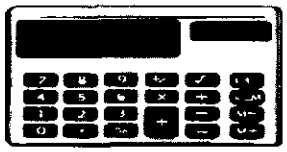
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2605

May/June 2013

DEPARTMENT OF DECISION SCIENCES
LINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS
FIRST DR S BELABBES
SECOND DR MP MULAUDZI

Programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of **five** pages and a sheet of graph paper

INSTRUCTIONS:

Answer all the questions.

Show all workings.

Marks will be allocated for intermediate steps and not for final answers only

Question 1 [10]

Remove the graph paper attached to this examination paper and use it to answer this question Write your student number and the module code on the graph paper and, after you have answered the question, place it inside your answer book

Consider the following LP model

$$\begin{aligned} &\text{Maximise} && Z = 70x_1 + 30x_2 \\ &\text{subject to} \\ & && 5x_1 + 6x_2 \leq 90 \\ & && 3x_1 + 2x_2 \leq 30 \\ & && x_1 + x_2 \geq 10 \\ &\text{and} && x_1, x_2 \geq 0 \end{aligned}$$

(a) Represent the constraints on a graph Indicate the feasible region clearly on your graph (7)

(b) Use the corner-point method to solve the LP model Write down your findings in detail (3)

Question 2 [4]

Determine the inverse of $F = \begin{bmatrix} 3 & 5 \\ 7 & 8 \end{bmatrix}$

Question 3 [8]

Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -3 \\ 13 & 8 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 2 & 6 \\ -1 & 7 & 5 \end{bmatrix}$

Where possible, compute the matrix represented by each of the following expressions State clearly when an operation is not defined, and explain why

(a) $CA + B$ (2)

(b) $(AC)^T$ (3)

(c) B^T (1)

(d) $|C|$ (2)

Question 4 [12]

For a telephonic survey, a marketing company needs to contact at least 150 wives, 120 husbands, 100 single adult males, and 110 single adult females It costs R8 to make a daytime call and R12 to make an evening call The following table lists the results

Person responding	Percentage of daytime calls	Percentage of evening calls
Wife	30%	30%
Husband	10%	30%
Single male	10%	15%
Single female	10%	20%
None	40%	5%

Because of limited staff, at most half of all phone calls can be evening calls

Formulate a LP to minimise the cost of completing the survey Define the variables clearly **DO NOT** solve your model

Question 5 [6]

Let $D = \begin{bmatrix} 3a-b & b+3c \\ 2b+c & -a+4b \end{bmatrix}$ and $E = \begin{bmatrix} 15 & 9 \\ 8 & 6 \end{bmatrix}$

What are the values of a, b and c in matrix D above if $D - E = 0$?

Question 6 [12]

A railway company wishes to purchase a new fleet of railway carriages The fleet must have the capacity to transport at least 2 800 tons of coal and at least 1 300 tons of lumber at any given moment Furthermore, at least a quarter of the carriages must be able to transport coal The company can buy three types of carriages models A, B and C

- One carriage of model A costs 5,8 million rand and can transport 21 tons of coal
- One carriage of model B costs 3,6 million rand and can transport 5 tons of lumber
- One carriage of model C costs 6,1 million rand and can transport either 17 tons of coal or 9 tons of lumber (not both simultaneously)

What is the least cost for which the company can buy a fleet with the desired capacity?

Formulate this problem as a linear programming model Define the variables clearly **DO NOT** solve your model

Question 7 [15]

The following systems of equations were obtained by applying the simplex method to different maximisation problems (x_1, x_2, x_3 are decision variables, s_1, s_2, s_3 are slack variables and Z indicates the value of the objective function)

In each case, state whether the given solution is optimal or not.

- If the solution is optimal, write down the complete solution and identify any special kind of solution or constraint Justify your answer
- If the solution is not optimal, determine the entering and leaving variables for the next iteration of the simplex method Justify your answer
- If there is no feasible solution, or if the solution is unbounded, state why

(a)

$$\begin{array}{rrrrrrrrrrr} & 2x_1 & + & x_2 & & + & 2s_1 & & & = & 1\,400 \\ & x_1 & & + & x_3 & & + & s_2 & & = & 600 \\ & 3x_1 & & + & 2x_3 & - & s_1 & + & s_3 & = & 650 \\ Z & -2x_1 & & - & 8x_3 & + & 6s_1 & & & = & 4\,200 \end{array}$$

(b)

$$\begin{array}{rrrrrrrrrrr} & 0{,}5x_1 & + & x_2 & & + & 0{,}5s_1 & & & = & 350 \\ & x_1 & & + & x_3 & & + & s_2 & & = & 600 \\ & 0{,}5x_1 & & & & - & 0{,}5s_1 & - & s_2 + s_3 & = & 50 \\ Z & +1{,}5x_1 & & & & + & 1{,}5s_1 & + & 2s_2 & = & 2\,250 \end{array}$$

(c)

$$\begin{array}{rrrrrrrrrrr} & -5x_1 & + & x_2 & & + & 3s_2 & & & = & 30 \\ & -2x_1 & & & & - & s_2 & + & s_3 & = & 30 \\ & -x_1 & & & + & s_1 & + & 20s_2 & & = & 20 \\ Z & -85x_1 & & & & & + & 20s_2 & & = & 600 \end{array}$$

Question 8 [13]

Consider the following LP model

$$\begin{aligned}
&\text{Maximise} \quad Z = 80x_1 + 40x_2 + 120x_3 - 10x_4 \\
&\text{subject to} \\
&\qquad\qquad x_1 + 2x_2 + x_3 + 5x_4 \leq 150 \\
&\qquad\quad 6x_1 + 7x_2 + 2x_3 - x_4 \geq 120 \\
&\qquad\qquad\quad x_2 - 4x_3 + 8x_4 = 70 \\
&\text{and} \qquad x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

(a) Write down the standard form of this LP model (2)

(b) Write down the augmented model that follows from (a) (4)

(c) Write down the initial basic variables with which the simplex method will be started, and write down the values of these variables (3)

(d) Which variable should enter the basis in the next iteration of the simplex method? Explain your answer (2)

(e) Which variable should leave the basis at the next iteration of the simplex method? Explain your answer. (2)

TOTAL [80]

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