UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2605

October/November 2012

DEPARTMENT OF DECISION SCIENCES
LINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS
FIRST DR S BELABBES
SECOND DR MP MULAUDZI

Programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of five pages and a sheet of graph paper

INSTRUCTIONS:

Answer all the questions.

Show all workings.

Marks will be allocated for intermediate steps and not for final answers only

[TURN OVER]

Question 1 **[10]**

Remove the graph paper attached to this examination paper and use it to answer this question. Write your student number and the module code on the graph paper and, after you have answered the question, place it inside your answer book

Consider the following LP model

$$\begin{array}{ll} \text{Minimise} & z = 4x_1 + 7x_2 \\ \text{subject to} & \\ & \begin{array}{rcrcr} 4x_1 & + & 3x_2 & \geq & 12 \\ 5x_1 & + & x_2 & \leq & 6 \\ x_1 & + & x_2 & \leq & 3 \end{array} \\ \text{and} & x_1, x_2 \geq 0 \end{array}$$

(a) Represent the constraints on a graph Indicate the solution set of each constraint clearly on your graph (7)

(b) Find the solution to the LP model Write down your findings in detail (3)

Question 2 **[4]**

Determine the inverse of $F = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$

Question 3 **[8]**

Let $A = \begin{bmatrix} -2 & 4 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 0 & 3 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & -4 \\ 5 & 0 & 1 \end{bmatrix}$.

Where possible, compute the matrix or value represented by each of the following expressions State clearly when an operation is not defined, and explain why

(a) $CA + B$ (3)

(b) $(AC)^T$ (2)

(c) B^T (1)

(d) $|C|$ (2)

Question 4 [10]

A South African company manufactures two types of office chairs, namely a business type (A) and an executive type (B) The production process at the company is divided into three distinct phases These phases are carpentry, finishing touches and packaging
The production of chair type (A) requires 7 hours of carpentry, 3 hours of finishing touches and 2 hours of packaging The production of chair type (B) requires 4 hours of carpentry, 2 hours of finishing touches and 3 hours of packaging
Due to the limited availability of skilled labour as well as machines and tools, 100 hours of carpentry, 30 hours of finishing touches and 20 hours of packaging are available each day
The profit for type (A) is R200 and for type (B) is R500 Management wants to determine how many chairs of type (A) and type (B) should be produced each day in order to maximise the total profit

Formulate this problem as a linear programming model Define the variables clearly
DO NOT solve your model

Question 5 [12]

Solve the following system of equations using the Gauss-Jordan method

$$\begin{array}{rcrcrcr} -2x & + & y & + & 3z & = & -12 \\ 3x & & & - & z & = & 8 \\ & & 3y & + & 2z & = & 10 \end{array}$$

Question 6 [12]

The Northern Coal Company operates two coal mines, Alpha and Beta It costs R75 000 per day to operate the Alpha mine and R60 000 per day to operate the Beta mine The coal ore is crushed and then processed to produce two grades of ore, premium grade and standard grade
The company has set weekly contracts with dealers To satisfy these contracts at least 300 tons of premium grade ore and at least 500 tons of standard grade ore must be produced per week
The Alpha mine averages a production output of 30 tons of premium grade ore and 75 tons of standard grade ore per day The Beta mine averages a production output of 60 tons of premium grade ore and 25 tons of standard grade ore per day
The company has labour contracts with its employees that guarantee them a full day's pay for each day or fraction of a day that the mine is open Management wants to determine how to operate the mines at the lowest possible cost

Formulate this problem as a linear programming model Define the variables clearly
DO NOT solve your model

[TURN OVER]

Question 7 [12]

The following systems of equations were obtained by applying the simplex method to different maximisation problems (x_1, x_2, x_3 are decision variables, s_1, s_2, s_3 are slack variables and Z indicates the value of the objective function)

In each case, state whether the given solution is optimal or not.

- If the solution is optimal, write down the complete solution and identify any special kind of solution or constraint Justify your answer
- If the solution is not optimal, determine the entering and leaving variables for the next iteration of the simplex method Justify your answer
- If there is no feasible solution, or if the solution is unbounded, state why.

(a)

$$\begin{array}{rrrrrrrl} & 25x_1 & + 50x_2 & - 3x_3 & + s_1 & & & = 95 \\ & 7x_1 & + 14x_2 & + x_3 & & + s_2 & & = 70 \\ & - x_1 & + 11x_2 & + 5x_3 & & & + s_3 & = 110 \\ Z & - 5x_1 & + 2x_2 & - 8x_3 & & & & = 50 \end{array}$$

(b)

$$\begin{array}{rrrrrrrl} & & - 4x_2 & & + 18s_1 & + 25s_2 & + s_3 & = 64 \\ & x_1 & - 2x_2 & & + 30s_1 & + s_2 & & = 90 \\ & & & x_3 & + 12s_1 & - 5s_2 & & = 120 \\ Z & & - 7x_2 & & - 6s_1 & + 2s_2 & & = 245 \end{array}$$

(c)

$$\begin{array}{rrrrrrl} & x_1 & & - 2s_1 & & & = 12 \\ & & 3x_2 & + 10s_1 & + 25s_2 & & = 75 \\ & & 10x_2 & - 6s_1 & - 5s_2 & + s_3 & = 120 \\ Z & & & + 3s_1 & + 4s_2 & & = 80 \end{array}$$

Question 8 [12]

Consider the following LP model

$$\begin{array}{ll} \text{Maximise} & Z = 2x_1 - x_2 + x_3 \\ \text{subject to} & \\ & \begin{array}{rcrcrcr} 3x_1 & + & x_2 & + & x_3 & \leq & 60 \\ x_1 & - & x_2 & + & 2x_3 & \leq & 10 \\ x_1 & + & x_2 & - & x_3 & \leq & 20 \end{array} \\ \text{and} & x_1, x_2, x_3 \geq 0. \end{array}$$

(a) Solve this linear programming model using the simplex method (9)

(b) Give the optimal solution in full (3)

TOTAL [80]