1+0.09) - (1-0.15) / (1+0.18)-0.85$
3	0.70	=0.727
✓ 4	0.73	=0.73

- 9 An asset manager wishes to enter into a two-year equity swap in which he will receive the rate of return on the FTSE 100 equity index in exchange for paying a fixed interest rate. The FTSE 100 equity index is at 1510.89 at the beginning of the swap. The swap has a notional principal of £100 million and calls for semi-annual payments. The current term structure of interest rates is

$$L_0(540) = 0.0642$$

$$L_0(720) = 0.0656$$

The present value discount factor for $L_0(540)$ would be closest to

- 1 0.8702
- 2 0.9122
- 3 0.9358
- 4 1.0642

- 10 An asset manager wishes to reduce his exposure to large-cap stocks and increase his exposure to small cap stocks. He seeks to do so using an equity swap. He agrees to pay a dealer the return on a large-cap index, and the dealer agrees to pay the manager the return on a small-cap index. The value of the small-cap index starts off at 712.70, and the large-cap index starts at 1230.10. In six months, the small-cap index is at 653.30 and the large-cap index is at 1270.92. Assume that payments are made semi-annually and the notional principal is \$5,000,000. The first overall payment and the party that makes the payment is most likely to be

1	R166,000	by the dealer to the asset manager
2	R166,000	by the asset manager to the dealer
✓ 3	R582,500	by the asset manager to the dealer
4	R582,500	by the dealer to the asset manager

(20)

$$\text{Small caps: } (653.3/712.70 - 1)(5000000) = -\$416725$$

Asset manager owes the dealer. If it was positive then dealer owed asset manager.

$$\text{Large caps: } (1270.92/1230.10 - 1)(5000000) = 165921$$

[TURN OVER]

Asset manager owes the dealer. If amount was negative the dealer owed Asset Manager.

Total $\$416725 + 165921 = 582646$. The asset manager owes the dealer.

SECTION B: FORWARDS AND FUTURES

(20 marks)

1 Adele, a treasury manager, wants to enter into a FRA that expires in 42 days and is based on the 162-day LIBOR. The dealer quotes a rate of 5.25% on this FRA. Assume that at expiration the 162-day LIBOR is 4.75% and the notional principal \$5,000,000.

1.1 What term is used to describe nonstandard FRAs?

(1)

Off the run FRA

1.2 Calculate the payoff of the FRA for the long position and indicate whether the long party makes a profit or a loss.

(2)

$$(0.0475 - 0.0525) \times (162/360) / (1 + 0.0475 \times (162/360)) \times 5,000,000$$

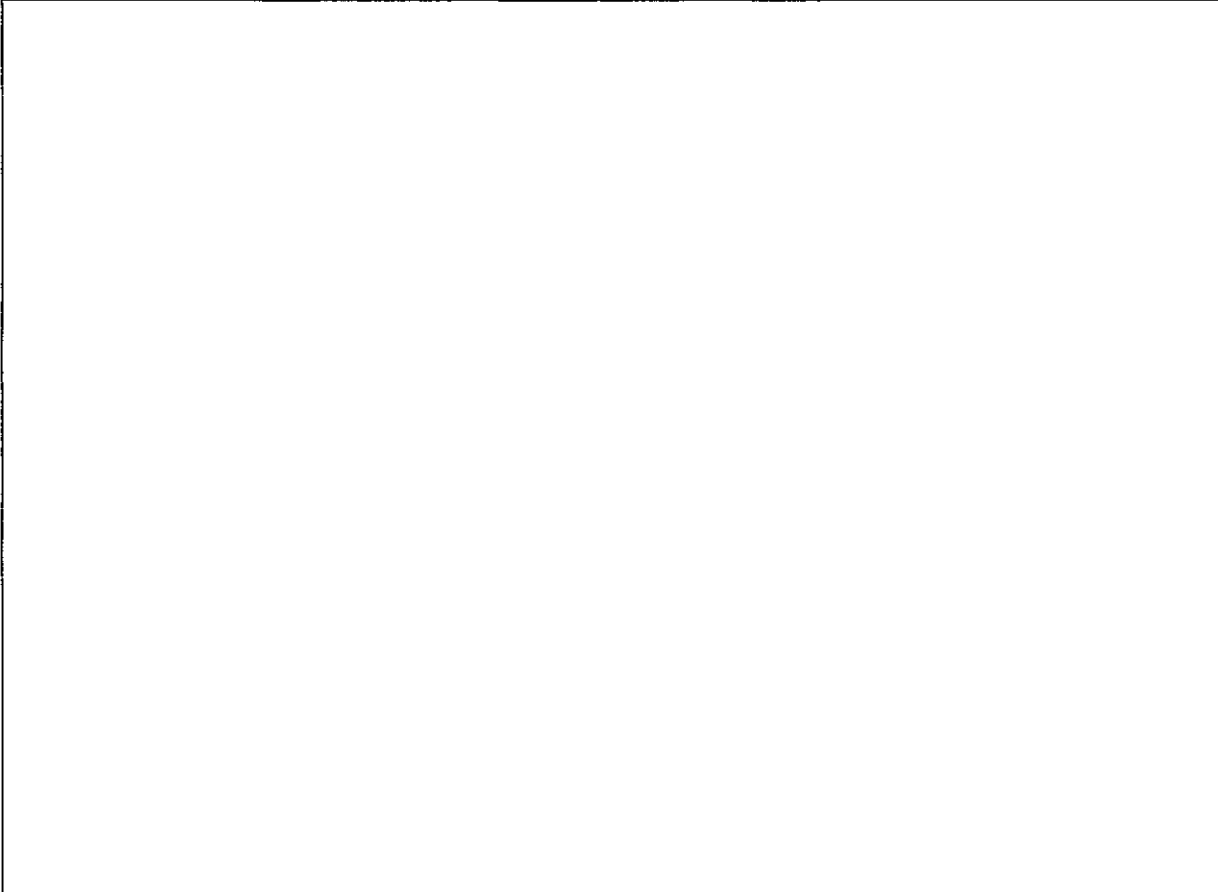
$$= -\$11,014.56$$

Because the party is long, the amount represents a loss.

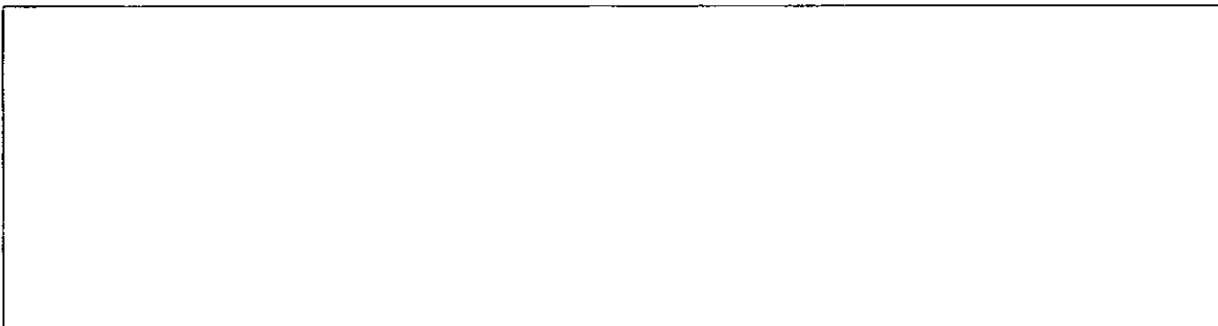
[TURN OVER]

2 A barrel of oil is currently priced at \$115 and the risk-free rate is 6%. Suppose a dealer was to offer you a contract in which the forward price of the barrel of oil with delivery in three months is \$109.

2.1 How would you take advantage of the situation to make an arbitrage profit? (3)



2.2 How much would your arbitrage profit be? (1)



[TURN OVER]

- 3 Edmund currently owns a share that pays regular dividends and trades at \$40. Edmund plans to sell the share in 320 days. In order to hedge against any possible price declines, Edmund takes a short position in a forward contract that expires in 320 days. The current risk-free rate is 6.5%. Over the next 320 days, the share will pay the following dividends:

Days to next dividend	Dividends per share
90	\$2.40
180	\$2.40
270	\$2.40

- 3.1 The initial price of the forward contract is \$34.89. Suppose it is 150 days after Edmund entered into the forward contract and the share price declined to \$32. What would the value of the contract be at this point and would the short position make a gain or loss? (6)

SIMILAR QUESTION PAGE 68 Q11 B

$S_t = \$32$ - share price declined to.

$F = 34.89$ (given) Initial price

$t = 150/365$

$T = 320/365$

$T-t = 170/365$

$r = 0.065$

We are 150 days later so it is only 30 days to the next dividend and $(270-150) = 120$ the next.

$2.40/(1.065)^{30/365} + 2.40/(1.065)^{120/365}$

$= 4.74$

$PV(D) = \$32 - 4.74 = -(34.89/(1.065)^{170/365})$

$= -\$6.62$

Negative value is a gain for the short.

[TURN OVER]

- 4 Copper is currently priced at \$90 per unit. A futures contract on the copper expires in 75 days and the risk-free rate is 7 per cent. Calculate the futures price of the copper if the storage cost at the futures expiration is equal to \$3 (2)

PAGE 114 QB

$$\$90(1.07)^{75/365} + 3 = \$94.26$$

- 5 The IMM index price was quoted yesterday at 94.23 for a March Eurodollar futures contract. Eurodollar futures contracts are each based on a \$1,000,000 notional principal and have 90-days to expiry.

- 5.1 Based on the rate of 94.23, what is the actual price of this futures contract? (2)

PAGE 144 Question 6 A & B

$$\begin{aligned} 100 - 94.23 &= 5.77\% \\ \$1,000,000(1 - 0.0577(90/360)) \\ &= \$985,575 \end{aligned}$$

[TURN OVER]

- 5.2 The IMM index price changed since yesterday, today the index is priced at 94.28. How much is the actual change in the futures price of the contract? (1)

$$100 - 94.28 = 5.72\%$$

$$1\,000\,000(1 - 0.0572(90/360)) = \$985\,700$$

Actual change: $985\,700 - 985\,575 = \$125$

OR

It changed 8 basis points

$$8 * \$25 (\text{always } 25) = 125$$

- 6 The current spot exchange rate for the British pound is \$1.3819/£1. The US interest rate is 3.6% and the UK interest rate is 2.8%. A futures contract on the exchange rate for the British pound expires in 340 days. Calculate the appropriate futures price of the forward contract. (2)

P146. Q18 A

$$\frac{S_0}{(1+r_f)^T} (1+r)^T$$

$$\frac{1.3819}{(1.028^{340/365})} (1+0.036)^{340/365}$$

$$= 1.3919$$

(20)

[TURN OVER]

SECTION C: OPTIONS

(15 marks)

1 A call option on an interest rate is based on a 180-day LIBOR rate of 7% and has a notional principal of \$1 000 000

1 1 Calculate the payoff at expiration for the call option if the exercise rate is 5% (2)
Similar to Page 244 Q3A

$$\begin{aligned} St &= 0.07 \\ (0.07 - 0.05) &\times (180/360) \times 1\,000\,000 \\ &= \$10\,000 \end{aligned}$$

1 2 Calculate the payoff at expiration for the call option if the exercise rate is 8% (1)

$$\begin{aligned} St &= 0.07 \\ (0.07 - 0.08) &\times (180/360) \times 1\,000\,000 \\ &= \$0 \text{ Can not be negative.} \end{aligned}$$

[TURN OVER]

- 2 Christie, an analyst at Brooklyn Investments collects the following information on call and put prices of a share

Call price	\$ 6 60
Put price	\$ 7 60
Share price	\$88 00
Exercise price	\$92 00
Risk-free rate	5 per cent
Days to expiration	329
Assume a 365 day year	

- 2.1 Use put-call parity to calculate the price of the synthetic put option (2)
PAGE 251 Q9 a

$$\text{Bond price: } 92/(1+0.05)^{329/365} = 88.04$$
$$6.60 + 88.04 - 88 = \$6.64$$

- 2.2 Use put-call parity to calculate the price of the synthetic call option (2)

$$7.60 + 88 - 88.04 = \$7.56$$

[TURN OVER]

- 3 Call and put options are available on a Netcare share (JSE NTC), the share is currently trading at R34 00 and the options have an exercise price of R35 00. The options expire in 310 days and the volatility is 0.40. The current continuously compounded risk-free rate is 5% and there are no dividends paid on this share. The exact call and put prices for this specific share according to the Black-Scholes option pricing model are as priced below.

Share price	Call price	Put price
R34 00	R1 19	R0 73

- 3.1 Calculate d_1 for the above share (Assume a 365 day year)

(3)

PAGE 237 Problem 12.

0.22 ONLINE CALCULATOR
EXERCISE PRICE IS SPOT PRICE

[TURN OVER]

- 3 2 Suppose the approximate delta value of the call option is 0.5871, what would the new price of the call option be if the Netcare share price changed with R3.00? (2)

Question 16 B page 248

$$0.5871(3) = 1.7613$$
$$1.19 + 1.7613 = 2.9513$$

- 4 Consider an asset that trades at \$30 today. Call and put options on this asset are available with an exercise price of \$34. The options expire in 300 days, and the volatility is 0.55. The continuously compounded risk-free rate is 4%. The asset has a dividend yield of 1%. Assume $N(d_1) = 0.5199$ and $N(d_2) = 0.3264$ and a 365-day year.

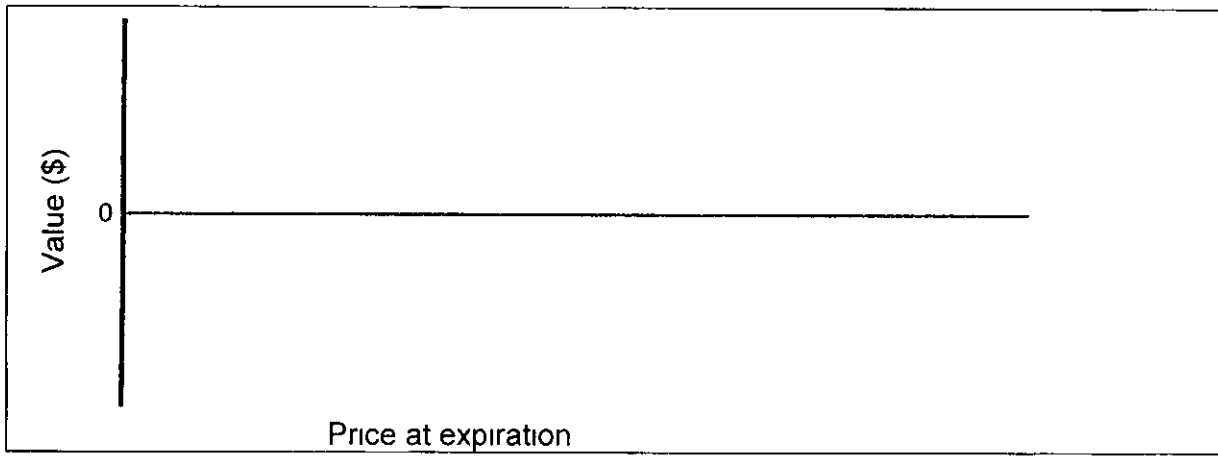
- 4 1 Use the Merton model to calculate the value of the European call option (2)

$$30(0.5199) - 34e^{-0.04(300/365)}(0.3264) = \$4.86$$

[TURN OVER]

5 Draw the option diagram of a short call on the following axis

(1)



(15)

[TURN OVER]

SECTION D: SWAPS

(10 marks)

- 1 Le Vu, a French company, issued a floating-rate bond and anticipates that interest rates will rise in the short term. Pierre, the treasurer, enters into a swap for a fixed payment with a dealer in order to hedge against the interest rate risk. The swap is on a notional principal of €5 000 000 and the company will pay a fixed rate of 6 per cent while receiving Euribor. The current Euribor rate is 5 per cent.
- 1.1 Calculate the first fixed and floating payments of the swap. Assume that the floating rate payments will be based on 90/360 days and the fixed rate at 90/365 days. (4)

Question4 page 319

Fixed payments: $5\,000\,000(0.06)(90/365) = \text{€}73972.60$ Floating payments: $5\,000\,000(0.05)(90/360) = \text{€}62500$

Net payment is €11472.6 made by the company paying the fixed

[TURN OVER]

- 2 An asset manager wishes to enter into a two-year equity swap in which he will receive the rate of return on the NYSE index (NYA IND) in exchange for paying a fixed interest rate. The NYSE index is at 10610.89 at the beginning of the swap. The swap has a notional principal of \$10 million and calls for semi-annual payments. The current term structure of interest rates is

$$L_0(180) = 0.0501$$

$$L_0(360) = 0.0556$$

$$L_0(540) = 0.0602$$

$$L_0(720) = 0.0656$$

The present value discount factors for 180, 360, 540 and 720 days are

$$B_0(180) = \frac{1}{1 + 0.0501 \left(\frac{180}{360} \right)} = 0.9756$$

$$B_0(360) = 0.9473$$

$$B_0(540) = 0.9172$$

$$B_0(720) = 0.8840$$

- 2.1 Calculate the annualised fixed rate on the swap per \$1 notional principal and Question 13 page 232

(2)

$$\begin{aligned} & 1 - 0.8840 / (0.9756 + 0.9473 + 0.9172 + 0.8840) \\ & = 0.0311 \\ & 0.0311 \times (360/180) \\ & = 0.0622 \end{aligned}$$

[TURN OVER]

Ninety days later the NYSE index is at 10908.16 and the present value factors of the term structure of interest are

$$B_{90}(180) = 0.9902$$

$$B_{90}(360) = 0.9615$$

$$B_{90}(540) = 0.9298$$

$$B_{90}(720) = 0.9024$$

2.2 Calculate the present value of the remaining fixed payment

(1)

$$0.0311(0.9902+0.9615+0.9298+0.9024)+1(0.9024)= 1.02 - \text{answer}$$

2.3 Calculate the value of the equity payment

(1)

$$(10908.16/10610.89)=1.028$$

[TURN OVER]

- 2.4 Calculate the market value of the swap and state whether the fixed rate payer pays or receives the market value (2)

$$(1.028 - 1.02) 10\,000\,000 = R80\,000$$

(10)

[TURN OVER]

SECTION E: THEORY AND APPLICATION

(5 marks)

1 Options are sensitive to a number of variables namely the spot price, strike price, volatility, time to maturity and interest rates

1 1 If the strike price of an option increases, would the value of the put option increase or decrease? (1)

Decrease

1 2 If the volatility of an option increases, would the value of the call option increase or decrease? (1)

Increase

2 Which type of derivative instrument can be considered as a variation of a forward contract as it is essentially equivalent to a series of forward contracts? (1)

SWAPS

3 Suppose an investor has a long position on gold futures that will expire in 6 months

3 1 If the investor wishes to close out the position before the expiry of the contacts, which transactions should the investor make? (1)

Enter into an opposite transaction. Go short. Offsetting

[TURN OVER]

3.2 What is this method called of closing out a futures position before expiry? (1)

Offset

(5)

TOTAL

[70 MARKS]

ROUGH WORK

(WILL NOT BE MARKED)

[TURN OVER]

ROUGH WORK

(WILL NOT BE MARKED)

[TURN OVER]

ROUGH WORK

(WILL NOT BE MARKED)

[TURN OVER]

Cumulative probabilities for a standard normal distribution

$$P(X \leq x) = N(x) \text{ for } x \geq 0 \text{ or } 1 - N(-x) \text{ for } x < 0$$

x	0 00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0 5000	0 5040	0 5080	0 5120	0 5160	0 5199	0 5239	0 5279	0 5319	0 5359
0.10	0 5398	0 5438	0 5478	0 5517	0 5557	0 5596	0 5636	0 5675	0 5714	0 5753
0.20	0 5793	0 5832	0 5871	0 5910	0 5948	0 5987	0 6026	0 6064	0 6103	0 6141
0.30	0 6179	0 6217	0 6255	0 6293	0 6331	0 6368	0 6406	0 6443	0 6480	0 6517
0.40	0 6554	0 6591	0 6628	0 6664	0 6700	0 6736	0 6772	0 6808	0 6844	0 6879
0.50	0 6915	0 6950	0 6985	0 7019	0 7054	0 7088	0 7123	0 7157	0 7190	0 7224
0.60	0 7257	0 7291	0 7324	0 7357	0 7389	0 7422	0 7454	0 7486	0 7517	0 7549
0.70	0 7580	0 7611	0 7642	0 7673	0 7704	0 7734	0 7764	0 7794	0 7823	0 7852
0.80	0 7881	0 7910	0 7939	0 7967	0 7995	0 8023	0 8051	0 8078	0 8106	0 8133
0.90	0 8159	0 8186	0 8212	0 8238	0 8264	0 8289	0 8315	0 8340	0 8365	0 8389
1.00	0 8413	0 8438	0 8461	0 8485	0 8508	0 8531	0 8554	0 8577	0 8599	0 8621
1.10	0 8643	0 8665	0 8686	0 8708	0 8729	0 8749	0 8770	0 8790	0 8810	0 8830
1.20	0 8849	0 8869	0 8888	0 8907	0 8925	0 8944	0 8962	0 8980	0 8997	0 9015
1.30	0 9032	0 9049	0 9066	0 9082	0 9099	0 9115	0 9131	0 9147	0 9162	0 9177
1.40	0 9192	0 9207	0 9222	0 9236	0 9251	0 9265	0 9279	0 9292	0 9306	0 9319
1.50	0 9332	0 9345	0 9357	0 9370	0 9382	0 9394	0 9406	0 9418	0 9429	0 9441
1.60	0 9452	0 9463	0 9474	0 9484	0 9495	0 9505	0 9515	0 9525	0 9535	0 9545
1.70	0 9554	0 9564	0 9573	0 9582	0 9591	0 9599	0 9608	0 9616	0 9625	0 9633
1.80	0 9641	0 9649	0 9656	0 9664	0 9671	0 9678	0 9686	0 9693	0 9699	0 9706
1.90	0 9713	0 9719	0 9726	0 9732	0 9738	0 9744	0 9750	0 9756	0 9761	0 9767
2.00	0 9772	0 9778	0 9783	0 9788	0 9793	0 9798	0 9803	0 9808	0 9812	0 9817
2.10	0 9821	0 9826	0 9830	0 9834	0 9838	0 9842	0 9846	0 9850	0 9854	0 9857
2.20	0 9861	0 9864	0 9868	0 9871	0 9875	0 9878	0 9881	0 9884	0 9887	0 9890
2.30	0 9893	0 9896	0 9898	0 9901	0 9904	0 9906	0 9909	0 9911	0 9913	0 9916
2.40	0 9918	0 9920	0 9922	0 9925	0 9927	0 9929	0 9931	0 9932	0 9934	0 9936
2.50	0 9938	0 9940	0 9941	0 9943	0 9945	0 9946	0 9948	0 9949	0 9951	0 9952
2.60	0 9953	0 9955	0 9956	0 9957	0 9959	0 9960	0 9961	0 9962	0 9963	0 9964
2.70	0 9965	0 9966	0 9967	0 9968	0 9969	0 9970	0 9971	0 9972	0 9973	0 9974
2.80	0 9974	0 9975	0 9976	0 9977	0 9977	0 9978	0 9979	0 9979	0 9980	0 9981
2.90	0 9981	0 9982	0 9982	0 9983	0 9984	0 9984	0 9985	0 9985	0 9986	0 9986
3.00	0 9987	0 9987	0 9987	0 9988	0 9988	0 9989	0 9989	0 9989	0 9990	0 9990

[TURN OVER]

PRICING AND VALUATION OF FORWARD

Discrete interest (r)

$$(1+r)^T$$

Continuous interest (r^c)

$$e^{r^c T}$$

Conversion ($r \leftrightarrow r^c$)

$$r^c = \ln(1+r)$$

$$r = e^{r^c} - 1$$

Discount interest

$$\text{Amount} \left[1 - r(d/360) \right]$$

Add-on interest

$$\text{Amount} \left[1 + r(d/360) \right]$$

Forward rate agreement (FRA)**Price/rate**

$$\text{FRA}_{\text{rate}} = \left[\frac{1 + L_o \left(\frac{h+m}{360} \right)}{1 + L_g \left(\frac{h}{360} \right)} - 1 \right] \left(\frac{360}{m} \right)$$

Value

$$V_0 = \left[\frac{1}{1 + L_g \left(\frac{h-g}{360} \right)} \right] - \left[\frac{1 + \text{FRA}_{\text{rate}} \left(\frac{m}{360} \right)}{1 + L_g \left(\frac{h+m-g}{360} \right)} \right]$$

Payoff

$$\text{FRA}_{\text{payoff}} = \text{NP} \left[\frac{(U_{\text{rate}} - \text{FRA}_{\text{rate}}) \left(\frac{U_{\text{days}}}{360} \right)}{1 + U_{\text{rate}} \left(\frac{U_{\text{days}}}{360} \right)} \right]$$

Forward contract – no cash flows**Price**

$$F_T = S_0 (1+r)^T$$

Value

$$V_0 = S_0 - \frac{F_T}{(1+r)^T} \quad V_T = S_T - \frac{F_T}{(1+r)^{T-1}}$$

Forward contract – equity**Price and value – discrete compounding**

$$F_T = [S_0 - \text{PV}(D)](1+r)^T$$

$$V_t = S_t - \text{PV}(D) - \left[\frac{F_T}{(1+r)^{T-t}} \right]$$

Price and value – continuous compounding

$$F_T = S_0 e^{(r^c - \delta^c)T}$$

$$V_t = S_t e^{-\delta(T-t)} - F_T e^{-r(T-t)}$$

Forward contract – fixed income**Price and value**

$$F_T = [B_0 - \text{PV}(C)](1+r)^T$$

$$V_t = B_t - \text{PV}(C) - \left[\frac{F_T}{(1+r)^{T-t}} \right]$$

Forward contract – currency**Price and value – discrete compounding**

$$F_T = S_0 \left[\frac{(1+r_d)^{d/365}}{(1+r_f)^{d/365}} \right]$$

$$V_t = \left[\frac{S_t}{(1+r_f)^{T-t}} \right] - \left[\frac{F_T}{(1+r_d)^{T-t}} \right]$$

Price and value – continuous compounding

$$F_T = S_0 e^{(r_d^c - r_f^c)(d/365)}$$

$$V_t = S_t e^{-r_f^c(T-t)} - F_T e^{-r_d^c(T-t)}$$

Interest rate parity (IRP)

$$(1+r_d)^{d/365} = (1+r_f)^{d/365} \left(\frac{F}{S} \right)$$

PRICING OF FUTURES**Futures price – no cost or benefit**

$$f_0(T) = S_0(1+r)^T$$

Futures price – net cost or benefit

$$f_0(T) = S_0(1+r)^T + FV(CB)$$

Futures price – stock

$$f_0(T) = S_0(1+r)^T - FV(D)$$

Futures price – stock index

$$f_0(T) = S_0 e^{(r-s)T}$$

Futures price – Treasury bill

$$f_0(T) = \frac{B_0(1+r)^T - FV(C)}{\text{Conversion Factor}}$$

Futures price – currency*Discrete compounding*

$$f_0(T) = S_0 \left[\frac{(1+r_d)^{d/365}}{(1+r_f)^{d/365}} \right]$$

Continuous compounding

$$f_0(T) = S_0 e^{(r_d - r_f)(d/365)}$$

PRICING AND VALUATION OF SWAPS**Net fixed payment**

$$NFP = (\text{swap rate} - \text{LIBOR}) \left(\frac{\text{days}}{360} \right) NP$$

Swap fixed rate

$$C = \left(\frac{1 - Z_4}{Z_1 + Z_2 + Z_3 + Z_4} \right)$$

Discount rate

$$Z_{\text{day}} = \frac{1}{1 + \left(R_{\text{day}} \times \frac{\text{days}}{360} \right)}$$

Market value of interest rate swap

$$MV_{\text{IRS}} = V_{\text{floating-rate bond}} - V_{\text{fixed-rate bond}}$$

Market value of currency swap

$$MV_{\text{CS}} = V_{\text{domestic bond}} - V_{\text{foreign bond}}$$

Return on equity

$$\text{Return} = \left(\frac{\text{Ending value}}{\text{Beginning value}} \right)$$

Yield on equity

$$\text{Yield} = \left(\frac{\text{Ending value}}{\text{Beginning value}} \right) - 1$$

Payment on equity position

$$PMT = \text{Yield} \times NP$$

Market value of equity swap

$$MV_{\text{ES}} = NP(\text{Return}_x - \text{Return}_y)$$

Swaption payoffs

$$\text{Payoff}_{\text{Swaption-payer}} = (\text{SFR} - X) \left(\frac{\text{days}}{360} \right) NP$$

$$\text{Payoff}_{\text{Swaption-receiver}} = (X - \text{SFR}) \left(\frac{\text{days}}{360} \right) NP$$

PRICING OF OPTION CONTRACTS**Intrinsic values**

$$c = \max[0, (S - X)]$$

$$p = \max[0, (X - S)]$$

Bounds – European options*No cash flows – upper and lower*

$$[S - X(1+r)^{-1}] \leq c \leq S$$

$$[X(1+r)^{-1} - S] \leq p \leq X(1+r)^{-1}$$

Cash flows – lower bounds

$$c \geq [S - PV(CF)] - PV(X)$$

$$p \geq PV(X) - [S - PV(X)]$$

Bounds – American options

$$[S - X(1+r)^{-1}] \leq C \leq S$$

$$(X - S) \leq P \leq X$$

Put-call parity – European options**No cash flows**

$$S + p = c + X(1+r)^{-1}$$

Cash flows

$$[S - PV(CF)] + p = c + PV(X)$$

Futures contracts

$$F_T(1+r)^{-1} + p = c + X(1+r)^{-1}$$

Put-call parity – American options

$$S - X \leq C - P \leq S - X(1+r)^{-1}$$

Binomial model

$$p = \frac{(1+r) - d}{u - d}$$

$$f = \frac{[(p)(f^+) + (1-p)(f^-)]}{(1+r)}$$

$$f = \frac{[(p)^2(f^{++}) + 2(p)(1-p)(f^{+-}) + (1-p)^2(f^{--})]}{(1+r)^2}$$

Black-Scholes Merton model**Black-Scholes model**

$$d_1 = \frac{\ln(S/X) + [r^c + (\sigma^2/2)]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = SN(d_1) - Xe^{-r^c T}N(d_2)$$

$$p = Xe^{-r^c T}N(-d_2) - SN(-d_1)$$

Merton's model

$$d_1 = \frac{\ln(S/X) + [(r^c - \delta) + (\sigma^2/2)]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = Se^{-\delta T}N(d_1) - Xe^{-r^c T}N(d_2)$$

$$p = Xe^{-r^c T}N(-d_2) - Se^{-\delta T}N(-d_1)$$

[$r^c = r_f^c$ and $\delta = r_f^c$ when pricing currency options]

Black's model

$$d_1 = \frac{\ln(F/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$c = e^{-r^c T} [FN(d_1) - XN(d_2)]$$

$$p = e^{-r^c T} [XN(-d_2) - FN(-d_1)]$$

Delta

$$\text{Delta} = \frac{f_1 - f_0}{S_1 - S_0} = N(d_1)$$

Interest rate options

$$IR_{\text{coll}} = NP(U_{\text{rate}} - X_{\text{rate}}) \left(\frac{d}{360} \right)$$

$$IR_{\text{put}} = NP(X_{\text{rate}} - U_{\text{rate}}) \left(\frac{d}{360} \right)$$

PART 1 (GENERAL/ALGEMEEN) DEEL 1

STUDY UNIT e.g. PSY100-X STUDIE-EENHEID by PSY100-X		INITIALS AND SURNAME VOORLETTERS EN VAN	
1		3	
PAPER NUMBER VRAESTELNOMMER		DATE OF EXAMINATION DATUM VAN EKSAMEN	
2		4	
STUDENT NUMBER STUDENTENOMMER		EXAMINATION CENTRE (E.G. PRETORIA) EKSAMENSENTRUM (BY PRETORIA)	
6		5	
7		8	
9		9	

For use by examination invigilator
Vir gebruik deur eksamenopsiener

- IMPORTANT**
1. USE ONLY AN HB PENCIL TO COMPLETE THIS SHEET
 2. MARK LIKE THIS
 3. CHECK THAT YOUR INITIALS AND SURNAME HAS BEEN FILLED IN CORRECTLY
 4. ENTER YOUR STUDENT NUMBER FROM LEFT TO RIGHT
 6. CHECK THAT YOUR STUDENT NUMBER HAS BEEN FILLED IN CORRECTLY
 6. CHECK THAT THE UNIQUE NUMBER HAS BEEN FILLED IN CORRECTLY
 7. CHECK THAT ONLY ONE ANSWER PER QUESTION HAS BEEN MARKED
 8. DO NOT FOLD

- BELANGRIK**
1. GEBRUIK SLEGS N HB POTLOOD OM HIERDIE BLAD TE VOLTOOI
 2. MERK AS VOLG
 3. KONTROLEER DAT U VOORLETTERS EN VAN REG INGEVUL IS
 4. VUL U STUDENTENOMMER VAN LINKS NA REGS IN
 6. KONTROLEER DAT U DIE KORREKTE STUDENTENOMMER VERSTREK HET
 6. KONTROLEER DAT DIE UNIEKE NOMMER REG INGEVUL IS
 7. MAAK SEKER DAT NET EEN ALTERNATIEF PER VRAAG GEMERK IS
 8. MOENIE YOU NIE

PART 2 (ANSWERS/ANTWOORDE) DEEL 2

1	2	3	4	5	36	37	38	39	40	71	72	73	74	75	106	107	108	109	110
6	7	8	9	10	41	42	43	44	45	76	77	78	79	80	111	112	113	114	115
11	12	13	14	15	46	47	48	49	50	81	82	83	84	85	116	117	118	119	120
16	17	18	19	20	51	52	53	54	55	86	87	88	89	90	121	122	123	124	125
21	22	23	24	25	56	57	58	59	60	91	92	93	94	95	126	127	128	129	130
26	27	28	29	30	61	62	63	64	65	96	97	98	99	100	131	132	133	134	135
31	32	33	34	35	66	67	68	69	70	101	102	103	104	105	136	137	138	139	140

Specimen only