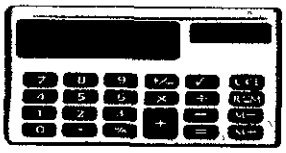
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2605

October/November 2014

LINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS :
FIRST DR S BELABBES
SECOND DR JE SINGLETON

Use of a non-programmable pocket calculator is permissible.

Closed book examination.

This paper consists of 4 pages and a sheet of graph paper

INSTRUCTIONS

Answer all the questions.

Show all workings.

Marks will be allocated for intermediate steps and not for final answers only

Question 1 [10]

Remove the graph paper attached to this examination paper and use it to answer this question Write your student number and the module code on the graph paper and, after you have answered the question, place it inside your answer book

Consider the following LP model

$$\begin{aligned} &\text{Minimise } z = 4x_1 + 7x_2 \\ &\text{subject to} \\ &\qquad 4x_1 + 3x_2 \geq 12 \\ &\qquad 5x_1 + x_2 \leq 6 \\ &\qquad x_1 + x_2 \leq 3 \\ &\text{and} \qquad x_1, x_2 \geq 0 \end{aligned}$$

(a) Represent the constraints on a graph Indicate the solution set of each constraint clearly on your graph (7)

(b) Find the solution to the LP model Write down your findings in detail (3)

Question 2 [3]

Prove that $B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

is the inverse of $A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

Question 3 [8]

Let $A = \begin{bmatrix} -2 & 4 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 0 & 3 \\ 2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & -4 \\ 5 & 0 & 1 \end{bmatrix}$

Where possible, compute the matrix or value represented by each of the following expressions State clearly when an operation is not defined, and explain why

(a) $CA + B$ (3)

(b) $(AC)^T$ (2)

(c) B^T (1)

(d) $|C|$ (2)

Question 4 [14]

A student would like to design a breakfast of corn flakes and milk that is as economical as possible On the basis of what he eats during his other meals, he decides that breakfast should supply him with at least 9 grams of protein, at least $\frac{1}{3}$ of the recommended daily allowance (RDA) of vitamin D, and at least $\frac{1}{4}$ of the RDA of calcium He finds the following nutritional information on the milk and corn flakes containers

	Milk ($\frac{1}{2}$ cup)	**Corn flakes (1 portion=30 g**
Cost	R1	75c
Protein	4 grams	2 grams
Vitamin D	$\frac{1}{8}$ of RDA	$\frac{1}{10}$ of RDA
Calcium	$\frac{1}{6}$ of RDA	None

In order not to make his mixture too soggy or too dry, the student decides to limit himself to mixtures that contain 1 to 3 portions of corn flakes per cup of milk What quantities of milk and corn flakes should he use to minimise the cost of his breakfast?

Formulate this problem as a linear programming model Define the variables clearly
DO NOT solve your model

Question 5 [8]

Use the Gauss-Jordan method to solve the following system of equations

$$\begin{aligned} x + y + z &= 3 \\ 2x + 3y + 7z &= 0 \\ x + 3y - 2z &= 17 \end{aligned}$$

Question 6 [12]

For a telephonic survey, a marketing company needs to contact at least 150 wives, 120 husbands, 100 single adult males, and 110 single adult females It costs R8 to make a daytime call and R12 to make an evening call The following table lists the results

Person responding	**Percentage of daytime calls**	**Percentage of evening calls**
Wife	30%	30%
Husband	10%	30%
Single male	10%	15%
Single female	10%	20%
None	40%	5%

Because of limited staff, at *most half* of all phone calls can be evening calls

Formulate a linear programming model to minimise the cost of completing the survey Define the variables clearly **DO NOT** solve your model

Question 7

[12]

Consider the following LP model

$$\begin{aligned} &\text{Maximise } Z = 5a + 7b \\ &\text{subject to} \\ &\qquad 10a + 10b \leq 100 \\ &\qquad 3a + 7b \leq 80 \\ &\qquad b \leq 40 \\ &\text{and} \quad a, b, \geq 0 \end{aligned}$$

(a) Do one iteration of the simplex method on the LP model (8)

(b) Is this the optimal solution? (1)

(c) Give the solution at this stage in full (3)

Question 8

[13]

Consider the following LP model

$$\begin{aligned} &\text{Maximise } Z = 80x_1 + 40x_2 + 120x_3 - 10x_4 \\ &\text{subject to} \\ &\qquad x_1 + 2x_2 + x_3 + 5x_4 \leq 150 \\ &\qquad 6x_1 + 7x_2 + 2x_3 - x_4 \geq 120 \\ &\qquad x_2 - 4x_3 + 8x_4 = 70 \\ &\text{and} \quad x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

(a) Write down the standard form of this LP model (2)

(b) Write down the augmented model that follows from (a) (4)

(c) Write down the initial basic variables with which the simplex method will be started, and write down the values of these variables (3)

(d) Which variable should enter the basis in the next iteration of the simplex method? Explain your answer (2)

(e) Which variable should leave the basis at the next iteration of the simplex method? Explain your answer (2)

TOTAL [80]