UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

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NONLINEAR MATHEMATICAL PROGRAMMING
Department of Decision Sciences

Duration 2 Hours 80 Marks

EXAMINERS :
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Programmable pocket calculator is permissible.

Closed book examination.

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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS

Question 1

Consider the following nonlinear programming (NLP) problem

$$\text{Maximise} \quad f(x) = 1 - e^{-x}$$

$$\text{subject to} \quad 1 \le x \le 4$$

1 1 Identify any horizontal or vertical asymptotes to the function $f(x)$ (1)

1 2 Examine the sign of the first and second derivative to determine whether the function $f(x)$ is

1 2 1 increasing or decreasing

1 2 2 convex or concave (6)

1 3 Write down the Kuhn-Tucker necessary conditions for this NLP problem (6)

1 4 Solve this NLP problem (4)

1 5 Determine the value of the Lagrange multipliers in the Kuhn-Tucker necessary conditions for the optimal solution to the NLP problem (2)

[19]

[TURN OVER]

Question 2

Consider the fifth power polinomial $g(x) = -x^5 + 80x + 32$

2.1 Approximate a zero of $g(x)$ by performing one iteration of Newton's method, starting with $x_0 = 0$ in the formula

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}, \quad n = 0, 1, 2,$$

(3)

2.2 Determine the stationary points and the inflection point(s) of the function $g(x)$. Determine the nature of the stationary points by applying the second derivative test (9)

2.3 Determine the interval(s) where the function $g(x)$ is

2.3.1 increasing

2.3.2 concave (4)

2.4 Use the bisection method to estimate an approximate integer value for a root of the equation $g(x) = 0$ in the interval $(0, 4)$ (6)

2.5 How many real roots does the equation $g(x) = 0$ have? (1)

2.6 Draw a graph of the function $g(x)$. Clearly indicate the stationary points, the scale and the intercept(s) on each axis (5)

2.7 Consider the following nonlinear programming (NLP) problem

$$\text{Maximise} \quad g(x) = -x^5 + 80x + 32$$

$$\text{subject to} \quad 1 \leq x \leq 3$$

Perform *one* iteration of the golden section search method to solve this problem (The golden section ratio is $r = \frac{\sqrt{5}-1}{2} = 0{,}618$.) (7)

[35]

Question 3

Suppose that a cylindrical soda can must have a volume of 340 cubic centimetres. If the soda company wants to minimise the surface area of the soda can, what should be the ratio of the height of the can to the radius of the can?

Hint: The volume of a circular cylinder is $\pi r^2 h$, and the surface area of a circular cylinder is $2\pi r^2 + 2\pi r h$, where r = the radius of the cylinder and h = the height of the cylinder (10)

[10]

Question 4

Suppose it costs R2,00 to purchase an hour of labour and R1,00 to purchase a unit of capital. If L hours of labour and K units of capital are purchased, then $L^{\frac{2}{3}}K^{\frac{1}{3}}$ machines can be produced.

The maximum number of machines that can be produced if R10,00 is available for the purchase of labour and capital must be determined.

4 1 Formulate this optimisation problem as a nonlinear mathematical programming (NLP) problem (3)

4 2 Formulate the Lagrangian NLP to this optimisation problem (1)

4 3 Solve this NLP problem (8)

[12]

Question 5

An efficiency study conducted for the Electra Electronics Company showed that the rate at which walkie-talkies are assembled by the average worker t hours after starting work at 8 00 in the morning, is given by the function

$$f(t) = -3t^2 + 12t + 15 \quad \text{for} \quad 0 \leq t \leq 4$$

Determine how many walkie-talkies can be assembled by the average worker in the second hour of the morning shift (4)

[4]

TOTAL: 80

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