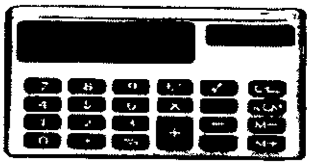
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2606

May/June 2013

NONLINEAR MATHEMATICAL PROGRAMMING
DEPARTMENT OF DECISION SCIENCES

Duration 2 Hours

80 Marks

EXAMINERS
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Programmable pocket calculator is permissible

Closed book examination

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This paper consists of 4 pages

ANSWER **ALL** THE QUESTIONS

Question 1

Compound interest arises in a transaction that extends over a period of time when interest due at the end of a payment period is not paid but is added to the principal. Thereafter the interest also earns interest, i.e. it is compounded.

The formula for compound interest is as follows

$$A = P(1+r)^t \quad \text{and} \quad I = A - P$$

where A is the amount which accrues when an initial principal of P is invested at interest rate r per term for t terms, and where I is the compound interest.

1.1 For which value(s) of t is A a linear function of r? (1)

1.2 For which value(s) of t is A a nonlinear function of r? (1)

1.3 Determine dA/dr (1)

1.4 Derive an expression for t in terms of A, P and r (3)

1.5 At what interest rate per annum must money be invested if the accrued principal must treble in ten years? (3)

[9]

[TURN OVER]

Question 2

The total cost $T(n)$ (in rand) of ordering, purchasing, and storing inventory for a year is given by the cost equation

$$T(n) = \frac{480\,000}{n} + 12n + 146\,400$$

where n is the order quantity per order

2 1 Prove that the function $T(n)$ is convex over the interval $(0, \infty)$ (4)

2 2 Determine the optimal order quantity (Which n minimises $T(n)$?) (2)

[6]

Question 3

Determine the optimal solution to the following nonlinear programming (NLP) problem

$$\text{Maximise} \quad f(x) = 1 - e^{-x}$$

$$\text{subject to} \quad 1 \leq x \leq 3$$

Show your reasoning (6)

[6]

Question 4

Consider the cubic function $f(x) = x^3 - 9x^2 + 15x + 74$

4 1 Determine the stationary points of the function $f(x)$ and prove their nature (relative minimum or maximum) by applying the second derivative test (9)

4 2 Determine the inflection point(s) of the function $f(x)$ (1)

4 3 Determine the interval(s) where the function $f(x)$ is

4 3 1 decreasing

4 3 2 convex (3)

4 4 Calculate $f(-1)$ en $f(-3)$ From this, derive a meaningful starting interval for applying the bisection algorithm in determining a real root of the equation $f(x) = 0$ (3)

4 5 Estimate an approximate integer value for a zero of $f(x)$ (1)

4 6 Draw a graph of the function $f(x)$ Clearly indicate the relative extrema and the intercepts on both axes (4)

[21]

Question 5

Find the point on the parabola $y = \frac{1}{2}x^2$ that is closest to the point $(\frac{27}{2}, 1)$ on the Euclidean plane (x, y) (8)

[8]

Question 6

The trapezoidal rule is given by

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[f(a) + f(b)] + h\sum_{i=1}^{n-1} f(x_i),$$

where the interval $[a, b]$ is divided into n subintervals $[x_i, x_{i+1}]$ of width $h = (b - a)/n$, with $x_0 = a$ and $x_n = b$

Estimate the area under the curve

$$f(x) = \sqrt{3x + 1}$$

on the interval $[1, 5]$ to three decimal places, by applying the trapezoidal rule with $n = 2$ intervals

By how much does the estimate differ from the actual area? (8)

[8]

Question 7

Consider the following nonlinear programming (NLP) problem

$$\begin{aligned} \text{Minimise} \quad & f(x,y) = \frac{50}{x} + \frac{20}{y} + xy \\ \text{subject to} \quad & x \geq 3, y \geq 2 \end{aligned}$$

7 1 Calculate the objective function value for the point $(10, 10)$ (1)

7 2 Is the point $(1, 1)$ a feasible solution to the NLP problem? Substantiate your answer (1)

7 3 The Kuhn-Tucker necessary conditions for this NLP problem are as follows

(1) $-50x^{-2} + y - \lambda_1 = 0$

(2) $-20y^{-2} + x - \lambda_2 = 0$

(3) $\lambda_1(-3 - x) = 0$

(4) $\lambda_2(-2 - y) = 0$

(5) $\lambda_1, \lambda_2 \geq 0$

Verify whether the point $(5, 2)$ satisfies

7 3 1 the Kuhn-Tucker necessary conditions

7 3 2 the Kuhn-Tucker sufficient conditions

What conclusion about optimality can you draw from this? (7)

[9]

Question 8

Use the method of Lagrange multipliers to solve the following nonlinear programming (NLP) problem

$$\text{Minimise } f(x,y) = 2x^2 + 3y^2 + x - 9y + 16$$
$$\text{subject to } \quad x + y = 5$$

(13)

[13]

TOTAL: 80

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