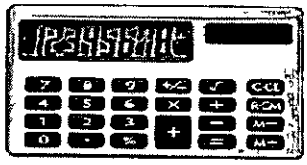
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2605

May/June 2014

LINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS

FIRST DR S BELABBES

SECOND PROF MP MULAUDZI

Programmable pocket calculator is permissible

Closed book examination

This paper consists of 4 pages and a sheet of graph paper.

INSTRUCTIONS

Answer all the questions.

Show all workings.

Marks will be allocated for intermediate steps and not for final answers only.

Question 1

Remove the graph paper attached to this examination paper and use it to answer this question Write your student number and the module code on the graph paper and, after you have answered the question, place it inside your answer book

Consider the following LP model

$$\begin{aligned} &\text{Minimise } z = 3x_1 + 2x_2 \\ &\text{subject to} \\ &\qquad x_1 - x_2 \leq 0 \\ &\qquad 2x_1 + 3x_2 \geq 12 \\ &\qquad x_2 \leq 7 \\ &\text{and} \qquad x_1, x_2 \geq 0 \end{aligned}$$

1 1 Represent the constraints on a graph Show the solution set of each constraint clearly on your graph Show the feasible region of the LP model clearly on your graph (7)

1 2 Use the corner-point method to solve the LP model Write down your findings in detail (3)

[10]

Question 2

Let $A = \begin{bmatrix} -3 & 0 \\ 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Where possible, compute the matrix or value represented by each of the following expressions State clearly when an operation is not defined

2 1 $A + A$ and $A + B$ (2)

2 2 AAA and BBB (4)

2 3 A^{-1} and B^{-1} (4)

2 4 A^T and B^T (2)

[12]

Question 3

Compute the determinant of $\begin{bmatrix} 2 & -4 & 5 \\ 1 & 0 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

[3]

Question 4

A cook on a ship wants to purchase a total of 70 litres of oils and fats for a voyage to Antarctica He wants to spend R1350 and has to pay R20 per litre for beef tallow, R35 per litre for olive oil, and R10 per litre for sunflower oil A dietitian advised him to use equal amounts of animal fat and plant oil on a journey to cold climates How much of each should he purchase?

[10]

Question 5

You have 100 units of aluminium and 200 units of wood available You can either make tables (which require 2 units of aluminium and 3 units of wood) or chairs (which require 4 units of aluminium and 2 units of wood)

Tables and chairs sell for the same amount Formulate a linear programming model to maximise income Define the variables clearly **DO NOT** solve your model

[8]

Question 6

A businessman has the option of investing R800 000 in two funds Plan Xeno offers a return of 7% for each full year of investment, while plan YYY offers 16% for money invested for two full years

He wants to invest his money into these funds for three years, after which he wants as much cash as possible for another project To diversify he requires that, at the start, nor more than 75% gets allocated to one of the plans

Formulate a linear programming model for this problem Define the variables clearly **DO NOT** solve your model

[12]

Question 7

Consider the following LP model

$$\begin{aligned} &\text{Minimise } z = 4x_1 + 4x_2 + x_3 \\ &\text{subject to} \\ &\qquad x_1 + x_2 + \quad x_3 \leq 2 \\ &\qquad 2x_1 + x_2 \qquad\quad \leq 3 \\ &\qquad 2x_1 + x_2 + \quad 3x_3 \geq 3 \\ &\text{and} \qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

Keep the model as a **minimisation problem** and answer the following questions

7 1 Write this LP model in the augmented form (2)

7 2 Rewrite the objective function in the form required to start the Big M method (2)

7 3 Write down the entering variable for the first iteration of the Big M method Explain your answer (1)

7 4 Write down the leaving variable for the first iteration of the Big M method Explain your answer (2)

7 5 Do one iteration of the Big M method (4)

7 6 Is this the optimal solution? Explain your answer (1)

7 7 Give the solution at this stage in full (3)

[15]

Question 8

Consider the following NLP model

$$\begin{aligned} &\text{Minimise } z = (x-1)^2 + (y-1)^2 \\ &\text{subject to} \\ &\qquad x - 2y \geq -2 \\ &\qquad x + 2y \leq 10 \\ &\qquad x - y \leq 4 \\ &\qquad x \geq 2 \\ &\qquad y \geq 0 \end{aligned}$$

8 1 Which two of the following points are not in the feasible area?

$$(6,1),(6,2),(3,1),(3,3),(2,1),(2,2)$$

(2)

8 2 For each of the feasible points, calculate the objective function value (2)

8 3 Can we conclude that $(2,1)$ is a local minimum? (2)

[6]

Question 9

Rewrite the following LP model in a form that can be entered into LINDO

$$\begin{aligned} &\text{Maximise } z = 300x_1 + 250x_2 \\ &\text{subject to} \\ &\qquad 2x_1 + x_2 \leq 40{,}5 \text{ (Labour time)} \\ &\qquad x_1 + 3x_2 \leq 45{,}25 \text{ (Machine time)} \\ &\qquad x_1 \leq 12{,}75 \text{ (Marketing)} \\ &\text{and} \quad x_1, x_2 \geq 0 \end{aligned}$$

[4]