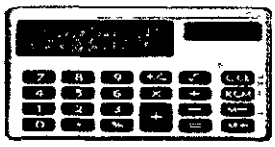
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC2606

October/November 2014

NONLINEAR MATHEMATICAL PROGRAMMING

Duration 2 Hours

80 Marks

EXAMINERS
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SECOND PROF WL FOUCHE

Programmable pocket calculator is permissible

Closed book examination.

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This paper consists of 3 pages

ANSWER **ALL** THE QUESTIONS

Question 1

Consider the function

$$f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x + 2$$

1 1 Determine the stationary points and inflection points of the function f, and determine where the function is convex and where it is concave (6)

1 2 Draw the graph of the function f Clearly indicate the stationary points, the inflection points, and the x and y intercepts (3)

1 3 Use the graph you drew in (b) to approximate integer values for the solution to the equation $f(x) = 0$ (2)

[11]

Question 2

Let

$$g(x) = (x - \sqrt{3})^3$$

2 1 Use two steps of the bisection method to approximate a root of the equation $g(x) = 0$ in the interval [1,2] (5)

2 2 Use two steps of Newton's method to approximate the square root of 3 Start with the initial point $x = 1$ (5)

2 3 Use the trapezoidal method to estimate the integral of the function g between $x = 2$ and $x = 3$ Use four equal subintervals of [2,3] to do the calculation Find the integral of the function through calculus and calculate the difference with the approximation (7)

[17]

Question 3

Consider the nonlinear program

$$\text{Maximise} \quad f(x) = 3x - e^{-2x}$$

$$\text{subject to} \quad 1 \leq x \leq 2$$

3 1 Use calculus to solve the program (4)

3 2 Use one iteration of the golden section method to estimate a solution for the program (4)

[8]

Question 4

A wire of length 50cm can be bent into a circle, bent into a square or cut into two pieces to make both a circle and a square Calculate how much wire should be used for the circle if the total area enclosed by the figures is to be a

4 1 maximum (8)

4 2 minimum (4)

[12]

Question 5

Consider the following NLP model

$$\text{Min } z = f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2)^2$$

subject to

$$-x_1 + x_2 = 1$$
$$x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

5 1 Write down the Kuhn-Tucker conditions for this NLP (13)

5 2 Use the Kuhn-Tucker conditions to find the optimal solution to this NLP (6)

[19]

Question 6

Use the method of Lagrange multipliers to solve the following nonlinear programming (NLP) problem

$$\text{Minimise } f(x,y) = 2x^2 + 3y^2 + x - 9y + 16$$
$$\text{subject to } \quad x + y = 5$$

(13)

[13]

TOTAL: 80