DSC1520 (499164) October/November 2011
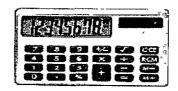
DEPARTMENT OF DECISION SCIENCES
QUANTITATIVE MODELLING I

Duration   2 Hours

100 Marks

EXAMINERS
FIRST   MRS MF IMMELMAN
SECOND   DR G DAVIE

Programmable pocket calculator is permissible

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This examination question paper consists of 17 pages, including 4 pages for rough work The paper comprises 25 questions that count a total of 100 marks

THE USE OF A PROGRAMMABLE CALCULATOR IS PERMITTED.

The paper is divided into two parts Section A and Section B

SECTION A   [60]

Answer ALL the questions in this section on the mark-reading sheet supplied Follow the instructions for completing the mark-reading sheet carefully
Also pay attention to the following information Suppose you are asked the question

$$3 + 2 \times -1 + 4 - 2 =$$

[1]   7
[2]   1
[3]   3
[4]   4
[5]   None of the above

The correct answer is 3 and therefore option [3] should be marked Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct If you mark more than one option, you will receive no marks for that question For a correct answer you will receive 3 MARKS Marks WILL NOT be deducted for incorrect answers

[TURN OVER]

Section A consists of 20 questions and counts 60 marks   Hand in the completed mark-reading sheet with your answers for Section B DO NOT STAPLE IT!

SECTION B

This section must be completed in the space provided below each question   Section B consists of five questions and counts 40 marks

Remember to include your MARK-READING SHEET IN YOUR ANSWER BOOK

SECTION A   [60]

Question 1

A company that produces washing machines decreased its production last year by 15% per week owing to a financial crisis If the present output is 425 washing machines per week, calculate the size of its production before the production cut

- [1]   362
- [2]   370
- [3]   489
- [4]   500
- [5]   None of the above

Question 2

Simplify

$$\frac{2}{3} \div \frac{1}{2} - 2 + \frac{2}{4}\left(-\frac{1}{2} + 2\frac{1}{6}\right)$$

- [1] $\frac{11}{12}$
- [2] $\frac{17}{24}$
- [3] $\frac{1}{6}$
- [4] $\frac{2}{3}$
- [5]   None of the above

Question 3

Determine the equation of the line that has a y-intercept of 3 and is parallel to the line

$$y = 4x + 1$$

- [1] $y = 3x + 1$
- [2] $y = 3$
- [3] $y = 4x + 3$
- [4] $y = x + 4$
- [5]   None of the above

[TURN OVER]

Question 4

The x-intercept of the line $2x = 3y - 5$ is equal to

[1] $\frac{5}{3}$

[2]   2

[3] -5

[4] $-\frac{5}{2}$

[5]   none of the above

Question 5

The demand function and cost function for a commodity are given by the equations

$$\text{Demand function} \quad Q = 6\,000 - 30P$$
$$\text{Cost function} \quad C(Q) = 72\,000 + 60Q$$

with P and Q the price and quantity respectively The total revenue function in terms of P is

[1]   Revenue $= 6\,000 - 30P$

[2]   Revenue $= 6\,000P - 30P^2$

[3]   Revenue $= -30P^2 + 7\,800P - 108\,000$

[4]   Revenue $= 360\,000 - 1\,800P$

[5]   none of the above

Question 6

If the demand function is equal to $P = 60 - 2,5Q$ with P and Q the price and quantity respectively, determine the expression for the price elasticity of demand if the price is equal to $P = 10$

[1] $-2,500$

[2] $-0,008$

[3] $-0,800$

[4] $-0,200$

[5]   None of the above

Question 7

Simplify

$$\left(\frac{3L^{0,5}}{L^{-2}}\right)^2$$

[1] $9L^5$

[2] $3L^{-3}$

[3] $3L$

[4] $3L^5$

[5]   None of the above

Question 8

The demand and supply functions for tent hire are given by the equations

$$\text{Demand function} \quad P_d = 400 - 5Q_d$$
$$\text{Supply function} \quad P_s = 3Q_s + 24$$

with P_d and P_s respectively the price demanded and supplied and Q_s and Q_d the quantity supplied and demanded. The equilibrium price and quantity are

[1] $P = 165, Q = 47$

[2] $P = 400, Q = 24$

[3] $P = 70, Q = 15$

[4] $P = 135, Q = 53$

[5]   none of the above

Question 9

Solve the following set of equations

$$x + 2y - z = 5$$
$$2x - y + z = 2$$
$$y + z = 2$$

[1]   Impossible to calculate

[2] $x = 8,5, y = 3,5, z = -1,5$

[3] $x = 1, y = 3, z = 2$

[4] $x = 1,75, y = 1,75 \ z = 0,25$

[5]   None of the above

[TURN OVER]

Question 10

The demand function of a specific product is $P = 80 - 2{,}5Q$ where P and Q represent the quantity and price respectively Calculate the consumer surplus for the demand function when the market price is $P = 15$

[1]   195

[2]   1 040

[3]   845

[4]   318,75

[5]   None of the above

Question 11

The roots of the function

$$y = x^2 + x - 6$$

are

[1] $x = 2$ and $x = -3$

[2] $x = 3$ and $y = 2{,}5$

[3] $x = -0{,}5$ and $x = -6{,}25$

[4] $x = -2$ and $x = 3$

[5]   None of the above

Question 12

A supplier supplies 50 sports hats when the price is R6 each and 90 hats when the price is R11 each Determine the equation of the linear supply function in the form $P = f(Q)$

[1] $P = 0{,}125Q - 0{,}25$

[2] $P = 8Q + 2$

[3] $P = 12{,}25 - 0{,}125Q$

[4] $P = 8 - 98Q$

[5]   None of the above

Question 13

Calculate $\log_5 \left(\frac{1200}{\sqrt{25}} \right)$ to four decimal places

[1]   2,3802

[2]   3,4053

[3]   4,8719

[4]   1,4180

[5]   None of the above

Question 14

The number of people who contracted a contagious disease t days after an epidemic started is approximated by the exponential equation

$$Q(t) = \frac{5\,000}{1 + 1\,249e^{-0,33t}}$$

Approximately how many people had contracted the disease after 15 days?

[1]   37

[2]   508

[3]   2 167

[4]   5 009

[5]   None of the above

Question 15

Differentiate the following function

$$f(x) = 3x^6 + x^4 + \sqrt{x} + 300$$

[1] $f'(x) = 3x^5 + x^3 + \frac{1}{\sqrt{x}}$

[2] $f'(x) = \frac{3}{7}x^7 + \frac{x^5}{5} + 2x + 300x$

[3] $f'(x) = x^7 + x^5 + x^{\frac{3}{2}} + 300x$

[4] $f'(x) = 18x^5 + 4x^3 + \frac{1}{2}x^{-\frac{1}{2}}$

[5]   None of the above

Question 16

What is the value of a company's maximum profit if the profit function (in millions of rand) is given by

$$P(x) = -x^2 + 6x + 7?$$

[1]   −R1 million

[2]   R3 million

[3]   R7 million

[4]   R16 million

[5]   None of the above

Question 17

Given the demand function $P = 20 - 0{,}2Q$ where P and Q are the price and quantity respectively, calculate the price elasticity of demand in terms of P only

[1] $\frac{P}{P-20}$

[2] $\frac{20}{P-0{,}2}$

[3] $\frac{P-20}{0{,}2}$

[4] $\frac{P}{P-10}$

[5] None of the above

Question 18

A company has a total cost function of $TC = 2Q^3 - Q^2 + 80Q + 150$ Determine the marginal cost if the quantity is equal to 10

[1] 80

[2] 118

[3] 660

[4] 2 850

[5] None of the above

Question 19

Determine the following indefinite integral

$$\int \left(2x + 4x^3\right) dx$$

[1] $2 + 12x^2 + c$

[2] $2x^2 + 4x^4 + c$

[3] $2x + x^4 + c$

[4] $x^2 + x^4 + c$

[5] None of the above

Question 20

Evaluate the following definite integral

$$\int_{x=0}^{x=4} (x^3 + 2x)dx$$

[1] 80

[2] 48

[3] 288

[4] 72

[5] None of the above

Please turn over for Section B.

SECTION B

Remember to include your **MARK-READING SHEET** in your answer paper.

Please fill in your answer to Section A in the spaces provided below before you move on to Section B in case you mark-reading sheet gets misplaced

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Option																				

This section must be completed in the space provided below each question.

Question 21 $\boxed{8}$

Pianni Beverages produce two ready-mixed cocktails drinks, the Zombie and the Skyjack Each is a mixture of vodka, vermouth and ginger It takes 3 litres of vodka, 6 litres of vermouth and one litre of ginger to make a container of Zombie and 5 litres of vodka, 3 litres of vermouth and 2 litres of ginger to make a container of Skyjack The maximum litres vodka and vermouth available per day are 1 500 litres vodka and 1 500 litres of vermouth The minimum litres ginger available per day is 400 litres of ginger If x is the number of containers of Zombie mixed and y the number of containers of skyjack mixed, determine the system of inequalities that best describes this situation Do not solve the system of inequalities

Question 22

6

A virus is thought to spread through a game reserve's impala population according to the equation

$$N = 125{,}5e^{0{,}12t}$$

where N is the number of impala infected after t days. After approximately how many days are 200 impala infected?

Question 23

$\boxed{10}$

(a) Draw a graph of the lines representing the following constraints on the grid below

$$-x + y \leq 3$$
$$x + y \leq 5$$
$$x \leq 3$$
$$x, y \geq 0$$

and indicate the region where all the constraints are satisfied (feasible region)   (6)

[TURN OVER]

(b) Determine the maximum value of $P = 2x + 4y$, subject to the constraints in (a)   (4)

Question 24   10

Lighting Warehouse produces q solar lamps at a fixed cost of R10 000 per week. Each lamp costs R150 to produce and is sold for R350

(a) Write down the equation of total revenue, total cost and profit functions   (6)

(b) Calculate the break-even quantity per week   (2)

(c) How many lamps were produced and sold if they made a loss of R2 000?   (2)

Question 25
$\boxed{6}$

Find the values of x for which the function $y = -x^3 + 9x^2 - 24x + 26$ has a minimum or maximum value

TOTAL

[100]

ROUGH WORK

ROUGH WORK

ROUGH WORK

ROUGH WORK