DSC2606

May/June 2014

NONLINEAR MATHEMATICAL PROGRAMMING
Department of Decision Sciences

Duration 2 Hours

80 Marks

EXAMINERS
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Programmable pocket calculator is permissible

Closed book examination.

This paper consists of 4 pages

ANSWER **ALL** THE QUESTIONS.

Question 1

Determine the stationary point(s) and the inflection point(s) of the function

$$f(x) = -x^5 + 80x + 32$$

Determine the nature of the stationary point(s) by applying the second derivative test (10)

[10]

Question 2

Determine all the real roots of the equation $h(x) = (x-2)(x^2-5) = 0$ (3)

[3]

Question 3

Approximate the positive zero of the function $f(x) = x^2 - 3$ by performing *one* iteration of Newton's method, starting with $x_0 = 0$ in the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots.$$

Calculate the difference between this approximate value and the real value of $\sqrt{3}$ (5)

[5]

Question 4

Ship Shape produces small cabin cruisers, the Neptune The cost function is given by

$$C(x) = 100\,000 + 6\,000x + 4x^2,$$

where x is the number of Neptunes produced per year The unit price of a Neptune is related to demand by the following demand function

$$p(x) = 30\,000 - 2x$$

How many Neptunes should be produced to maximise profit? Find the maximum annual profit and the optimal selling price (12)

[12]

Question 5

Estimate the area under the curve

$$f(x) = \sqrt{3x+1}$$

on the interval $[1, 5]$ to three decimal places, by applying the trapezoidal rule with $n = 2$ intervals

By how much does this estimate differ from the actual area (calculated by applying the fundamental theorem of calculus)?

The trapezoidal rule is given by

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[f(a) + f(b)] + h\sum_{i=1}^{n-1} f(x_i),$$

where the interval $[a, b]$ is divided into n subintervals $[x_i, x_{i+1}]$ of width $h = (b-a)/n$, with $x_0 = a$ and $x_n = b$ (9)

[9]

Question 6

The total cost (in rand) of ordering, purchasing, and storing inventory for a year is given by the cost equation

$$K(Q) = \frac{480\,000}{Q} + 12Q + 146\,400$$

where Q is the order quantity per order

6 1 Prove that the function $K(Q)$ is convex over the interval $(0, \infty)$ (4)

6 2 Determine the optimal order quantity
(Which Q minimises $K(Q)$?) (2)

[6]

Question 7

Consider the following nonlinear programming (NLP) problem

$$\text{Maximise} \quad g(x) = \frac{\ln x}{x}$$
$$\text{subject to} \quad 1 \leq x \leq 6$$

Perform *one* iteration of the golden section search method to solve this problem (The golden section ratio is $r = \frac{\sqrt{5}-1}{2} = 0{,}618$) (7)

[7]

Question 8

Consider the following nonlinear programming (NLP) problem

$$\text{Maximise} \quad f(x) = e^{-x}$$
$$\text{subject to} \quad 1 \leq x \leq 4$$

8 1 Determine the first derivative of the function $f(x)$ and from that draw a conclusion about the nature of the function (2)

8 2 Draw a graph of the function $f(x)$ and solve this NLP problem (6)

8 3 The Kuhn-Tucker necessary conditions for this NLP problem are as follows

$$\begin{aligned} -e^{-x} - \lambda_1 + \lambda_2 &= 0 \qquad (1) \\ \lambda_1(4 - x) &= 0 \qquad (2) \\ \lambda_2(x - 1) &= 0 \qquad (3) \\ \lambda_1,\ \lambda_2,\ &\geq 0 \qquad (4) \end{aligned}$$

Determine the value of the Lagrange multipliers in the optimal solution to the NLP problem (2)

[10]

Question 9

Suppose it costs R2,00 to purchase an hour of labour and R1,00 to purchase a unit of capital If L hours of labour and K units of capital are purchased, then $L^{\frac{2}{3}}K^{\frac{1}{3}}$ machines can be produced

The maximum number of machines that can be produced if R10,00 is available for the purchase of labour and capital must be determined

9 1 Formulate this optimisation problem as a nonlinear mathematical programming (NLP) problem (3)

9 2 Determine the first-order partial derivatives of the function $f(L, K) = L^{\frac{2}{3}}K^{\frac{1}{3}}$ (2)

9 3 Determine the second-order partial derivatives of the function $f(L, K) = L^{\frac{2}{3}} K^{\frac{1}{3}}$ (4)

9 4 Formulate and solve the Lagrangian NLP to this optimisation problem (9)

[18]