

# Tutorial Letter 101/3/2015

Quantitative Modelling

DSC1520

Semesters 1 and 2

Department of  
Decision Sciences

**Important Information:**

This tutorial letter contains important information  
about your module.

Bar code

## Contents

|           |  |           |
|-----------|--|-----------|
| <b>1</b>  | <b>Introduction and welcome</b>                  | <b>4</b>  |
| <b>2</b>  | <b>Purpose of module</b>                         | <b>4</b>  |
| <b>3</b>  | <b>Lecturer</b>                                  | <b>4</b>  |
| <b>4</b>  | <b>Communication with the University</b>         | <b>4</b>  |
| <b>5</b>  | <b>Student support system</b>                    | <b>5</b>  |
| 5.1       | Study groups . . . . .                           | 5         |
| 5.2       | E-tutor . . . . .                                | 5         |
| 5.3       | Tutor classes . . . . .                          | 5         |
| 5.4       | Online services . . . . .                        | 5         |
| <b>6</b>  | <b>Study material</b>                            | <b>6</b>  |
| <b>7</b>  | <b>Syllabus for the module</b>                   | <b>7</b>  |
| <b>8</b>  | <b>Relevant sections of the text book</b>        | <b>7</b>  |
| <b>9</b>  | <b>Learning outcomes and assessment criteria</b> | <b>9</b>  |
| <b>10</b> | <b>Calculator</b>                                | <b>11</b> |
| <b>11</b> | <b>Study approach</b>                            | <b>11</b> |
| 11.1      | Study time . . . . .                             | 11        |
| 11.2      | Study method . . . . .                           | 11        |
| 11.3      | Study plan . . . . .                             | 11        |
| 11.3.1    | Study unit 1: Preliminaries . . . . .            | 12        |
| 11.3.2    | Study unit 2: Linear functions . . . . .         | 13        |
| 11.3.3    | Study unit 3: Linear algebra . . . . .           | 14        |
| 11.3.4    | Study unit 4: Non-linear functions . . . . .     | 15        |
| 11.3.5    | Study unit 5: Beginning calculus . . . . .       | 16        |
| <b>12</b> | <b>Assessment</b>                                | <b>18</b> |
| 12.1      | Examination . . . . .                            | 18        |
| 12.2      | Assignment 1, 2 and 3 (COMPULSORY!) . . . . .    | 18        |
| 12.3      | Evaluation exercises . . . . .                   | 19        |

---

|           |  |           |
|-----------|--|-----------|
| 12.3.1    | How to attempt the evaluation exercises . . . . .    | 20        |
| 12.3.2    | Evaluating your answers . . . . .                    | 20        |
| <b>13</b> | <b>ASSIGNMENTS: SEMESTER 1</b>                       | <b>21</b> |
| 13.1      | Assignment 01 (COMPULSORY): MCQ format . . . . .     | 21        |
| 13.2      | Assignment 02 (COMPULSORY): Written format . . . . . | 25        |
| 13.3      | Assignment 03 (COMPULSORY): MCQ format . . . . .     | 27        |
| <b>14</b> | <b>ASSIGNMENTS: SEMESTER 2</b>                       | <b>30</b> |
| 14.1      | Assignment 01 (COMPULSORY): MCQ format . . . . .     | 30        |
| 14.2      | Assignment 02 (COMPULSORY): Written format . . . . . | 34        |
| 14.3      | Assignment 03 (COMPULSORY): MCQ format . . . . .     | 37        |
| <b>15</b> | <b>Solutions: Self-evaluation exercises</b>          | <b>40</b> |
| 15.1      | Self-Evaluation Exercise 1 : Unit 1 . . . . .        | 40        |
| 15.2      | Self-Evaluation Exercise 2: Unit 2 . . . . .         | 50        |
| 15.3      | Self-Evaluation Exercise 3 : Unit 3 . . . . .        | 69        |
| 15.4      | Self-Evaluation Exercise 4: Unit 4 . . . . .         | 92        |
| 15.5      | Self-Evaluation Exercise 5: Unit 5 . . . . .         | 108       |

## 1 Introduction and welcome

It is a pleasure to welcome you to this module: **QUANTITATIVE MODELLING**.

We hope that you will enjoy this module and complete it successfully.

It is essential that you read this tutorial letter, Tutorial Letter 101, 2015, as well as Tutorial Letter 301, 2015, very carefully.

- Tutorial Letter 301 contains general information relevant to all undergraduate students in the Department of Decision Sciences.
- Tutorial letter 101 contains information about this particular module, including the compulsory assignments for this module.

For other detailed information and requirements see *myStudies@Unisa*, which you received with your tutorial matter.

## 2 Purpose of module

To introduce the learner to basic mathematical concepts and computational skills for application in the business world.

## 3 Lecturer

You will find the name of the lecturer responsible for this module in Tutorial letter 301. Transfer the name, email address and telephone number of the lecturer to the space provided below.

All queries about the content of this module should be directed to the lecturer.

## 4 Communication with the University

If you need to contact the University about matters **not related to the content of this module**, please consult the publication *myStudies@Unisa*, which you received with your study material. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

## 5 Student support system

For information on the various student support systems and services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the publication *myStudies@Unisa*, which you received with your study material.

### 5.1 Study groups

It is advisable to have contact with fellow students. One way to do this is to form study groups. The addresses of students in your area may be obtained by contacting the Student Administration and Registration department. See the publication *myStudies@Unisa* for details.

### 5.2 E-tutor

The University facilitates e-tutors to promote student success and student interaction. The e-tutor will help with, for example, solving of problems, lectures on the study material or exercise and solutions regarding the study material.

### 5.3 Tutor classes

The University facilitates a tutor service in some centres to assist students. At present students in DSC1520 can benefit from this support service. Interest from a minimum of 15 students is required for a tutor to be appointed. **There is a fee payable to attend these classes.** To find out more about the tutorial services in your area you can phone the regional office of Unisa nearest to you.

### 5.4 Online services

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University, and communicate electronically with the University and fellow students.

As a registered Unisa student you have free access to *myUnisa*, Unisa's learning management system and *myLife*, a free email address.

You can access *myUnisa* and *myLife* via the internet using an internet browser such as Internet Explorer or Mozilla Firefox etc., but to do this your computer must be linked to the internet.

If you do not have your own internet access, you may need to visit an internet cafe, library or learning centre in your area. These centres provide access to the internet at a small fee.

In line with Open Distance learning (ODL) principles, Unisa has established relations with Multipurpose Community Centres across the country in areas identified as remote. Registered Unisa students across South Africa's rural areas and townships can access free internet for academic purposes (access to *myUnisa*, emails, digital library, internet research and other computer based training modules) courtesy of Unisa.

For a contact centre close to you, see the publication *myStudies @ unisa* for details.

To use your *myUnisa* and *myLife* account you first need to register on the *myUnisa* website (<http://my.unisa.ac.za/>). During this process you will be issued with a username and choose your own passwords. Note that you first have to activate your *myLife* email account before you can activate your *myUnisa* account.

- **myLife**

*myLife* is a web-based email service that you can use to access your email from anywhere in the world using an internet browser. To activate your *myLife* email box, follow the following steps:

- Go to *myUnisa* at <http://my.unisa.ac.za/> and click on the “Claim *myLife* email” link.
- Provide your details by completing the e-form on the screen. This is done for verification purposes.
- Receive your *myLife* address and password.
- To access your email account, use the link <http://www.outlook.com/>, your *myLife* username (studentnumber@mylife.unisa.ac.za) and your chosen password.

If you prefer to use another email account, you can configure your *myLife* account to forward emails automatically. See *myStudies@Unisa* for details.

- **myUnisa**

The *myUnisa* learning management system is Unisa’s online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa – all through the computer and the internet. You will be able to join online discussion forums, submit your assignments and access a number of other resources. Before you can activate your *myUnisa* account you have to activate your *myLife* email account. To activate your *myUnisa* account follow the following steps:

- Create your free *myLife* email account before you join *myUnisa* as discussed before.
- Go to <http://my.unisa.ac.za/> and click on the “Join *myUnisa*” link.
- Complete the verification process and choose your own password.
- To log in to *myUnisa*, type in your student number and chosen password in the space provided on the top right-hand corner of the *myUnisa* opening page.

If you have any problems with *myUnisa* you may send an email to [myunisahelp@unisa.ac.za](mailto:myunisahelp@unisa.ac.za).

**Please consult *myUnisa* on a regular basis as the lecturer, from time to time, post additional information on *myUnisa*. This may include errata on study material, announcements or additional notes to help you better understand a certain part of the study material.**

## 6 Study material

Your study material consists of the following:

1. A prescribed book, which forms the basis of the study material in this module. It is advisable to purchase the text book as soon as possible. The text book is your most important source of reference in this module. **It is impossible to pass this module without the textbook.**

Teresa Bradley

**Essential Mathematics for Economics and Business**

John Wiley and Sons Publishing, Fourth Edition, 2013.

**OR**

Teresa Bradley and Paul Patton:  
**Essential Mathematics for Economics and Business**  
 John Wiley and Sons Publishing, Third Edition, 2008.

Please consult the list of booksellers and their addresses in *myStudies@Unisa*. If you have any difficulties with obtaining books from these bookshops, please contact the Section Prescribe Books as soon as possible by sending an email to *vospresc@unisa.ac.za*.

2. This tutorial letter, Tutorial Letter 101, 2015
3. Any additional tutorial letters that may be sent to you during the semester.

The Department of Despatch should supply you with the following study material, when you register:

- Tutorial letter 101, 2015
- Tutorial letter 301, 2015
- Booklet: *myStudies@Unisa*.

## 7 Syllabus for the module

The material has been subdivided into five units. The chapters of the textbook that make up the unit are stated adjacent to the unit.

| UNIT | TOPIC                | CHAPTERS |
|------|----------------------|----------|
| 1    | Preliminaries        | 1        |
| 2    | Linear functions     | 2        |
| 3    | Linear algebra       | 3 and 9  |
| 4    | Non-linear functions | 4        |
| 5    | Beginning calculus   | 6 and 8  |

The textbook forms the basis of your study material.

## 8 Relevant sections of the text book

Please find the sections in each chapter of the text book that are relevant for the module in the table below. Please see the section Study plan for a detailed plan for your studies.

| Study Unit              | Chapter in text book | Sections in text book                                | Progress Exercise in edition 3 / edition 4   |
|-------------------------|----------------------|--|--|
| 1. Preliminaries        | 1                    | 1.2; 1.3;<br>1.4; 1.5<br>1.6; 1.7<br>1.8             | 1.1, page 7/10<br>1.2, page 14/17<br>1.3, page 20/23<br>Make sure you know how to use your calculator  |
| 2. Linear Function      | 2                    | 2.1<br>2.2-2.3<br><br>2.4<br>2.6                     | 2.2, page 55/54<br>2.3, page 69/68<br>2.4, page 75/75<br>2.5, page 81/81<br>2.7, page 91/91  |
| 3. Linear Algebra       | 3<br><br><br>9       | 3.1<br>3.2.1<br>3.2.5<br>3.3<br>9.1                  | 3.1, page 110/111<br>3.2, page 116/117<br>3.3, page 125/126<br>3.4, page 131/131<br>9.1, page 485/487  |
| 4. Non-linear Functions | 4                    | 4.1<br><br>4.2<br><br>4.3<br><br>4.4                 | 4.1, page 152/152<br>4.2, page 158/158<br>4.3, page 163/163<br>4.4, page 170/170<br>4.5, page 176/177<br>4.6, page 179/179<br>4.8, page 183/184<br>4.9, page 186/186<br>4.10, page 188/188<br>4.11, page 195/196<br>4.12, page 199/199<br>4.13, page 201/201 |
| 5. Beginning Calculus   | 6<br><br><br>8       | 6.1<br>6.2.1<br><br>6.3.1<br>6.3.2<br>8.1-8.2<br>8.5 | 6.1, page 266/268<br>6.3, page 278/280<br>(Just Marginal problems)<br>6.5, page 287/289<br>6.6, page 292/295<br>8.1, page 433/435<br>8.3, page 445/447   |



## 9 Learning outcomes and assessment criteria

### Specific outcome 1:

Students understand and can apply mathematical concepts to do basic modelling.

#### Range:

The context is using linear functions solving elementary quantitative business problems such as demand, supply, cost and revenue functions and elasticity of demand and supply.

#### Assessment criteria:

1. Explain the concept of a function.
2. Explain the different characteristics of a linear function.
3. Determine the equation of a linear equation given slope, intercept, or two points on the line or combinations thereof.
4. Graphically represent a linear function by using its slope, intercept, equation or two points on the line.
5. Apply linear functions to problems in the business world, for example demand, supply, cost and revenue.
6. Describe and plot linear demand, supply, cost and revenue functions.
7. Describe the concept and calculate the price elasticity of demand and supply for linear demand and supply functions.

### Specific outcome 2:

Students are able to apply the basic concepts to solve equations and inequalities in practical problems.

#### Range:

The context is using sets of linear functions and inequalities to solve elementary quantitative business problems such as break-even analysis, market equilibrium, profit and loss functions and optimisation problems using linear programming.

#### Assessment criteria:

1. Solve a system of linear equations algebraically and graphically.
2. Use a set of equations to solve business problems for example break-even, equilibrium and profit and loss.
3. Determine the consumer surplus and producer surplus.
4. Graphically solve a system of linear inequalities with two variables.
5. Formulate the constraints and objective function of an optimising business problem by using linear programming.
6. Solve a linear programming problem graphically.

**Specific outcome 3:**

Students can apply the basic concepts of non-linear functions to solve practical problems.

**Range:**

Characteristics and properties of non-linear functions such as quadratic, cubic, logarithmic and exponential functions are used to solve elementary quantitative business problems like supply and demand functions, break-even analysis, market equilibrium and minimum and maximum values.

**Assessment criteria:**

1. Explain the different characteristics of a quadratic function.
2. Calculate the vertex, roots and  $y$ -intercepts of a quadratic function.
3. Graphically represent a quadratic function by using its vertex, intercept and roots.
4. Apply characteristics of a quadratic function to solve business problems, for example supply and demand, break-even and equilibrium problems.
5. Explain the different characteristics of the cubic, exponential and logarithmic functions.
6. Simplify and solve exponential and logarithmic expressions and equations by using exponential and logarithmic rules.
7. Graphically represent a non-linear function by using a substitution table to calculate different values of the function.
8. Solve one variable of a non-linear function given the value of the other variable of the non-linear function.

**Specific outcome 4:**

Students can apply the basic techniques of calculus to solve problems.

**Range:**

Apply rules of differentiation and integration to solve elementary business problems by determining minimum and maximum values of a function and the area underneath a curve.

**Assessment criteria:**

1. Define and determine the slope of a tangent line to a curve.
2. Determine the equation of the line tangent to a curve.
3. Determine the derivative of a basic function by applying a differentiation rule.
4. Determine marginal functions.
5. Calculate the rate at which a function changes.
6. Determine the maximum or minimum value of a function by using differentiation.
7. Explain what is meant by integration of a function.
8. Determine an indefinite integral of a basic function by applying an integration rule.
9. Determine the definite integral (area underneath a curve between  $x = a$  and  $x = b$ ) of a basic function by applying an integration rule.

## 10 Calculator

You will be allowed to use **any** scientific or financial pocket calculator in the examination. A programmable calculator will be permitted.

## 11 Study approach

### 11.1 Study time

With the semester system a student cannot afford to fall behind with his or her studies. Owing to the limited study time, it is essential that you plan your study program carefully. Keep in mind that a **semester is not longer than about 15 weeks**. The study material has been subdivided into five units and you should therefore give yourself, on average, **not more than three weeks to master each unit**. The earlier units might contain some material that you are already familiar with and therefore you should try to master them more quickly. You will have to **work consistently throughout the semester if you wish to be successful in this module**. Work out your own schedule of dates by which you aim to complete each topic. Plan your studies in such a way that there will be enough time left for revision before the examination.

### 11.2 Study method

We suggest that you approach the study material as follows:

1. Study each unit in the syllabus by working through the section in the text book. Each unit contains examples, exercises and problems, together with solutions. You are expected to work through all of these.
2. **Please contact your lecturers at once if you need any help with the study material, before you carry on with a new study unit.**
3. Do the evaluation exercises of the study unit as specified in the section: "Study plan". You are also welcome to work through the additional progress exercises not specified under the self-evaluation exercises.
4. Only proceed to the next study unit once you've mastered all the work of a study unit and have worked through all the exercises and examples.

Remember the text book forms the basis of the study material in this module.

### 11.3 Study plan

Below, we explain in detail which parts of the text book, as well as evaluation exercises you need to study and in what order.

Please remember to contact the lecturer immediately if you need help regarding the study material. Only once you've mastered a study unit and worked through all the examples and exercises do you proceed to the next study unit.

### 11.3.1 Study unit 1: Preliminaries

Practically everything in this unit should be revision only and you should therefore be able to complete it quite quickly. It is nevertheless important to do the exercises and to be absolutely sure that you have mastered all the concepts that appear in this part of the study material. Learning mathematics is like building a house: if the foundation is not solid, the house cannot stand.

- **Study material sources**

Start with Chapter 1 of the textbook. Work through the following sections and examples in the sections

- 1.2 Arithmetic Operations
- 1.3 Fractions
- 1.4 Solving Equations
- 1.5 Currency Conversions
- 1.6 Simple Inequalities
- 1.7 Calculating Percentages
- 1.8 Make sure you know how to use your calculator

**Please contact the lecturer immediately if you need help regarding the study material.**

**Once you've mastered all the study material of the study unit and worked through all the examples proceed to the evaluation exercises for the study unit.**

- **Evaluation exercises**

**The page numbers of Edition 3 of the textbook are mentioned first and then those of Edition 4.** Do the following self-evaluating Evaluation Exercises for Study unit 1:

1. Progress Exercise 1.1, Question 1, page 7 / *page 10*
2. Progress Exercise 1.1, Question 4, page 7 / *page 10*
3. Progress Exercise 1.1, Question 5, page 8 / *page 10*
4. Progress Exercise 1.1, Question 9, page 8 / *page 10*
5. Progress Exercise 1.1, Question 10, page 8 / *page 10*
6. Progress Exercise 1.1, Question 11, page 8 / *page 10*
7. Progress Exercise 1.1, Question 14, page 8 / *page 10*
8. Progress Exercise 1.2, Question 1, page 14 / *page 17*
9. Progress Exercise 1.2, Question 6, page 14 / *page 17*
10. Progress Exercise 1.2, Question 9, page 14 / *page 17*
11. Progress Exercise 1.2, Question 10, page 14 / *page 17*
12. Progress Exercise 1.2, Question 15, page 14 / *page 17*
13. Progress Exercise 1.3, Question 1, page 20 / *page 23*
14. Progress Exercise 1.3, Question 2, page 20 / *page 23*
15. Progress Exercise 1.3, Question 3, page 20 / *page 23*

16. Progress Exercise 1.3, Question 4, page 20 / *page 23*
17. Progress Exercise 1.3, Question 7, page 20 / *page 23*
18. Progress Exercise 1.3, Question 9, page 20 / *page 23*
19. Test Exercise 1, Question 8, part (b) page 34 / *page 34*

**Remember you can find the solutions in Section 15 of this Tutorial letter.**

### 11.3.2 Study unit 2: Linear functions

In this unit you will come across new applications (principally in economic models) of ideas, such as the graph of a straight line, that should already be familiar to you.

- **Study material sources**

1. Work through the material and examples of the following section of Chapter 2 of the textbook:
  - 2.1 The straight line
2. Work through the material and examples of the following section of Chapter 2 of the textbook:
  - 2.2 Mathematical Modelling
  - 2.3 Applications: demand, supply, cost, revenue
  - 2.4 More mathematics of the straight line
  - 2.6 Elasticity of demand, supply and income

**Please contact the lecturer immediately if you need help regarding the study material.**

**Once you've mastered all the study material of the study unit and worked through all the examples proceed to the evaluation exercises for the study unit.**

- **Evaluation Exercises**

**The page numbers of Edition 3 of the textbook are mentioned first and then those of *Edition 4*. Do the following Self-evaluation Exercises for Study unit 2:**

1. Progress Exercise 2.1, Question 2, page 43 / *page 43*
2. Progress Exercise 2.2, Question 2, page 55 / *page 54*
3. Progress Exercise 2.2, Question 3, page 55 / *page 54*
4. Progress Exercise 2.2, Question 4, page 55 / *page 54*
5. Progress Exercise 2.2, Question 6, page 55 / *page 54*
6. Progress Exercise 2.2, Question 8, page 55 / *page 54*
7. Progress Exercise 2.3, Question 2, page 69 / *page 68*
8. Progress Exercise 2.3, Question 4, page 70 / *page 69*
9. Progress Exercise 2.3, Question 6, page 70 / *page 69*
10. Progress Exercise 2.3, Question 7, page 70 / *page 69*
11. Progress Exercise 2.4, Question 2, page 75 / *page 75*
12. Progress Exercise 2.4, Question 3, page 75 / *page 75*

13. Progress Exercise 2.4, Question 4, page 75 / *page 75*
14. Progress Exercise 2.5, Question 4, page 81 / *page 81*
15. Progress Exercise 2.5, Question 5, page 81 / *page 81*
16. Progress Exercise 2.7, Question 5, page 91 / *page 91*
17. Progress Exercise 2.7, Question 6, page 91 / *page 91*
18. Progress Exercise 2.7, Question 7, page 91 / *page 91*
19. Test Exercise 2, Question 6 page 99 / *page 99*
20. Test Exercise 2, Question 7 page 99 / *page 99*
21. Test Exercise 2, Question 8 page 99 / *page 99*

**Remember you can find the solutions in Section 15 of this Tutorial letter.**

### 11.3.3 Study unit 3: Linear algebra

You have probably encountered some of the mathematical techniques in this unit. As in the previous unit, there are a number of new applications in the economic sciences. You will also briefly encounter the powerful modelling tool called linear programming.

- **Study material sources**

1. Work through the material and examples of the following section of Chapter 3 of the textbook:
  - 3.1 Solving Simultaneous linear equations
2. Work through the material and examples of the following section of Chapter 3 of the textbook:
  - 3.2 Equilibrium and break-even
    - \* 3.2.1 Equilibrium in the goods and labour markets
    - \* 3.2.5 Break-even analysis
  - 3.3 Consumer and Producer surplus
3. Work through the material and examples of the following section of Chapter 9 of the textbook:
  - 9.1 Linear programming

**Please contact the lecturer immediately if you need help regarding the study material.**

**Once you've mastered all the study material of the study unit and worked through all the examples proceed to the evaluation exercises for the study unit.**

- **Evaluation Exercises**

**The page numbers of Edition 3 of the textbook are mentioned first and then those of Edition 4.** Do the following Self-evaluation Exercises for Study unit 3:

1. Progress Exercise 3.1, Question 3, page 110 / *page 111*
2. Progress Exercise 3.1, Question 4, page 110 / *page 111*
3. Progress Exercise 3.1, Question 6, page 110 / *page 111*
4. Progress Exercise 3.1, Question 9, page 110 / *page 111*

5. Progress Exercise 3.1, Question 10, page 110 / *page 111*
6. Progress Exercise 3.1, Question 14, page 110 / *page 111*
7. Progress Exercise 3.1, Question 15, page 110 / *page 111*
8. Progress Exercise 3.2, Question 5, page 117 / *page 118*
9. Progress Exercise 3.2, Question 6, page 117 / *page 118*
10. Progress Exercise 3.2, Question 7, page 117 / *page 118*
11. Progress Exercise 3.3, Question 1, page 125 / *page 126*
12. Progress Exercise 3.3, Question 7, page 126 / *page 127*
13. Progress Exercise 3.3, Question 9, page 127 / *page 127*
14. Progress Exercise 3.3, Question 10, page 127 / *page 127*
15. Progress Exercise 3.4, Question 2, page 131 / *page 131*
16. Progress Exercise 3.4, Question 3, page 131 / *page 132*
17. Progress Exercise 9.1, Question 1, page 485 / *page 487*
18. Progress Exercise 9.1, Question 3, page 485 / *page 487*
19. Progress Exercise 9.1, Question 6, page 485 / *page 487*
20. Progress Exercise 9.1, Question 7, page 485 / *page 487*
21. Progress Exercise 9.1, Question 10, page 485 / *page 487*

**Remember you can find the solutions in Section 15 of this Tutorial letter.**

#### 11.3.4 Study unit 4: Non-linear functions

The work in this unit involves quite a bit more algebra, with important applications in economics and business.

- **Study material sources**

Work through the material and examples of the following section of Chapter 4 of the text book:

- 4.1 Quadratic, cubic and other polynomial functions
- 4.2 Exponential functions
- 4.3 Logarithmic functions
- 4.4 Hyperbolic functions of the form  $a/(bx + c)$

**Please contact the lecturer immediately if you need help regarding the study material.**

**Once you've mastered all the study material of the study unit and worked through all the examples proceed to the evaluation exercises for the study unit.**

- **Evaluation Exercises**

**The page numbers of Edition 3 of the textbook are mentioned first and then those of *Edition 4*.** Do the following Self-evaluation Exercises for Study unit 4:

1. Progress Exercise 4.1, Question 1, page 152 / *page 152*

2. Progress Exercise 4.1, Question 4, page 152 / *page 152*
3. Progress Exercise 4.1, Question 8, page 152 / *page 152*
4. Progress Exercise 4.2, Question 1, page 158 / *page 158*
5. Progress Exercise 4.2, Question 6, page 158 / *page 158*
6. Progress Exercise 4.3, Question 2, page 163 / *page 163*
7. Progress Exercise 4.3, Question 3, page 164 / *page 164*
8. Progress Exercise 4.3, Question 4, page 164 / *page 164*
9. Progress Exercise 4.4, Question 3, page 170 / *page 170*
10. Progress Exercise 4.4, Question 4, page 170 / *page 170*
11. Progress Exercise 4.5, Question 1, page 176 / *page 177*
12. Progress Exercise 4.5, Question 7, page 177 / *page 177*
13. Progress Exercise 4.5, Question 13, page 177 / *page 177*
14. Progress Exercise 4.6, Question 1, page 179 / *page 179*
15. Progress Exercise 4.6, Question 4, page 179 / *page 179*
16. Progress Exercise 4.6, Question 5, page 179 / *page 179*
17. Progress Exercise 4.6, Question 7, page 179 / *page 179*
18. Progress Exercise 4.6, Question 8, page 179 / *page 179*
19. Progress Exercise 4.6, Question 20, page 179 / *page 179*
20. Progress Exercise 4.8, Question 2, page 184 / *page 184*
21. Progress Exercise 4.10, Question 11, page 188 / *page 189*
22. Progress Exercise 4.13, Question 8, page 201 / *page 202*

**Remember you can find the solutions in Section 15 of this Tutorial letter.**

### 11.3.5 Study unit 5: Beginning calculus

“Calculus” means pebble in Latin and alludes to the roots of the subject in Greek mathematics of the pre-Christian era, and the earlier use of small stones in calculations. While mathematical development in Europe languished during the Middle Ages, Indian mathematicians in Kerala developed much of the theory. The invention of the modern version of the theory is attributed to Sir Isaac Newton and, independently and roughly simultaneously, the German philosopher Gottfried Leibniz. It is one of the most powerful tools used by large numbers of applied scientists, economists and engineers. Since this is the most advanced topic in the course, you should plan on spending a bit more time on this unit than on the others.

#### • Study material sources

1. Work through the material and examples of the following section of Chapter 6 of the text book:
  - 6.1 *Slope of a curve and differentiation*
  - 6.2.1 *Marginal functions: an introduction*
  - 6.3.1 *Slope of a curve and turning points*
  - 6.3.2 *Determining maximum and minimum turning points*
2. Work through the material and examples of the following section of Chapter 8 of the text book:



- 8.1 Integration as the reverse of differentiation
- 8.2 The power rule for integration
- 8.5 The definite integral and the area under a curve

**Please contact the lecturer immediately if you need help regarding the study material.**

**Once you've mastered all the study material of the study unit and worked through all the examples proceed to the evaluation exercises for the study unit.**

- **Evaluation Exercises**

**The page numbers of Edition 3 of the textbook are mentioned first and then those of Edition 4.** Do the following Self-evaluation Exercises for Study unit 5:

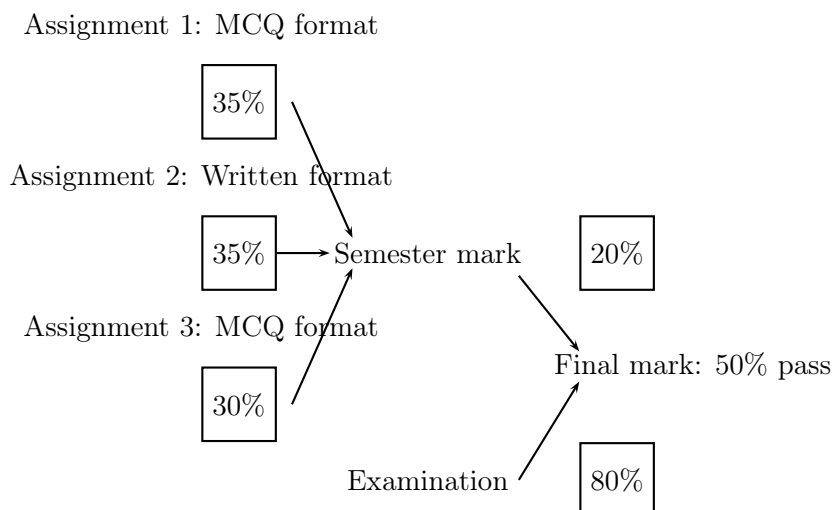
1. Progress Exercise 6.1, Question 1, page 266 / page 268
2. Progress Exercise 6.1, Question 3, part (c), page 267 / page 269
3. Progress Exercise 6.1, Question 3, part (e), page 267 / page 269
4. Progress Exercise 6.3, Question 1, page 278 / page 280
5. Progress Exercise 6.3, Question 2, page 278 / page 280
6. Progress Exercise 6.5, Question 1, page 287 / page 289
7. Progress Exercise 6.5, Question 6, page 287 / page 289
8. Progress Exercise 6.5, Question 7, page 287 / page 289
9. Progress Exercise 6.5, Question 10, page 287 / page 289
10. Progress Exercise 6.9, Question 3, page 315 / page 318
11. Progress Exercise 6.9, Question 4, page 315 / page 318
12. Progress Exercise 6.9, Question 5, page 315 / page 318
13. Progress Exercise 6.17, Question 1, page 352 / page 355
14. Progress Exercise 6.17, Question 2, page 352 / page 355
15. Progress Exercise 6.17, Question 7, page 352 / page 355
16. Progress Exercise 8.1, Question 1, page 433 / page 435
17. Progress Exercise 8.1, Question 9, page 433 / page 435
18. Progress Exercise 8.1, Question 11, page 433 / page 435
19. Progress Exercise 8.1, Question 17, page 433 / page 435
20. Progress Exercise 8.3, Question 1, page 445 / page 447
21. Progress Exercise 8.3, Question 4, page 445 / page 447
22. Progress Exercise 8.3, Question 20, page 446 / page 448
23. Progress Exercise 8.3, Question 22, page 446 / page 448

**Remember you can find the solutions in Section 15 of this Tutorial letter.**

## 12 Assessment

This module is assessed by means of a written examination contributing 80% of the final mark and three **compulsory assignments** that contribute 20% of the final mark.

Assignment 01 will contribute 35%, Assignment 02 will contribute 35% and Assignment 03 will contribute 30% to the semester mark. Together these assignments will contribute 20% to your final mark for this module.



### 12.1 Examination

**You are required to submit Assignment 01 to obtain admission to the examination.** Admission will only be obtained by *submitting* the first assignment *on time*, and not by the marks you obtain for it. Please ensure that the first assignment reaches the University before the due date. Students who register for the first semester will write the examination in May/June and students who register for the second semester will write the examination in October/November.

The duration of the examination is two hours. See Tutorial letter 102, that you will receive during the semester, for examination details.

**You will be allowed to use any scientific pocket calculator in the examination. A programmable calculator will be permitted.** You may take only your writing materials and your pocket calculator into the examination hall.

**You need at least 50% from your combined assignments and examination mark in order to pass the module. Note that your assignment marks will only be considered if you obtain at least 40% in the examination.**

### 12.2 Assignment 1, 2 and 3 (COMPULSORY!)

For students to benefit fully from formative tuition and assessment the Management of the University decided to introduce compulsory assignments in all modules.

All three the assignments are **compulsory for all students** in this module. Failure to submit the assignments through the proper channels by the **due date** may result in admission to the examination **not** being granted. Answer the questions to the best of your ability.

The assignments consists of two multiple choice assignments and one written assignment. You may submit your assignment either by post or electronically via *myUnisa*. Assignments may **not** be submitted by fax or email.

To submit an assignment via *myUnisa*, follow the steps below.

- Go to <https://my.unisa.ac.za/>
- Log in with your student number and password.
- Select this course from the orange bar.
- Click on “Assignments” in the left-hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions.

**Make sure your assignment has reached Unisa. You can check *myUnisa* to see if your assignment has reached Unisa by selecting the Assignment option and entering your student number and course.**

Note that neither the Department nor the School of Economic Sciences will be able to confirm whether the University has received your assignment or not.

The **due dates** and **unique numbers** for the compulsory assignments are:

|                        |               |                  |        |
|------------------------|---------------|------------------|--------|
| <b>FIRST semester</b>  | Assignment 01 | 3 March 2015     | 504642 |
|                        | Assignment 02 | 31 March 2015    | 504680 |
|                        | Assignment 03 | 22 April 2015    | 504696 |
| <b>SECOND semester</b> | Assignment 01 | 18 August 2015   | 504731 |
|                        | Assignment 02 | 8 September 2015 | 504755 |
|                        | Assignment 03 | 6 October 2015   | 504776 |

The solutions to the compulsory assignments will be mailed to all students after the due date of the assignment. It will also be posted on *myUnisa* after the due date. You are welcome to download it in advance. Remember that the marks obtained in your assignments are accessible under the Assignment option of *myUnisa* after it has been marked.

### 12.3 Evaluation exercises

Evaluation exercises are given on each unit of the study material and are important for the following reasons:

- (a) Evaluation exercises assist you in understanding and mastering the study material and its practical implications. They are therefore an integral part of the study material.
- (b) Evaluation exercises test your knowledge and understanding of the study material. They are a way of evaluating your progress.

### 12.3.1 How to attempt the evaluation exercises

**You must work through the prescribed study material for a section thoroughly before you start with the evaluation exercises, in fact before you read the questions for the first time.** The process of understanding and mastering the study material takes time and you should set aside plenty of time for it. The evaluation exercises consist of just a few questions. Do not let this fool you into thinking that you can complete these questions quickly. You will need to devote enough time to answer them.

### 12.3.2 Evaluating your answers

You are responsible for correcting your own evaluation exercises. When marking your exercises, you should compare your answers with the model solutions. Each calculation and detail of your answer should be checked against the model answer. This will assist you in understanding each question. The solutions often contain helpful explanations and remarks. This process of self-evaluation will also ensure that you take note of the extra information. The solutions often contain helpful explanations and remarks. This process of self-evaluation will also ensure that you take note of the extra information.

Unless stated otherwise, **all exercises are from the text book.**

**If you have any questions you should not hesitate to contact the lecturers at once.**

## 13 ASSIGNMENTS: SEMESTER 1

### 13.1 Assignment 01 (COMPULSORY): MCQ format

| Semester | Unique Number | Due Date     |
|----------|---------------|--------------|
| One      | 504642        | 3 March 2015 |

**Instructions:** Answer all the questions on the mark-reading sheet. Work through the study material of study units 1 and 2 in your textbook before attempting this assignment.

#### Question 1

Simplify  $\frac{2}{3} \div \frac{5}{6} + 5 \div \frac{5}{4} - 1\frac{1}{3} \times 6$ .

- [1] -3
- [2]  $\frac{3}{10}$
- [3]  $-\frac{19}{20}$
- [4]  $-3\frac{1}{5}$
- [5] None of the above.

#### Question 2

A house is valued at R600 000,00. This is 20% higher than the price paid for the house. What was the price paid?

- [1] R500 000,00
- [2] R480 000,00
- [3] R720 000,00
- [4] R120 000,00
- [5] None of the above.

#### Question 3

The slope of the line  $2y + 20x = 10x - 5$  equals

- [1] 10
- [2] 5
- [3]  $-\frac{5}{2}$
- [4] -5
- [5] None of the above.

**Question 4**

Determine the value of  $x$  that solves the inequality:

$$-3(x + 1) + 6\left(x + \frac{1}{3}\right) \leq 4\left(x - \frac{1}{2}\right).$$

- [1]  $x \leq -\frac{3}{7}$
- [2]  $x \leq -1$
- [3]  $x \geq 1$
- [4]  $x \leq -\frac{1}{2}$
- [5] None of the above.

**Question 5**

Find the equation of the straight line passing through the points (2;1) and (1;2).

- [1]  $y = -x + 3$
- [2]  $y = -x + 6$
- [3]  $y = 2x + 4$
- [4]  $y = x + 3$
- [5] None of the above.

**Question 6**

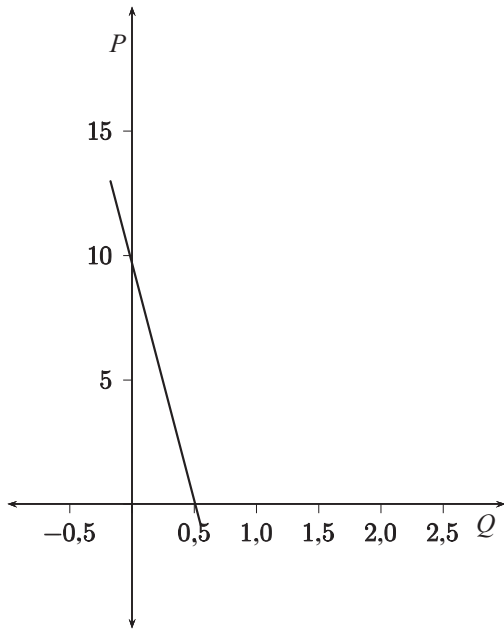
The slope of the line  $2y = 1 + 4x$  is

- [1]  $-\frac{7}{2}$ .
- [2] 7.
- [3] 2.
- [4] 4.
- [5] none of the above.

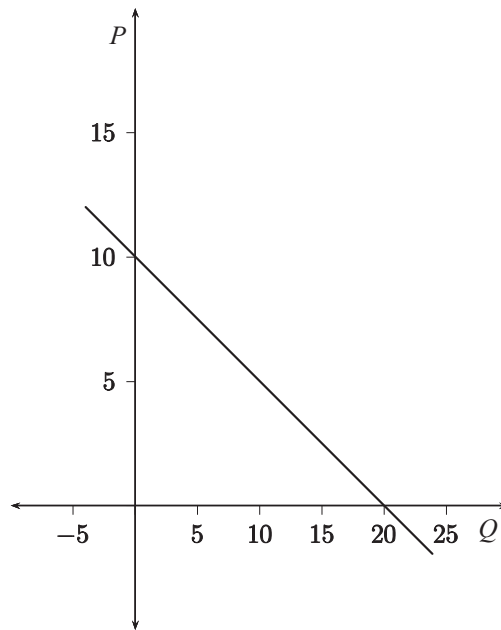
### Question 7

The linear function  $2P = 20 - Q$  can be graphically represented as:

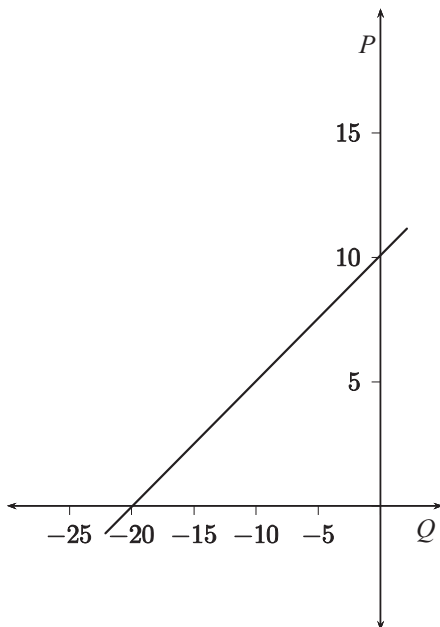
[1]



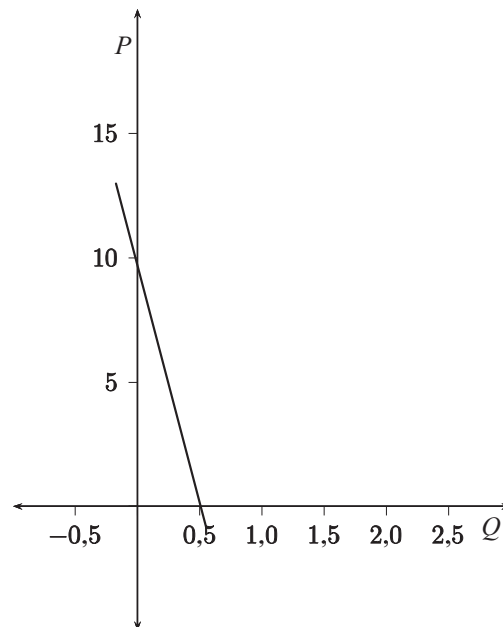
[2]



[3]



[4]



[5] none of the above.

**Question 8**

Find the value of quantity  $Q$  for the demand function  $P = 60 - 4Q$  when the market price is  $P = 24$ .

- [1] 8
- [2] 9
- [3] 10
- [4] 11
- [5] None of the above.

**Question 9**

The cost  $y$  (in rands) of manufacturing  $x$  bicycles is

$$y = 250x + 720.$$

How many bicycles have been manufactured if the cost is  $37\,320 - 50x$ ?

- [1] 120
- [2] 122
- [3] 123
- [4] 125
- [5] None of the above.

**Question 10**

If the demand function is  $P = 250 - 5Q$  where  $P$  and  $Q$  are the price and quantity respectively, give an expression for the price elasticity of demand in terms of  $P$  only.

- [1]  $\frac{P - 250}{P}$
- [2]  $\frac{P - 5}{P}$
- [3]  $\frac{P}{P - 250}$
- [4]  $\frac{P}{P - 5}$
- [5] None of the above.



**13.2 Assignment 02 (COMPULSORY): Written format**

| Semester | Unique Number | Due Date      |
|----------|---------------|---------------|
| One      | 504680        | 31 March 2015 |

**Instructions:** Answer all the questions. Work through the study material of study unit 3 in your textbook before attempting this assignment. Only a selected number of questions will be marked. You will receive the solutions to all the questions.

**Question 1**

Determine the coordinates of the intersection of the two lines:

$$\begin{aligned} 3x + 2y - 3 &= 0 \\ x + y + 1 &= 0. \end{aligned}$$

**Question 2**

Solve the following system of linear equations:

$$\begin{aligned} x - y + z &= 0 \\ 2y - 2z &= 2 \\ -x + 2y + 2z &= 29. \end{aligned}$$

**Question 3**

Determine graphically the coordinates of the intersection of the two lines:

$$\begin{aligned} y + 2x &= 3 \\ y - x &= 2. \end{aligned}$$

**Question 4**

In the following market:

$$\begin{aligned} \text{Demand function: } Q &= 80 - P \\ \text{Supply function: } Q &= -8 + \frac{1}{3}P, \end{aligned}$$

where  $P$  and  $Q$  are the price and quantity respectively. Calculate the equilibrium price and quantity.

**Question 5**

Calculate the consumer surplus for the demand function

$$P = 50 - 4Q$$

when the market price is  $P = 10$ .

**Question 6**

Suppose the cost of manufacturing 10 units of a product is R40 and the cost of manufacturing 20 units is R70. If the cost  $C$  is linearly related to output  $Q$  (units produced), calculate the cost of producing 35 items.

### Question 7

A manufacturer of leather articles produces boots and jackets. The manufacturing process consists of two activities:

- Making (cutting and stitching)
- Finishing

There are 800 hours available for making the articles and 1 200 hours available for finishing them. It takes 4 hours to make and 3 hours to finish a pair of boots, and 2 hours to make and 4 hours to finish a jacket. Market experience requires the production of boots to be a minimum of 150 pairs per month. Write down a system of linear inequalities that describe the appropriate constraints if  $x$  is the number of pairs of boots and  $y$  the number of jackets manufactured.

### Question 8

Find the equation of the straight line passing through the points  $(4; 2)$  and  $(2; 4)$ .

### Question 9

Draw the following set of inequalities and indicate the feasible region where all the inequalities are satisfied simultaneously:

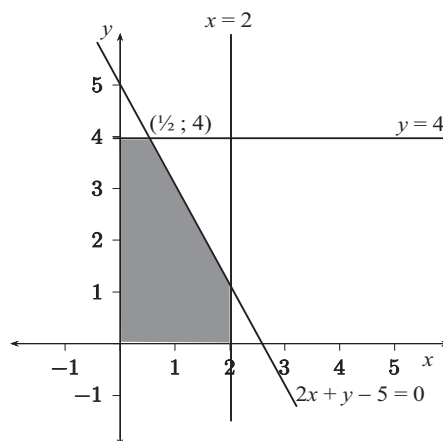
$$\begin{aligned}x + 8y &\leq 400 & (1) \\x + 2y &\geq 200 & (2) \\x \geq 0, \quad y &\geq 0\end{aligned}$$

### Question 10

In the graph below the following set of inequalities

$$\begin{aligned}2x + y - 5 &\leq 0 & (1) \\x - 2 &\leq 0 & (2) \\y - 4 &\leq 0 & (3) \\x, y &\geq 0\end{aligned}$$

were drawn and the feasible region of the set of inequalities shaded in grey. Determine the maximum value of the function  $C = 20x + 30y$  subject to the set of inequalities above.



**13.3 Assignment 03 (COMPULSORY): MCQ format**

|          |               |               |
|----------|---------------|---------------|
| Semester | Unique Number | Due Date      |
| One      | 504696        | 22 April 2015 |

**Instructions:** Answer all the questions on the mark-reading sheet. Work through the study material of study units 4 and 5 in your textbook before attempting this assignment.

**Question 1**

Simplify:

$$1 + \frac{36}{45} \times \frac{5}{12} \div \frac{2}{3}$$

- [1]  $\frac{3}{2}$
- [2]  $\frac{2}{3}$
- [3]  $\frac{1}{2}$
- [4]  $\frac{81}{41}$
- [5] None of the above.

**Question 2**

The roots of the function  $y + 6 = 2x^2 + x$  are

- [1]  $x = 2$  and  $x = -3$ .
- [2]  $x = 3$  and  $y = 2,5$ .
- [3]  $x = -0,5$  and  $x = -6,25$ .
- [4]  $x = -2$  and  $x = 1,5$ .
- [5] none of the above.

**Question 3**

How many units must be produced to maximise a profit defined by the function

$$2y = -4x^2 + 16x - 12?$$

- [1] 1 unit
- [2] 2 units
- [3] 3 units
- [4] 4 units
- [5] None of the above

**Question 4**Simplify  $\sqrt{(x^8)^8}$ 

- [1]  $x^4$ .
- [2]  $x^8$ .
- [3]  $x^{32}$ .
- [4]  $x^{16}$ .
- [5] none of the above.

**Question 5** $\log_5\left(\frac{15}{0,45}\right)$  is equal to

- [1]  $-2,107$ .
- [2]  $2,179$ .
- [3]  $3,739$ .
- [4]  $0,701$ .
- [5] none of the above.

**Question 6**

The number of people who contracted a contagious disease  $t$  days after an epidemic started is approximated by the exponential equation

$$Q(t) = \frac{5\,000}{1 + 1\,249e^{-0,33t}}$$

Approximately how many people had contracted the disease after 15 days?

- [1] 508
- [2] 37
- [3] 2 167
- [4] 5 009
- [5] None of the above.

**Question 7**

Find the derivative of the function:  $G(x) = x(x^2 - 4\sqrt{x} + 4)$

- [1]  $G'(x) = x^3 - 4x^{\frac{3}{2}} + 4x$
- [2]  $G'(x) = 3x^2 - \frac{3}{2}x^{\frac{1}{2}} + 4$
- [3]  $G'(x) = 3x^4 - 6x^{\frac{5}{2}} + 4x^2$
- [4]  $G'(x) = 3x^2 - 6x^{\frac{1}{2}} + 4$
- [5] None of the above

**Question 8**

Evaluate

$$\int x^3 \left(1 + \frac{1}{x^2} + \frac{1}{\sqrt{x^2}}\right) dx$$

- [1]  $x^3 + \frac{x^3}{x^2} + \frac{x^3}{\sqrt{x^2}} + c.$
- [2]  $\frac{x^4}{4} + \frac{x^2}{2} + \frac{x^3}{3} + c.$
- [3]  $4x^4 + 2x^2 + 3x^3 + c.$
- [4]  $\frac{x^4}{4} + \frac{x^2}{2} + \frac{1}{3\sqrt{x^2}} + c.$
- [5] none of the above.

**Question 9**Calculate  $\int_1^5 \frac{1}{x}(2x^2 + x^4) dx$ 

- [1] 180.
- [2] 69,25.
- [3] 180,50.
- [4] 132.
- [5] none of the above.

**Question 10**What is the marginal cost when  $Q = 9$  if the total cost is given by

$$TC = 3Q^3 - Q^2 + 70Q + 800?$$

- [1] 574
- [2] 781
- [3] 504
- [4] 434
- [5] None of the above.

## 14 ASSIGNMENTS: SEMESTER 2

### 14.1 Assignment 01 (COMPULSORY): MCQ format

|          |               |                |
|----------|---------------|----------------|
| Semester | Unique Number | Due Date       |
| Two      | 504731        | 18 August 2015 |

**Instructions:** Answer all the questions on the mark-reading sheet. Work through the study material of study units 1 and 2 in your textbook before attempting this assignment.

#### Question 1

Find the slope of the line  $0 = 6 + 3x - 2y$ .

- [1]  $\frac{2}{3}$
- [2]  $\frac{3}{2}$
- [3] 3
- [4] 2
- [5] None of the above.

#### Question 2

Simplify the following

$$\frac{(x-4)(x+3)}{(x-4)(x+5)}$$

- [1]  $\frac{x-3}{x-5}$
- [2]  $\frac{x+3}{x+5}$
- [3]  $\frac{x+4}{x-4}$
- [4]  $\frac{3}{5}$
- [5] None of the above.

**Question 3**

The  $x$ -intercept of the line

$$2y - 10x + 5 = 0$$

equals

- [1]  $-0,5$ .
- [2]  $0,5$ .
- [3]  $10$ .
- [4]  $20$ .
- [5] none of the above.

**Question 4**

A suit costs R800 in 2012. The price of the suit was increased by 21% in 2013. In 2014 the price of a suit was increased by 25% of the 2013 price. What was the price of the suit in 2014?

- [1] R1 210,00
- [2] R1 000,00
- [3] R1 168,00
- [4] R968,00
- [5] None of the above.

**Question 5**

Find the equation of the line passing through the points  $(3;1)$  and  $(\frac{4}{3}; 2)$ .

- [1]  $y = -\frac{3}{5}x + \frac{14}{5}$
- [2]  $y = -\frac{3}{5}x + 1$
- [3]  $y = x + \frac{14}{5}$
- [4]  $y = -\frac{3}{5}x - \frac{14}{5}$
- [5] None of the above.

**Question 6**

A car was valued at R170 000 in 2013 and R140 000 in 2014. Calculate the percentage decrease in the value of the car between 2013 and 2014.

- [1] 17,65%
- [2] 17%
- [3] 18,15%
- [4] 18%
- [5] None of the above.

**Question 7**

Simplify

$$\frac{3}{4} \div 2 \left( 1\frac{5}{6} - \frac{1}{2} \right) + \frac{3}{2} \times \frac{5}{2}.$$

- [1]  $4\frac{1}{32}$
- [2]  $5\frac{3}{4}$
- [3]  $\frac{3}{4}$
- [4]  $3\frac{9}{10}$
- [5] None of the above.

**Question 8**

A swimming club provides  $x$  number of swimming lessons per day. The club has a daily fixed cost of R1 250 when offering lessons. The variable cost is  $(30 + 20x)$  for each lesson given. Write down the linear equation for the total cost of the club per day.

- [1] Cost =  $20x + 1\,280$
- [2] Cost = 1 280
- [3] Cost =  $1\,280x$
- [4] Cost =  $1\,280x + 20$
- [5] None of the above.

**Question 9**

A company manufactures radios. If  $x$  is the number of radios that retailers are likely to purchase at a price  $p(x) = 5 - \frac{x}{1\,000}$  rand per unit and the cost function is given by  $c(x) = 5\,000 + 2x$ , what is the revenue function of the manufacturing company?

- [1]  $R(x) = 5 - \frac{x}{1\,000}$
- [2]  $R(x) = 5 - \frac{x}{1\,000} - 5\,000 + 2x$
- [3]  $R(x) = 5x - 0,001x^2$
- [4]  $R(x) = 5\,000 - x$
- [5] None of the above.



**Question 10**

If the demand function is  $P = 50 - \frac{1}{2}Q$  where  $P$  and  $Q$  are the price and quantity respectively, give an expression for the price elasticity of demand in terms of  $P$  only.

- [1]  $\frac{P}{P-\frac{1}{2}}$
- [2]  $\frac{P-\frac{1}{2}}{P}$
- [3]  $\frac{P}{P-50}$
- [4]  $\frac{P-50}{P}$
- [5] None of the above.

**14.2 Assignment 02 (COMPULSORY): Written format**

|                 |                      |                          |
|-----------------|----------------------|--------------------------|
| <b>Semester</b> | <b>Unique Number</b> | <b>Due Date</b>          |
| <b>Two</b>      | <b>504755</b>        | <b>08 September 2015</b> |

**Instructions:** Answer all the questions. Work through the study material of study unit 3 in your textbook before attempting this assignment. Only a selected number of questions will be marked. You will receive the solutions to all the questions.

**Question 1**

Determine the coordinates of the intersection of the two lines:

$$\begin{aligned}y + 2x &= 3 \\y - x &= 2.\end{aligned}$$

**Question 2**

Solve the following systems of equations:

$$\begin{aligned}x + 2y - z &= 5 & (1) \\2x - y + z &= 2 & (2) \\y + z &= 2 & (3)\end{aligned}$$

**Question 3**

Consider the market for rugby balls defined by:

$$\begin{aligned}\text{Demand function: } Q &= 400 - \frac{1}{2}P \\ \text{Supply function: } Q &= 5 + \frac{1}{8}P\end{aligned}$$

where  $P$  and  $Q$  are the price and quantity respectively. Calculate the equilibrium price and quantity.

**Question 4**

The driving school provides  $x$  number of lessons per day. The school has a daily fixed cost of R1 250 when offering lessons. The variable cost is given as R335 for each lesson given. If the revenue function of the school per day is given as  $9\,000 + 25x$ , how many lessons should be provided in order to break even?

**Question 5**

- Find the equation of the line passing through the points (1; 20) and (5; 60).
- Draw the graph of the line  $y = 10x + 10$ .

### Question 6

Calculate the consumer surplus for the demand function

$$P = 90 - 5Q$$

when the market price is  $P = 20$ .

### Question 7

Draw the following set of inequalities and indicate the feasible region where all the inequalities are satisfied simultaneously:

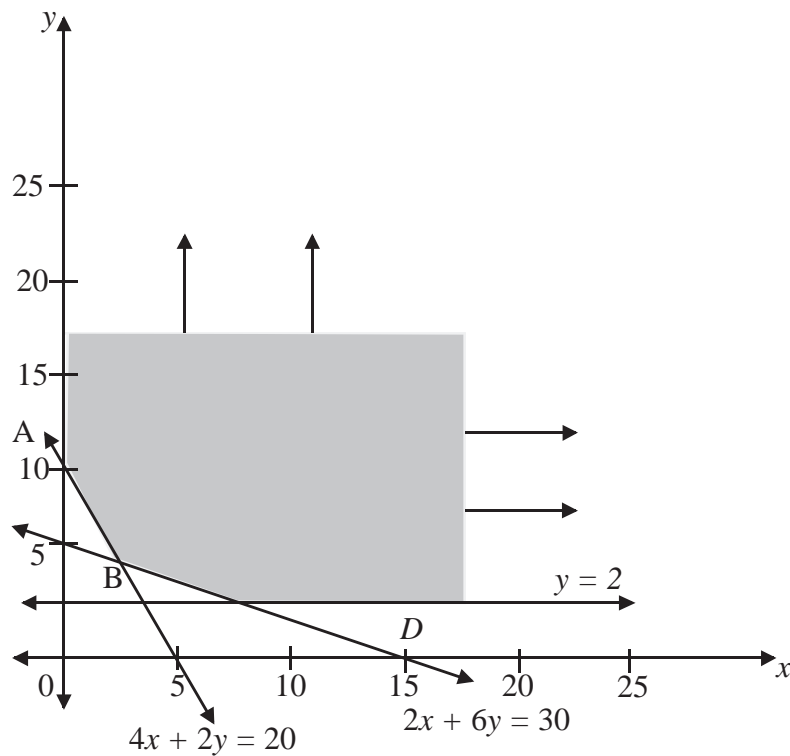
$$\begin{aligned} -x_1 + x_2 &\leq 3 \\ x_1 + x_2 &\geq 5 \\ x_1 &\leq 3 \\ x_1; x_2 &\geq 0. \end{aligned}$$

### Question 8

In the graph below the set of inequalities

$$\begin{aligned} 2x + 6y &\geq 30 \quad (1) \\ 4x + 2y &\geq 20 \quad (2) \\ y &\geq 2 \quad (3) \\ x, y &\geq 0 \end{aligned}$$

were drawn and the feasible region of the set of inequalities shaded in grey. Determine the minimum value



of the function  $Z = 18x + 12y$  subject to the set of inequalities above.

**Question 9**

Pianni Beverages produces two ready-mixed cocktail drinks, the Zombie and the Skyjack. Each is a mixture of vodka, vermouth and ginger ale. It takes 3 litres of vodka, 6 litres of vermouth and 1 litre of ginger ale to make a container of Zombie and 5 litres of vodka, 3 litres of vermouth and 2 litres of ginger ale to make a container of Skyjack. The maximum litres of vodka and vermouth available per day are 1 500 litres of vodka and 1 500 litres of vermouth. The minimum litres of ginger ale available per day is 400 litres of ginger ale. If  $x$  is the number of containers of Zombie mixed and  $y$  the number of containers of Skyjack mixed, determine the system of inequalities that best describes the situation. Do not solve the system of inequalities.

**Question 10**

The price of a T-shirt including a 20% markup is R36. What is the price of the T-shirt before the markup?

**14.3 Assignment 03 (COMPULSORY): MCQ format**

|                 |                      |                       |
|-----------------|----------------------|-----------------------|
| <b>Semester</b> | <b>Unique Number</b> | <b>Due Date</b>       |
| <b>Two</b>      | <b>504776</b>        | <b>6 October 2015</b> |

**Instructions:** Answer all the questions on the mark-reading sheet. Work through the study material of study units 4 and 5 in your textbook before attempting this assignment.

**Question 1**

$\log_3\left(\frac{3}{\sqrt{3}}\right)$ , to four decimal places, equals

- [1] -0,0795.
- [2] 0,0795.
- [3] 2,0000.
- [4] 0,5000.
- [5] none of the above.

**Question 2**

Solve the inequality  $x^2 - 3x \geq 6 - 2x$

- [1]  $-2 \leq x \leq 3$ .
- [2]  $-6 \leq x \leq 1$ .
- [3]  $x \leq -2; x \geq 3$ .
- [4]  $x \leq -3; x \geq 2$ .
- [5] none of the above.

**Question 3**

Determine the roots of  $6x^2 + 5x - 1$ .

- [1]  $x = \frac{1}{6}; x = 1$
- [2]  $x = \frac{1}{6}; x = 1$
- [3]  $x = -\frac{1}{4}; x = 1$
- [4]  $x = \frac{1}{6}; x = -1$
- [5] None of the above

**Question 4**

If  $y = 2^{-x}$ , find  $x$  if  $y = 0,0625$ .

- [1]  $x = -2$
- [2]  $x = 3$
- [3]  $x = 4$
- [4]  $x = 5$
- [5] None of the above.

**Question 5**

Evaluate the following definite integral:

$$\int_{-2}^2 (x^2 - 3) dx$$

- [1]  $6\frac{2}{3}$
- [2]  $-6\frac{2}{3}$
- [3]  $3\frac{1}{3}$
- [4]  $-3\frac{1}{3}$
- [5] None of the above

**Question 6**

Evaluate

$$\int x^2 \left(1 + \frac{1}{x^2}\right) dx.$$

- [1]  $x^3 + x + c$
- [2]  $\frac{1}{3}x^3 + x + c$
- [3]  $x^2 + 1$
- [4]  $\frac{1}{2}x^2 + x + c$
- [5] None of the above.

**Question 7**

Simplify

$$\frac{d}{dx} \left[ \frac{x - x^2}{\sqrt{x}} \right].$$

- [1]  $\frac{2}{3}\sqrt{x} + \frac{1}{2\sqrt{x}}$
- [2]  $\frac{1}{2\sqrt{x}} - \frac{3}{2}\sqrt{x}$
- [3]  $\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$
- [4]  $\frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}}$
- [5] None of the above.

**Question 8**

The demand function of a firm is  $Q = 150 - 0,5P$ , where  $P$  and  $Q$  represent the quantity and price respectively. At what value of  $Q$  is marginal revenue equal to zero?

- [1] 150
- [2] 75
- [3] 113
- [4] 0
- [5] None of the above

**Question 9**

The coordinates of the turning point of the function  $y = x^2 - 2x + 3$  are

- [1] (1; 1).
- [2] (1; 2).
- [3] (1; 3).
- [4] (1; 4).
- [5] none of the above.

**Question 10**

What is the marginal cost when  $Q = 10$  if the total cost is given by:

$$TC = Q^4 - 30Q^2 + 300Q + 500?$$

- [1] 3 700
- [2] 4 900
- [3] 4 200
- [4] 6 400
- [5] None of the above.

## 15 Solutions: Self-evaluation exercises

### Evaluating your answers

You are responsible for correcting your own self-evaluation exercises. When marking your exercises, you should compare your answers to the model solutions. Each calculation and detail of your answer should be checked against the model answer. This will assist you in understanding each **Question**. The solutions often contain helpful explanations and remarks. This process of self-evaluation will also ensure that you take note of the extra information.

Unless stated otherwise, all exercises are from the text book. Exercises' page numbers from Edition 3 of the text book are quoted first and those of *Edition 4* second.

### 15.1 Self-Evaluation Exercise 1 : Unit 1

#### 1. Progress Exercises 1.1, page 7 / page 10

##### (i) Question 1

$$\begin{aligned} 2x + 3x + 5(2x - 3) &= 2x + 3x + 10x - 15 && \text{(Removing brackets)} \\ &= 15x - 15 && \text{(putting like terms together)} \\ &= 15(x - 1) && \text{(factoring out common 15)} \end{aligned}$$

##### (ii) Question 2

$$\begin{aligned} 4x^2 + 7x + 2x(4x - 5) &= 4x^2 + 7x + 8x^2 - 10x \\ &= 12x^2 - 3x \\ &= 3x(4x - 1) \end{aligned}$$

##### (iii) Question 3

$$\begin{aligned} 2x(y + 2) - 2y(x + 2) &= 2xy + 4x - 2xy - 4y \\ &= 4x - 4y \\ &= 4(x - y) \end{aligned}$$

##### (iv) Question 4

$$\begin{aligned} (x + 2)(x - 4) - 2(x - 4) &= x^2 - 4x + 2x - 8 - 2x + 8 && \text{(Removing brackets)} \\ &= x^2 - 4x && \text{(putting like terms together)} \\ &= x(x - 4) && \text{(factoring out common } x) \end{aligned}$$

##### (v) Question 5

$$\begin{aligned} (x + 2)(y - 2) + (x - 3)(y + 2) &= xy - 2x + 2y - 4 + xy + 2x - 3y - 6 \\ &\quad \text{(putting like terms together)} \\ &= 2xy - y - 10 \end{aligned}$$

##### (vi) Question 6

$$\begin{aligned} (x + 2)^2 + (x - 2)^2 &= x^2 + 4x + 4 + x^2 - 4x + 4 \\ &= 2x^2 + 8 \\ &= 2(x^2 + 4) \end{aligned}$$



(vii) **Question 7**

$$\begin{aligned}
 (x+2)^2 - (x-2)^2 &= (x^2 + 4x + 4) - (x^2 - 4x + 4) \\
 &= x^2 + 4x + 4 - x^2 + 4x - 4 \\
 &= 8x
 \end{aligned}$$

(viii) **Question 8**

$$\begin{aligned}
 (x+2)^2 - x(x+2) &= x^2 + 4x + 4 - x^2 - 2x \\
 &= 2x + 4 \\
 &= 2(x+2)
 \end{aligned}$$

(ix) **Question 9**

$$\begin{aligned}
 \frac{1}{3} + \frac{3}{5} + \frac{5}{7} &= \frac{(1 \times 5 \times 7) + (3 \times 3 \times 7) + (5 \times 3 \times 5)}{3 \times 5 \times 7} \quad (\text{Common denominator}) \\
 &= \frac{35 + 63 + 75}{105} \\
 &= \frac{173}{105} \\
 &= 1\frac{68}{105}
 \end{aligned}$$

(x) **Question 10**

$$\begin{aligned}
 \frac{x}{2} - \frac{x}{3} &= \frac{3x - 2x}{2 \times 3} \quad (\text{Common denominator}) \\
 &= \frac{x}{6} \quad (\text{Simplifying})
 \end{aligned}$$

(xi) **Question 11**

$$\begin{aligned}
 \frac{(\frac{2}{3})}{(\frac{1}{5})} &= \frac{2}{3} \div \frac{1}{5} \\
 &= \frac{2}{3} \times \frac{5}{1} \quad (\text{Invert denominator and multiply}) \\
 &= \frac{10}{3}
 \end{aligned}$$

(xii) **Question 12**

$$\begin{aligned}
 \frac{(\frac{2}{7})}{3} &= \frac{2}{7} \div 3 \\
 &= \frac{2}{7} \times \frac{1}{3} \\
 &= \frac{2}{21}
 \end{aligned}$$

(xiii) **Question 13**

$$\begin{aligned}
 2\left(\frac{2}{x} - \frac{x}{2}\right) &= 2\frac{(4-x^2)}{2x} \quad (\text{Common denominator}) \\
 &= \frac{4-x^2}{x} \quad (\text{Cancelling out common factors}) \\
 &= \frac{4}{x} - \frac{x^2}{x} \\
 &= \frac{4}{x} - x
 \end{aligned}$$

(xiv) **Question 14**

$$\begin{aligned} \frac{-12}{p} \left( \frac{3p}{2} + \frac{p}{2} \right) &= \frac{-12}{p} \left( \frac{3p+p}{2} \right) && \text{(Common denominator)} \\ &= \frac{-12}{p} \left( \frac{4p}{2} \right) \\ &= -24 && \text{(Cancelling out common terms)} \end{aligned}$$

(xv) **Question 15**

$$\begin{aligned} \frac{\left(\frac{3}{x}\right)}{x+3} &= \frac{3}{x} \times \frac{1}{x+3} && \text{(invert and multiply)} \\ &= \frac{3}{x(x+3)} \end{aligned}$$

(xvi) **Question 16**

$$\begin{aligned} \frac{\left(\frac{5Q}{P+2}\right)}{\left(\frac{1}{P+2}\right)} &= \left(\frac{5Q}{P+2}\right) \times \left(\frac{P+2}{1}\right) && \text{(invert and multiply)} \\ &= 5Q && \text{(since } P+2 \text{ cancels out)} \end{aligned}$$

**2. Progress Exercises 1.2, page 14 / page 17**(i) **Question 1**

$$\begin{aligned} 2x + 3x + 5(2x - 3) &= 30 \\ 2x + 3x + 10x - 15 &= 30 && \text{(Removing brackets)} \\ 15x - 15 &= 30 && \text{(like terms together)} \\ 15x &= 30 + 15 && \text{(like terms together)} \\ 15x &= 45 \\ \frac{15x}{15} &= \frac{45}{15} && \text{(dividing by 15 both sides)} \\ x &= 3 \end{aligned}$$

(ii) **Question 2**

$$\begin{aligned} 4x^2 + 7x - 2x(2x - 5) &= 17 \\ 4x^2 + 7x - 4x^2 + 10x &= 17 && \text{(Removing brackets)} \\ 17x &= 17 && \text{(like terms together)} \\ x &= 1 && \text{(dividing both sides by 17)} \end{aligned}$$

(iii) **Question 3**

$$\begin{aligned} (x-2)(x+4) &= 0 \\ \text{Either } x-2 &= 0 \text{ or } x+4 = 0 \\ x &= 2 \text{ or } x = -4 \end{aligned}$$

(iv) **Question 4**

$$\begin{aligned}
 (x-2)(x+4) &= 2x \\
 x(x+4) - 2(x+4) - 2x &= 0 && \text{(Expanding brackets)} \\
 x^2 + 4x - 2x - 8 - 2x &= 0 \\
 x^2 - 8 &= 0 && \text{(like terms together)} \\
 x^2 &= 8 \\
 x &= \pm\sqrt{8} && \text{(taking square-root of both sides)}
 \end{aligned}$$

(v) **Question 5**

$$\begin{aligned}
 (x-2)(x+4) &= -8 \\
 x(x+4) - 2(x+4) + 8 &= 0 && \text{(Expanding brackets)} \\
 x^2 + 4x - 2x - 8 + 8 &= 0 \\
 x^2 + 2x &= 0 \\
 x(x+2) &= 0 && \text{(factorising)}
 \end{aligned}$$

Either  $x = 0$  or  $x + 2 = 0$   
 $x = 0$  or  $x = -2$

(vi) **Question 6**

$$\begin{aligned}
 x(x-2)(x+4) &= 0 \\
 \text{Either } x &= 0, \quad x-2=0 \quad \text{or} \quad x+4=0 && \text{(Equating each bracket to zero).} \\
 \text{Therefore } x &= 0, \quad x=2 \quad \quad \quad x=-4.
 \end{aligned}$$

(vii) **Question 7**

$$\begin{aligned}
 4x(x-2)(x-2) &= 0 \\
 \text{Either } 4x &= 0, \quad x-2=0 \quad \text{or} \quad x-2=0 && \text{(Equating each term to zero).} \\
 x &= 0, \quad x=2
 \end{aligned}$$

(viii) **Question 8**

$$\begin{aligned}
 2x(y+2) - 2y(x+2) &= 0 \\
 2xy + 4x - 2xy - 4y &= 0 && \text{(Expanding each bracket)} \\
 4x - 4y &= 0 \\
 4x &= 4y \\
 x &= y && \text{(dividing both sides by 4)}
 \end{aligned}$$

There are many solutions.

(ix) **Question 9**

$$(x + 2)(y + 2) = 0$$

Either  $x + 2 = 0$  or  $y + 2 = 0$  (Equating each bracket to zero).

$$\text{Either } x = -2 \text{ or } y = -2.$$

(x) **Question 10**

$$(x + 2)(y + 2) + (x - 3)(y + 2) = 0$$

$$(y + 2)[x + 2 + x - 3] = 0 \quad (\text{Factoring out } y + 2.)$$

$$(y + 2)(2x - 1) = 0 \quad (\text{simplifying the } x \text{ terms}).$$

Either  $y + 2 = 0$  or  $2x - 1 = 0$

Therefore  $y = -2$  or  $2x = 1$

$$x = \frac{1}{2}$$

(xi) **Question 11**

$$(x - 2)(x + 4) - 2(x - 4) = 0$$

$$x(x + 4) - 2(x + 4) - 2(x - 4) = 0 \quad (\text{Expanding brackets})$$

$$x^2 + 4x - 2x - 8 - 2x + 8 = 0$$

$$x^2 = 0 \quad (\text{adding like terms together})$$

$$x = 0 \quad (\text{finding square-root of both sides})$$

(xii) **Question 12**

$$(x + 2)^2 + (x - 2)^2 = 0$$

$$(x + 2)(x + 2) + (x - 2)(x - 2) = 0$$

$$x(x + 2) + 2(x + 2) + x(x - 2) - 2(x - 2) = 0$$

$$x^2 + 2x + 2x + 4 + x^2 - 2x - 2x + 4 = 0$$

$$2x^2 + 8 = 0$$

$$2x^2 = -8$$

$$x^2 = -4$$

$x = \sqrt{-4}$  or  $x = -\sqrt{-4}$ . These are complex numbers and beyond the scope of this module.

(xiii) **Question 13**

$$(x + 2)^2 - (x - 2)^2 = 0$$

$$(x + 2)(x + 2) - (x - 2)(x - 2) = 0$$

$$x(x + 2) + 2(x + 2) - x(x - 2) - 2(x - 2) = 0 \quad (\text{Expanding each bracket})$$

$$x^2 + 2x + 2x + 4 - x^2 + 2x + 2x - 4 = 0$$

$$8x = 0 \quad (\text{adding like terms})$$

$$x = 0 \quad (\text{dividing both sides by 8})$$

(xiv) **Question 14**

$$x(x^2 + 2) = 0$$

Either  $x = 0$  or  $x^2 + 2 = 0$   
 $x = 0$  or  $x^2 = -2$   
 $x = 0$  or  $x = \sqrt{-2}$  or  $x = -\sqrt{-2}$ . These are complex numbers beyond the scope of this module.

(xv) **Question 15**

$$\begin{aligned} \frac{x}{3} - \frac{x}{2} &= \frac{2}{3} \\ \frac{2x - 3x}{3 \times 2} &= \frac{2}{3} && \text{(finding common denominator of LHS)} \\ -\frac{x}{6} &= \frac{2}{3} \\ x &= -\frac{2}{3} \times 6 && \text{(multiplying both sides by 6)} \\ x &= -4 \end{aligned}$$

(xvi) **Question 16**

$$\begin{aligned} \frac{x}{3} &= 2x \\ x &= 6x && \text{(cross-multiplying )} \\ x - 6x &= 0 && \text{(like terms together )} \\ -5x &= 0 \\ x &= 0 && \text{(dividing by } -5 \text{ both sides )} \end{aligned}$$

(xvii) **Question 17**

$$\begin{aligned} \frac{2}{x} - \frac{3}{2x} &= 1 \\ \frac{4 - 3}{2x} &= 1 && \text{(common denominator )} \\ 1 &= 2x && \text{(cross multiplying )} \\ \text{or } 2x &= 1 \\ x &= \frac{1}{2} && \text{(dividing both sides by 2 )} \end{aligned}$$

(xviii) **Question 18**

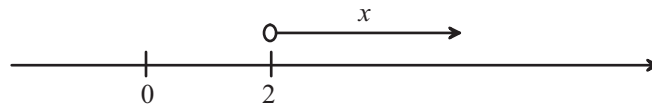
$$\frac{4x(x-4)(x+3,8)}{x^4 - 4x^3 + 7x^2 - 5x + 102} = 0$$

$$4x(x-4)(x+3,8) = 0 \quad \text{(cross-multiplication )}$$

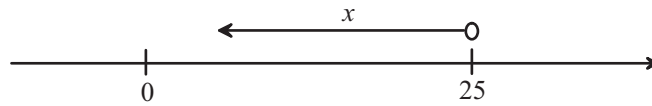
$4x = 0, \quad x - 4 = 0, \quad x + 3,8 = 0$  (equating each bracket to zero )  
 $x = 0, \quad x = 4, \quad \text{or} \quad x = -3,8$

## 3. Progress Exercises 1.3, Question 1, page 20 / page 23

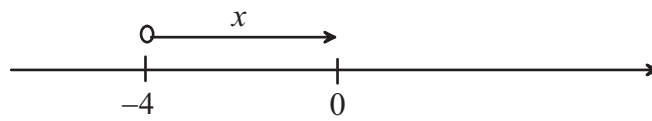
(a)  $x > 2$



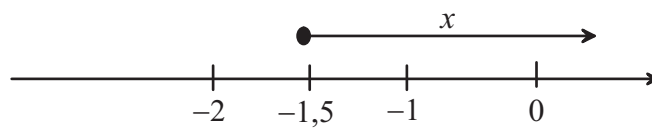
(b)  $x < 25$



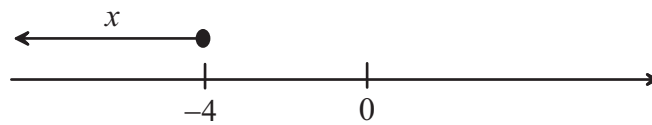
(c)  $x > -4$



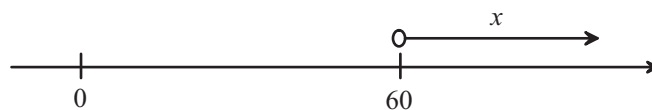
(d)  $x \geq -1,5$



(e)  $-4 \geq x$

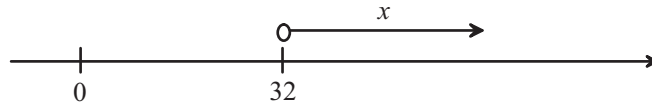


(f)  $60 < x$



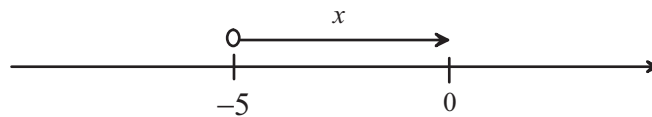
## 4. Progress Exercises 1.3, Question 2, page 20 / page 23

$$\begin{aligned} \text{(a)} \quad x - 25 &> 7 \\ x &> 7 + 25 \\ x &> 32 \end{aligned}$$



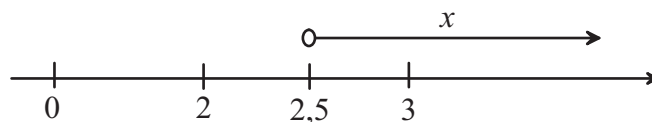
$x$  is greater than 32.

$$\begin{aligned} \text{(b)} \quad 5 &< 2x + 15 \\ 5 - 15 &< 2x \\ -10 &< 2x \\ -5 &< x \\ \text{or} \quad x &> -5 \end{aligned}$$



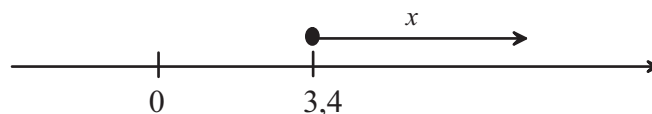
$x$  is greater than -5.

$$\begin{aligned} \text{(c)} \quad \frac{25}{x} &< 10 \\ 25 &< 10x \\ \frac{25}{10} &< x \\ 2,5 &< x \\ \text{or} \quad x &> 2,5 \end{aligned}$$



$x$  is greater than 2,5.

$$\begin{aligned} \text{(d)} \quad \frac{x}{2} + \frac{x}{3} &\geq \frac{17}{6} \\ 6 \times \frac{x}{2} + 6 \times \frac{x}{3} &\geq 6 \times \frac{17}{6} \quad (\text{Multiplying throughout by the lowest common multiple, 6}). \\ 3x + 2x &\geq 17 \\ 5x &\geq 17 \\ x &\geq \frac{17}{5} \\ x &\geq 3,4 \end{aligned}$$



$x$  is greater or equal to 3,4.

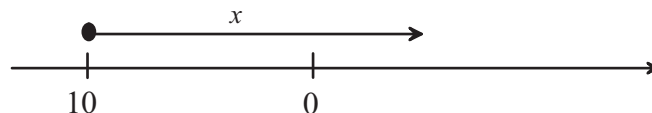
$$(e) \quad 3x - 29 \leq 7x + 11$$

$$3x - 7x \leq 11 + 29 \quad (\text{like terms together}).$$

$$-4x \leq 40$$

$$\frac{-4x}{-4} \geq \frac{40}{-4} \quad (\text{divide by } -4 \text{ both sides}).$$

$$x \geq -10 \quad (\text{inequality changes direction when} \\ \text{dividing by a negative number}).$$



$x$  is greater or equal to  $-10$ .

5. **Progress Exercises 1.3, Question 3, page 20 / page 23**

$$(a) \quad 12\% \text{ of } 5\,432,7 = \frac{12}{100} \times 5\,432,7 \\ = 651,924$$

$$(b) \quad 85\% \text{ of } 23,65 = 0,85 \times 23,65 \\ = 20,1025$$

$$(c) \quad 11,5\% \text{ of } 6,5 = 0,115 \times 6,5 \\ = 0,7475$$

6. **Progress Exercises 1.3, Question 4, page 20 / page 23**

(a)

$$\begin{aligned} \text{The increase in the hourly rate} &= 0,14 \times 5,65 \\ &= 0,791 \end{aligned}$$

The increase is £0,791.

(b)

$$\begin{aligned} \text{The new hourly rate} &= \text{old hourly rate} + \text{increase} \\ &= 5,65 + 0,791 \\ &= 6,441 \end{aligned}$$

The new hourly rate is £6,411.

7. **Progress Exercises 1.3, Question 7, page 20 / page 23**

Week 1

$$\begin{aligned} \text{Number of cars produced} &= 400 - 0,2 \times 400 \\ &= 400 - 80 \\ &= 320. \end{aligned}$$

Week 2

$$\begin{aligned} \text{Number of cars produced} &= 320 - 0,2 \times 320 \\ &= 320 - 64 \\ &= 256. \end{aligned}$$



$$\begin{aligned}
 \text{Week 3} \quad \text{Number of cars produced} &= 256 - 0,2 \times 256 \\
 &= 256 - 51,2 \\
 &\approx 256 - 51 \\
 &\approx 205.
 \end{aligned}$$

$$\begin{aligned}
 \text{Week 4} \quad \text{Number of cars produced} &= 205 - 0,2 \times 205 \\
 &= 205 - 41 \\
 &= 164.
 \end{aligned}$$

$$\begin{aligned}
 \text{Week 5} \quad \text{Number of cars produced} &= 164 - 0,2 \times 164 \\
 &= 164 - 32,8 \\
 &\approx 164 - 33 \\
 &\approx 131.
 \end{aligned}$$

$$\begin{aligned}
 \text{Week 6} \quad \text{Number of cars produced} &= 131 - 0,2 \times 131 \\
 &= 131 - 26,2 \\
 &\approx 131 - 26 \\
 &\approx 105.
 \end{aligned}$$

**8. Progress Exercises 1.3, Question 9, page 20 / page 23**

$$\begin{aligned}
 \text{Profit} &= \text{Selling Price} - \text{Cost Price} \\
 &= 658 - 480 \\
 &= 178
 \end{aligned}$$

$$\begin{aligned}
 \text{Profit as a percentage of cost} &= \frac{\text{Profit}}{\text{Cost}} \\
 &= \frac{178}{480} \\
 &= 0,3708333333 \\
 &\approx 37,08\%.
 \end{aligned}$$

## 9. Test Exercise 1, Question 8, part (b), page 34 / page 35

$$\begin{aligned} \text{Total tonnage of tea} &= 400 + 580 + 250 + 120 \\ &= 1\,350\text{tons.} \end{aligned}$$

$$\begin{aligned} \text{Percentage from India} &= \frac{400}{1350} \\ &= 0,2962962963 \\ &\approx 29,63\% \end{aligned}$$

$$\begin{aligned} \text{Percentage from China} &= \frac{580}{1350} \\ &= 0,4296296296 \\ &\approx 42,96\% \end{aligned}$$

$$\begin{aligned} \text{Percentage from Sri Lanka} &= \frac{250}{1350} \\ &= 0,1851851852 \\ &\approx 18,52\% \end{aligned}$$

$$\begin{aligned} \text{Percentage from Burma} &= \frac{120}{1350} \\ &= 0,888888889 \\ &\approx 8,89\%. \end{aligned}$$

$$\text{Check: } 29,63 + 42,96 + 18,52 + 8,89 = 100$$

## 15.2 Self-Evaluation Exercise 2: Unit 2

## 1. Progress Exercises 2.1, Question 2, page 43 / page 43

We may use any two points on the straight line to compute the slope of the line, for example  $(-1; 7)$  and  $(5; 1)$ :

$$\Delta x = 5 - (-1) = 6 \quad \Delta y = 1 - 7 = -6$$

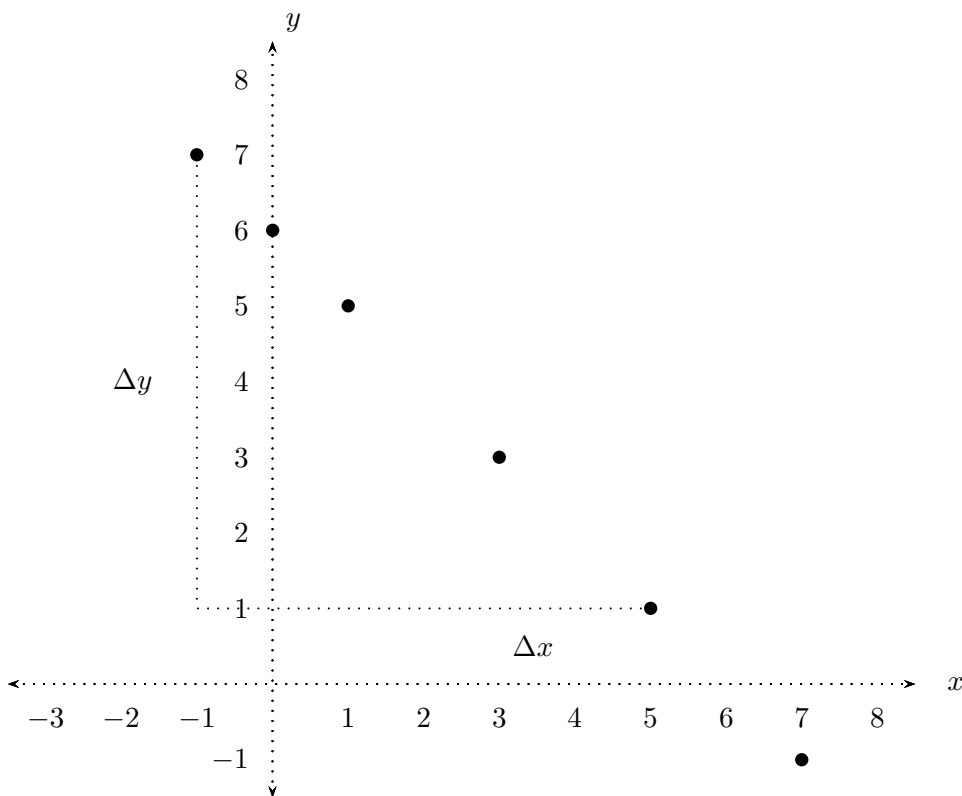
so that

$$m = \frac{\Delta y}{\Delta x} = \frac{-6}{6} = -1.$$

## 2. Progress Exercises 2.2, Question 2, page 55 / page 54

(a)  $y = x + 2 = 1 \cdot x + 2$

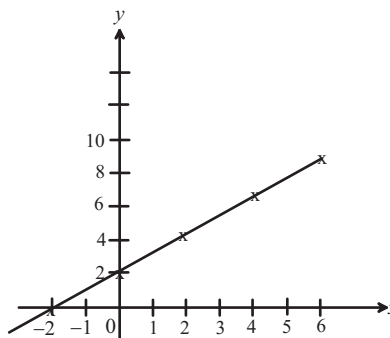
(i) Slope = 1.



- (ii) If  $x = 0$ ,  $y = 2$ . Therefore  $y$ -intercept = 2.  
 If  $y = 0$ ,  $0 = x + 2$   
 $-2 = x$   
 or  $x = -2$ . Therefore  $x$ -intercept =  $-2$ .

(iii)

| $x$ | $y$          |
|-----|--------------|
| -2  | $-2 + 2 = 0$ |
| 0   | $0 + 2 = 2$  |
| 2   | $2 + 2 = 4$  |
| 4   | $4 + 2 = 6$  |
| 6   | $6 + 2 = 8$  |



- (b)  $y = -4x + 3 = -4 \times x + 3$   
 (i) Slope =  $-4$ .

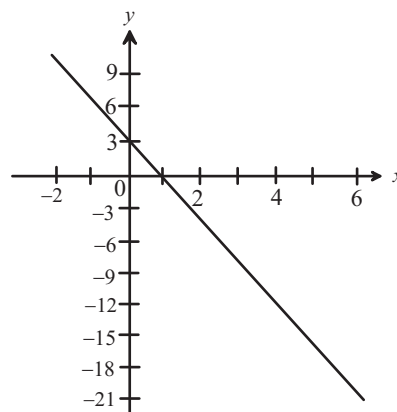
(ii) If  $x = 0$ ,  $y = -4 \times 0 + 3 = 3$ . Therefore  $y$ -intercept = 3.

$$\begin{aligned} \text{If } y = 0, \quad 0 &= -4x + 3 \\ 4x &= 3 \\ \frac{4x}{4} &= \frac{3}{4} \\ x &= \frac{3}{4} \\ x &= 0,75 \end{aligned}$$

Therefore  $x$ -intercept = 0,75.

(iii)

| $x$ | $y$                               |
|-----|-----------------------------------|
| -2  | $-4 \times -2 + 3 = 8 + 3 = 11$   |
| 0   | $-4 \times 0 + 3 = 0 + 3 = 3$     |
| 2   | $-4 \times 2 + 3 = -8 + 3 = -5$   |
| 4   | $-4 \times 4 + 3 = -16 + 3 = -13$ |
| 6   | $-4 \times 6 + 3 = -24 + 3 = -21$ |



(c)  $y = 0,5x - 2 = 0,5 \times x - 2$ .

(i) Slope = 0,5.

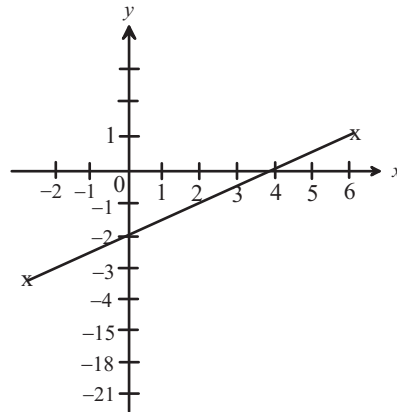
(ii) If  $x = 0$ ,  $y = 0,5 \times 0 - 2 = -2$ . Therefore  $y$ -intercept = -2.

$$\begin{aligned} \text{If } y = 0, \quad 0 &= 0,5x - 2 \\ 2 &= 0,5x \\ \frac{2}{0,5} &= x \\ \text{or } x &= 4. \end{aligned}$$

Therefore  $x$ -intercept = 4.

(iii)

| $x$ | $y$                               |
|-----|-----------------------------------|
| -2  | $0,5 \times -2 - 2 = -1 - 2 = -3$ |
| 0   | $0,5 \times 0 - 2 = 0 - 2 = -2$   |
| 2   | $0,5 \times 2 - 2 = 1 - 2 = -1$   |
| 4   | $0,5 \times 4 - 2 = 2 - 2 = 0$    |
| 6   | $0,5 \times 6 - 2 = 3 - 2 = 1$    |



(d)

$$\begin{aligned}
 2y &= 6x + 4 \\
 \frac{2y}{2} &= \frac{6x}{2} + \frac{4}{2} && \text{(dividing by 2 both sides)} \\
 y &= 3x + 2.
 \end{aligned}$$

(i) Slope = 3.

(ii) If  $x = 0$ ,  $y = 3 \times 0 + 2 = 0 + 2 = 2$ . Therefore  $y$ -intercept = 2.

$$\text{If } y = 0, \quad 3x + 2 = 0$$

$$3x = 0 - 2$$

$$3x = -2$$

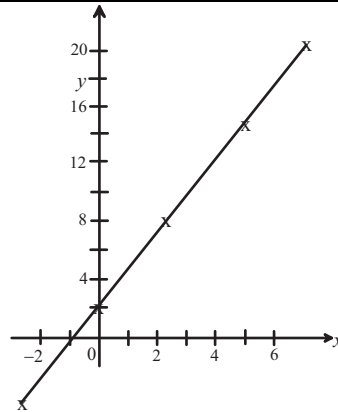
$$\frac{3x}{3} = -\frac{2}{3}$$

$$\text{or } x = -\frac{2}{3}$$

Therefore  $x$ -intercept =  $-\frac{2}{3}$ .

(iii)

| $x$ | $y$               |            |        |
|-----|-------------------|------------|--------|
| -2  | $3 \times -2 + 2$ | $= -6 + 2$ | $= -4$ |
| 0   | $3 \times 0 + 2$  | $= 0 + 2$  | $= 2$  |
| 2   | $3 \times 2 + 2$  | $= 6 + 2$  | $= 8$  |
| 4   | $3 \times 4 + 2$  | $= 12 + 2$ | $= 14$ |
| 6   | $3 \times 6 + 2$  | $= 18 + 2$ | $= 20$ |



## 3. Progress Exercises 2.2, Question 3, page 55 / page 54

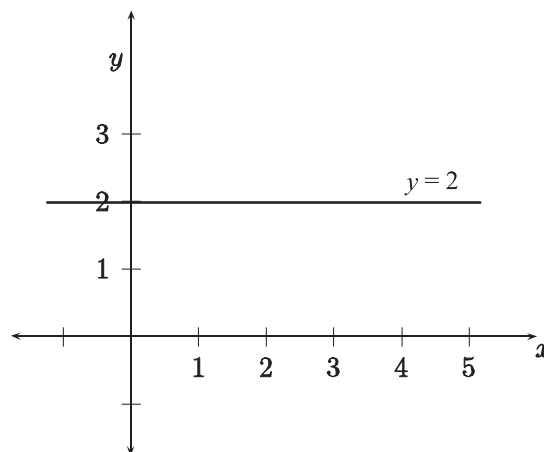
$$y = f(x) = mx + c.$$

(a)

$$y = 2 = 0 \times x + 2$$

Slope = 0 (horizontal line).

If  $x = 0$ ,  $y = 2$ . ( $y$ -intercept)

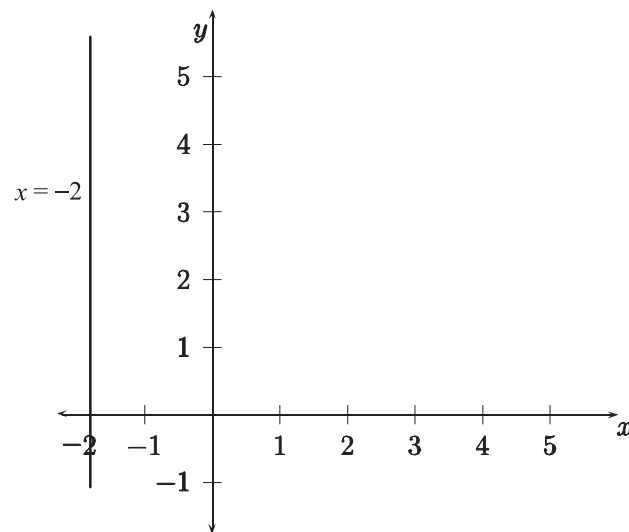


(b)

$$x = -2$$

Slope =  $\infty$  (vertical line).

If  $y = 0$ ,  $x = -2$  ( $x$ -intercept)



(c)

$$\begin{aligned} 5x + y + 4 &= 0 \\ \text{or } y &= -5x - 4 \\ \text{Slope} &= -5 \end{aligned}$$

$$\text{If } x = 0, \quad y = -5 \times 0 - 4 = 0 - 4 = -4 \quad (y\text{-intercept})$$

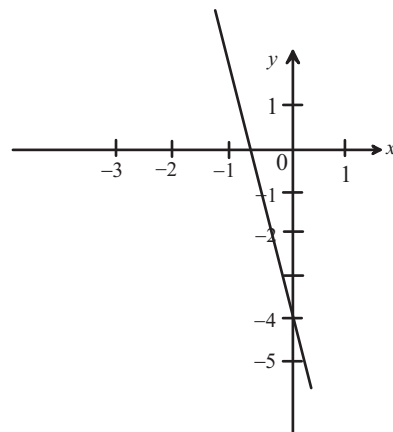
$$\text{If } y = 0 \quad 0 = -5x - 4$$

$$5x = -4$$

$$\frac{5x}{5} = \frac{-4}{5}$$

$$x = -\frac{4}{5}$$

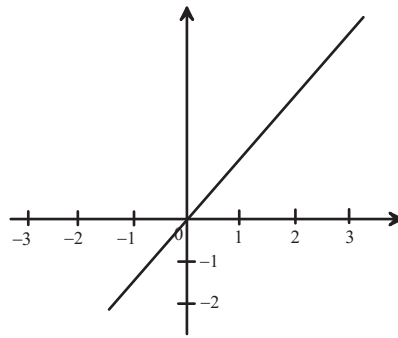
$$\text{or } \quad x = -0,8 \quad (x\text{-intercept})$$



(d)

$$\begin{aligned} y = x &= 1 \times x + 0 \\ \text{Slope} &= 1. \\ \text{If } x = 0, \quad y &= 0 \quad (y\text{-intercept}) \\ \text{If } y = 0, \quad x &= 0 \quad (x\text{-intercept}) \end{aligned}$$

Therefore, line passes through the origin.

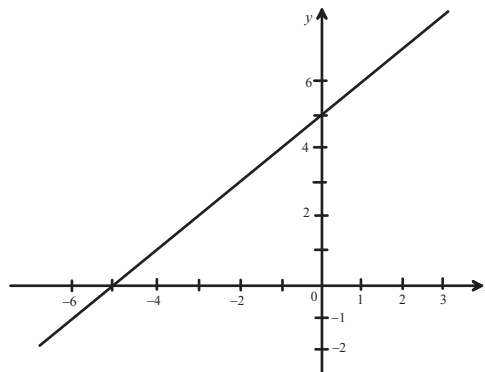


(e)

$$\begin{aligned}x - y + 5 &= 0 \\x + 5 &= y \\ \text{or } y &= x + 5 \\ \text{Slope} &= 1.\end{aligned}$$

$$\text{If } x = 0, \quad y = 5 \quad (y\text{-intercept})$$

$$\begin{aligned}\text{If } y = 0, \quad x + 5 &= 0 \\ x &= -5 \quad (x\text{-intercept})\end{aligned}$$



## 4. Progress Exercises 2.2, Question 4, page 55 / page 54

(a)  $2y - 5x + 10 = 0$

(i)

$$2y = 5x - 10$$

$$y = \frac{5}{2}x - \frac{10}{2}$$

$$y = 2,5x - 5$$

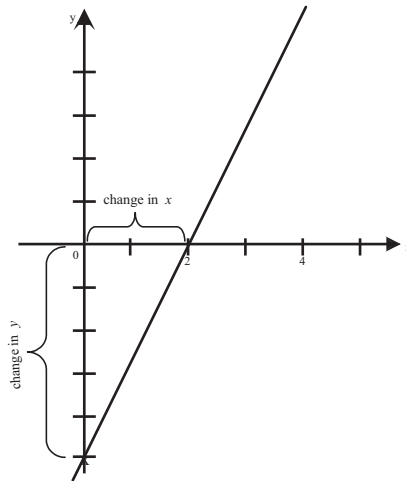
(ii)

$$\text{If } x = 0, \quad y = -5 \quad y\text{-intercept.}$$

$$\begin{aligned}\text{If } y = 0, \quad 2,5x - 5 &= 0 \\ 2,5x &= 5\end{aligned}$$

$$x = 2 \quad x\text{-intercept.}$$





- (iii) Magnitude of change in  $x = 2$   
 Magnitude of change in  $y = 5$

$$\begin{aligned} \text{magnitude of slope} &= \frac{\text{magnitude of change in } y}{\text{magnitude of change in } x} \\ &= \frac{5}{2} \\ &= 2,5 \end{aligned}$$

(b)  $x = 10 - 2y$

(i)

$$\begin{aligned} 2y &= -x + 10 \\ y &= -\frac{x}{2} + \frac{10}{2} \\ y &= -0,5x + 5 \end{aligned}$$

(ii)

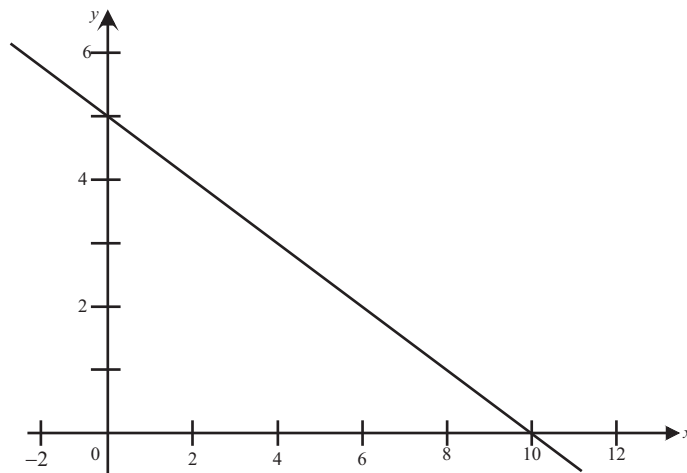
If  $x = 0$ ,  $y = 5$   $y$ -intercept.

If  $y = 0$ ,  $0 = -0,5x + 5$

$$0,5x = 5$$

$$x = \frac{5}{0,5}$$

$x = 10$   $x$ -intercept.



- (iii) Change in  $y = 0 - 5 = -5$   
 Change in  $x = 10 - 0 = 10$

$$\begin{aligned} \text{magnitude of slope} &= \frac{\text{magnitude of change in } y}{\text{magnitude of change in } x} \\ &= \frac{-5}{10} \\ &= -0,5 \end{aligned}$$

(c)  $y + 5x = 15$

(i)  $y = -5x + 15$

(ii)

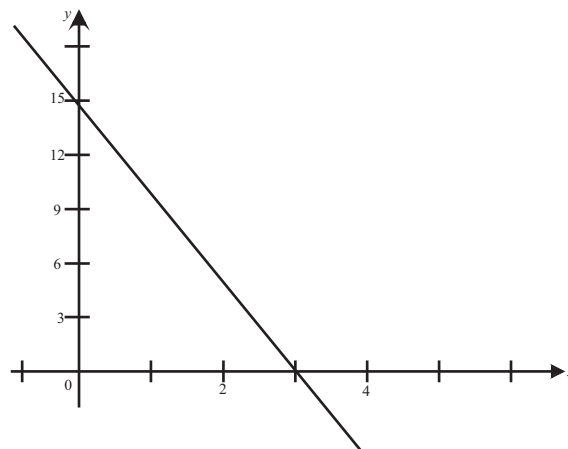
If  $x = 0$ ,  $y = 15$   $y$ -intercept.

If  $y = 0$ ,  $0 = -0,5x + 15$

$$5x = 15$$

$$\frac{5x}{5} = \frac{15}{5}$$

$x = 3$   $x$ -intercept.



- (iii) Change in  $y = 0 - 15 = -15$   
 Change in  $x = 3 - 0 = 3$

$$\begin{aligned} \text{magnitude of slope} &= \frac{\text{magnitude of change in } y}{\text{magnitude of change in } x} \\ &= \frac{-15}{3} \\ &= -5 \end{aligned}$$

5. Progress Exercises 2.2, Question 6, page 55 / page 54

(a)  $y = 2x + 1$

- (i) If  $x = 1$ ,  $y = 2 \times 1 + 1 = 2 + 1 = 3$ .  
 Therefore  $A(1; 3)$  lies on the line  $y = 2x + 1$
- (ii) If  $x = -1$ ,  $y = 2 \times -1 + 1 = -2 + 1 = -1$ .  
 Therefore  $B(-1; -1)$  lies on the line  $y = 2x + 1$
- (iii) If  $x = 0$ ,  $y = 2 \times 0 + 1 = 0 + 1 = 1$ .  
 Therefore  $C(0; 1)$  lies on the line  $y = 2x + 1$

(b)  $Q = 50 - 0,5P$

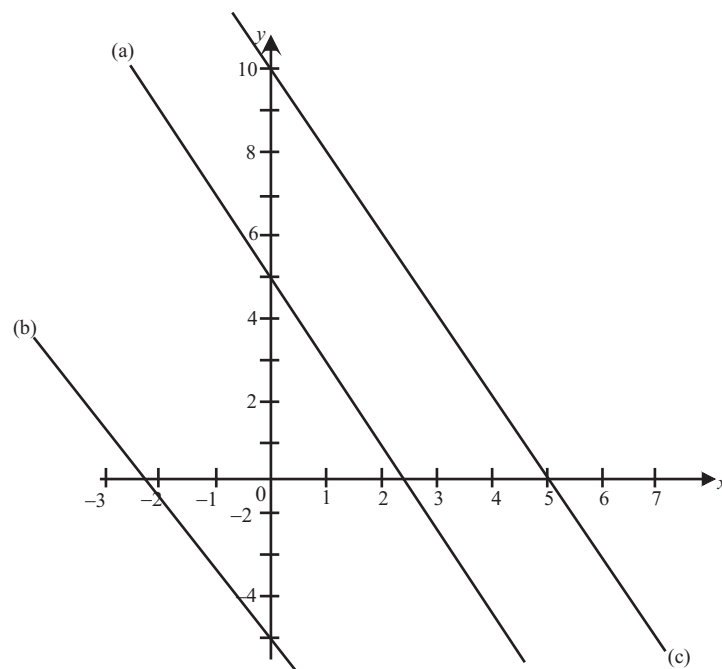
- (i) If  $P = 90$ ,  $Q = 50 - 0,5 \times 90 = 50 - 45 = 5$ .  
 Therefore  $A(90; 5)$  lies on the line  $Q = 50 - 0,5P$
- (ii) If  $P = 8$ ,  $Q = 50 - 0,5 \times 8 = 50 - 4 = 46$ .  
 Therefore  $B(8; 10)$  does not lie on the line  $Q = 50 - 0,5P$
- (iii) If  $P = 70$ ,  $Q = 50 - 0,5 \times 70 = 50 - 35 = 15$ .  
 Therefore  $C(70; 15)$  lies on the line  $Q = 50 - 0,5P$

(c)  $TC = 10 + 2Q$

- (i) If  $Q = 2$ ,  $TC = 10 + 2 \times 2 = 10 + 4 = 14$   
 Therefore  $A(2; 14)$  lies on the line  $TC = 10 + 2Q$
- (ii) If  $Q = 14$ ,  $TC = 10 + 2 \times 14 = 10 + 28 = 38$   
 Therefore  $B(14; 18)$  does not lie on the line  $TC = 10 + 2Q$
- (iii) If  $Q = 6$ ,  $TC = 10 + 2 \times 6 = 10 + 12 = 22$   
 Therefore  $C(6; 22)$  lies on the line  $TC = 10 + 2Q$ .

6. Progress Exercises 2.2, Question 8, page 55 / page 54

|                |  |   |                        |
|----------------|--|---|------------------------|
| Equation       | $y = -2x + 5$                                    | $y + 2x + 5 = 0$                        | $0,2y + 0,4x = 2$      |
| $x$ -intercept | $0 = -2x + 5, 2x = 5$<br>$x = \frac{5}{2} = 2,5$ | $2x + 5 = 0$<br>$2x = -5$<br>$x = -2,5$ | $0,4x = 2$<br>$x = 5$  |
| $y$ -intercept | $y = 0 + 5$<br>$y = 5$                           | $y + 5 = 0$<br>$y = -5$                 | $0,2y = 2$<br>$y = 10$ |



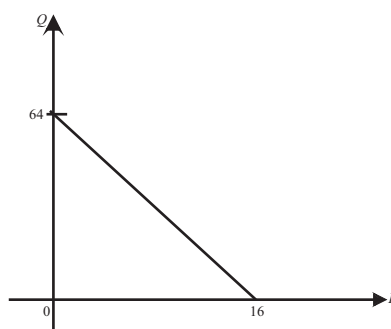
- (i) lines are parallel.  
 (ii) No, this property does not change.

7. **Progress Exercises 2.3, Question 2, page 69 / page 68**

$$Q = 64 - 4P$$

(a)

$$\begin{aligned} \text{If } P = 0, \quad Q &= 64 \\ \text{If } Q = 0, \quad 0 &= 64 - 4P \\ &4P = 64 \\ &P = \frac{64}{4} \\ &P = 16. \end{aligned}$$



(b) Change in demand when price increases by 1 unit is the same as the slope of the line

$$Q = 64 - 4P. \quad C = -4.$$

Therefore, demand will decrease by 4 units if price increases by 1 unit.

(c) If  $P = 0$ , Demand = 64.

(d) If  $Q = 0$ , Price =  $\frac{64}{4} = 16$ .

8. Progress Exercises 2.3, Question 4, page 70 / page 69

$$P = 500 + 2Q$$

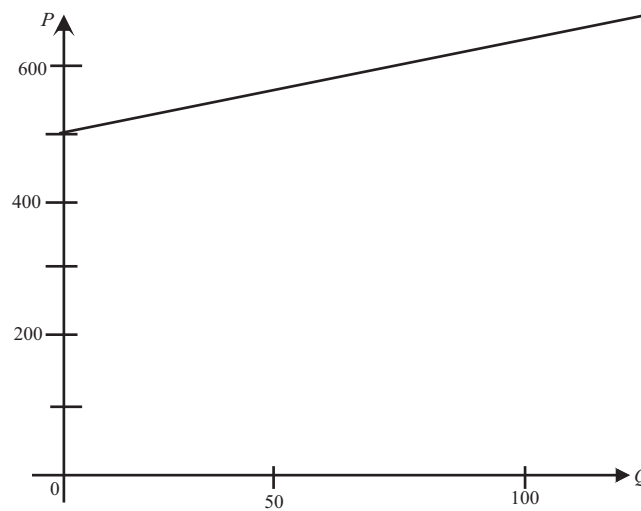
(a)

$$\text{If } Q = 0, \quad P = 500$$

$$\text{If } Q = 100, \quad P = 500 + 2 \times 100$$

$$P = 500 + 200$$

$$P = 700$$



(b) If  $P = 600$ , then

$$500 + 2Q = 600$$

$$2Q = 600 - 500$$

$$2Q = 100$$

$$Q = 50$$

If price is 600 francs, then 50 litre of cognac are supplied.

(c) If  $P = 0$ , then

$$P = 500 + 2 \times 20$$

$$= 500 + 40$$

$$P = 540$$

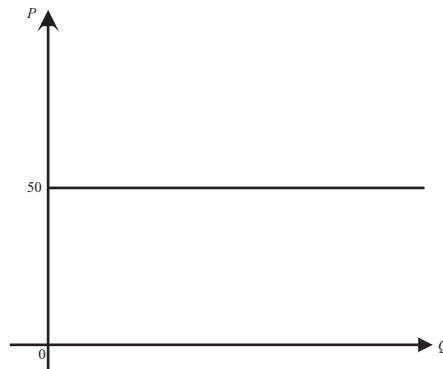
If 20 litre of Cognac are supplied, the price of each bottle will be 540 francs.

9. Progress Exercises 2.3, Question 6, page 70 / page 69

$$p = 50$$

(a) Slope = 0 (horizontal line).

(b)



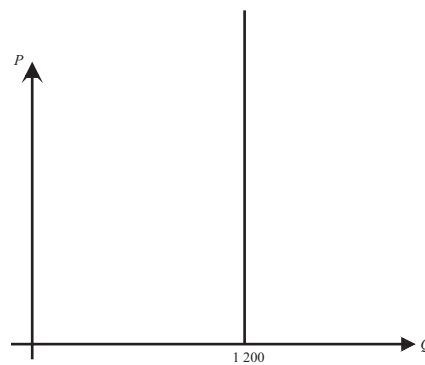
If quantity changes by 10 units, price does not change.

10. Progress Exercises 2.3, Question 7, page 70 / page 69

$$Q = 1200$$

(a) Slope =  $\infty$  (vertical line).

(b)



Regardless of the price, 1200 dinners will be supplied every day.

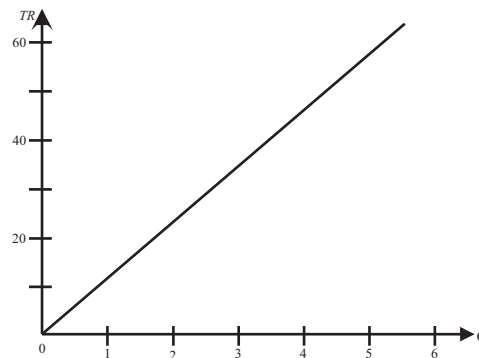
11. Progress Exercises 2.4, Question 2, page 75 / page 75

(a)

$$TR = \text{price} \times \text{quantity}$$

$$TR = 10Q$$

(b)



Therefore, price = 10.

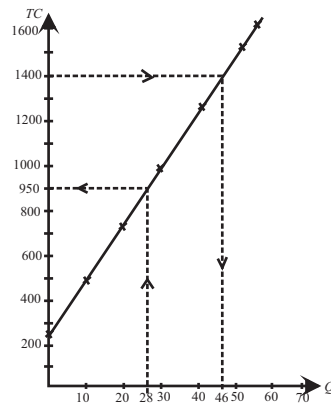
12. **Progress Exercises 2.4, Question 3, page 75 / page 75**

(a)

$$\begin{aligned} \text{Total cost} &= \text{Fixed cost} + \text{Variable cost} \\ &= \text{Fixed cost} + \text{Variable cost per unit} \times \text{Quantity} \end{aligned}$$

Therefore,  $TC = 250 + 25Q$ .

| Q  | TC                                       |
|----|--|
| 0  | $250 + 25 \times 0 = 250 + 0 = 250$      |
| 10 | $250 + 25 \times 10 = 250 + 250 = 500$   |
| 20 | $250 + 25 \times 20 = 250 + 500 = 750$   |
| 30 | $250 + 25 \times 30 = 250 + 750 = 1000$  |
| 40 | $250 + 25 \times 40 = 250 + 1000 = 1250$ |
| 50 | $250 + 25 \times 50 = 250 + 1250 = 1500$ |
| 60 | $250 + 25 \times 60 = 250 + 1500 = 1750$ |



(b)

$$\begin{aligned} \text{If } Q = 28 \quad TC &= 250 + 25 \times 28 \\ &= 250 + 700 \\ &= 950 \end{aligned}$$

(c)

$$\begin{aligned} \text{If } TC = 1400, \quad 250 + 25Q &= 1400 \\ 25Q &= 1400 - 250 \\ 25Q &= 1150 \\ Q &= 46. \end{aligned}$$

(c) See the dotted lines on the graph.

13. **Progress Exercises 2.4, Question 4, page 75 / page 75**

$$\text{Total revenue} = \text{price} \times \text{quantity}$$

(a) Therefore,  $TR = 32Q$

(b)

$$\begin{aligned} \text{If } TR = 1024, \text{ then } 32Q &= 1024 \\ Q &= \frac{1024}{32} \\ Q &= 32. \end{aligned}$$

There are 32 students.

(c)

$$\begin{aligned} \text{If } TR = 44, \text{ then } TR &= 32 \times 44 \\ TR &= 1408 \\ \\ TC &= 250 + 25 \times 44 \\ &= 250 + 1100 \\ TC &= 1350. \end{aligned}$$

Therefore, revenue exceeds costs by  $1408 - 1350 = 58$ .

#### 14. Progress Exercises 2.5, Question 4, page 81 / page 81

Let  $Q$  = number of units of lunch.  
 $P$  = price of lunch

(a) If  $Q = 80$  when  $P = \pounds 5$  and  $Q = 45$  when  $P = \pounds 12$ , then

$$\begin{aligned} \frac{Q - 80}{P - 5} &= \frac{45 - 80}{12 - 5} && \text{(From the formula of a straight line given two points).} \\ Q - 80 &= -\frac{35}{7}(P - 5) \\ Q &= -5(P - 5) + 80 \\ Q &= -5P + 25 + 80 \\ Q &= 105 - 5P \end{aligned}$$

(b)

(i) If price increases by  $\pounds 3$ , demand decreases by  $5 \times 3 = 15$  units.

(ii) If price decreases by  $\pounds 2$ , demand increases by  $5 \times 2 = 10$  units.

(c)

$$\begin{aligned} \text{If } Q &= 105 - 5P \\ 5P &= 105 - Q \\ \frac{5P}{5} &= \frac{105}{5} - \frac{Q}{5} \\ P &= 21 - 0,2Q \end{aligned}$$

If  $Q$  increases by 15 units, then the price will decrease by  $\pounds 3(0,2 \times 15 = 3)$

#### 15. Progress Exercises 2.5, Question 5, page 81 / page 81

Let  $Q$  = number of scarves  
 $P$  = price of scarves, in pounds.



(a) If  $Q = 50$  when price = £6 and  $Q = 90$  when price is £11 then

$$\frac{P - 6}{Q - 50} = \frac{11 - 6}{90 - 50} \quad (\text{Using the formula of the linear function given two points}).$$

$$P - 6 = \frac{5}{40}(Q - 50)$$

$$P = 0,125Q - 6,25 + 6$$

$$P = 0,125Q - 0,25$$

(b) For each £1 increase in price,  $\frac{1}{0,125} = 8$  more scarves are supplied.

(c) If  $P = \text{£}8,50$ ,

$$\begin{aligned} 0,125Q - 0,25 &= 8,50 \\ 0,125Q &= 8,50 + 0,25 \\ 0,125Q &= 8,75 \\ Q &= \frac{8,75}{0,125} \\ Q &= 70 \end{aligned}$$

(d) If  $Q = 120$ , then

$$P = 0,125 \times 120 - 0,25$$

$$P = 15 - 0,25$$

$$P = 14,75.$$

(e) If  $Q = 0$ , then  $P = -0,25$ .

### 16. Progress Exercises 2.7, Question 5, page 91 / page 91

$$\begin{aligned} P &= 90 - 0,05Q \quad \text{and} \quad 0,05Q = 90 - P \\ \text{or} \quad Q &= 1800 - 20P \end{aligned}$$

(a)

$$\begin{aligned} \text{(i)} \quad \varepsilon_d &= -\frac{1}{b} \times \frac{P}{Q} \\ &= -\frac{1}{0,05} \times \frac{P}{1800 - 20P} \\ \varepsilon_d &= \frac{P}{P - 90}. \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \varepsilon_d &= -\frac{1}{b} \times \frac{P}{Q} \\ &= -\frac{1}{0,05} \times \frac{90 - 0,05Q}{Q} \\ \varepsilon_d &= \frac{Q - 1800}{Q}. \end{aligned}$$

(b)

$$\text{(i) If } P = 20, \text{ then } \varepsilon_d = \frac{20}{20-90} = -\frac{20}{70} = -0,2857.$$

$$\text{(ii) If } P = 30, \text{ then } \varepsilon_d = \frac{30}{30-90} = -\frac{30}{60} = -0,5.$$

(iii) If  $P = 70$ , then  $\varepsilon_d = \frac{70}{70-90} = -\frac{70}{20} = -3,5$ .

(c)

$$\begin{aligned} \text{(i) If } \varepsilon_d = -1, \text{ then } \quad & \frac{Q - 1800}{Q} = -1 \\ & Q - 1800 = -Q \\ & Q + Q = 1800 \\ & 2Q = 1800 \\ & Q = \frac{1800}{2} \\ & Q = 900. \end{aligned}$$

$$\begin{aligned} \text{(ii) If } \varepsilon_d = 0, \text{ then } \quad & \frac{Q - 1800}{Q} = 0 \\ & Q - 1800 = 0 \\ & Q = 0 + 1800 \\ & Q = 1800. \end{aligned}$$

### 17. Progress Exercises 2.7, Question 6, page 91 / page 91

(a)

$$\text{Slope} = \frac{\text{Change in quantity}}{\text{Change in price}} = \frac{\Delta Q}{\Delta P}$$

For a linear demand function, this is the same at every point.

$$\begin{aligned} \varepsilon_d &= \frac{\% \text{change in quantity}}{\% \text{change in price}} \\ &= \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} \\ &= \frac{\Delta Q}{Q} \times \frac{P}{\Delta P} \\ &= \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} \end{aligned}$$

$\varepsilon_d$  is different at every point.

(b)

| price, $P$                  | 20      | 30    | 45    | 70    | 90        |
|-----------------------------|---------|-------|-------|-------|-----------|
| slope, $m$                  | -0,05   | -0,05 | -0,05 | -0,05 | -0,05     |
| elasticity, $\varepsilon_d$ | -0,2857 | -0,5  | -1,0  | -3,5  | $-\infty$ |
| % $\Delta P$ (fixed)        | 10%     | 10%   | 10%   | 10%   | 10%       |
| % $\Delta Q$                | -2,857% | -5%   | -10%  | -35%  | $-\infty$ |

Where  $\% \Delta Q = \% \Delta P \times \varepsilon_d$

### 18. Progress Exercises 2.7, Question 7, page 91 / page 91

(a)

$$\begin{aligned}
 P &= 20 + 0,5Q \quad \text{or} \quad 0,5Q = -20 + P \\
 Q &= -\frac{20}{0,5} + \frac{P}{0,5} \\
 Q &= -40 + 2P.
 \end{aligned}$$

If  $P = 40$ , then  $Q = -40 + 2 \times 40 = -40 + 80 = 40$

If  $P = 60$ , then  $Q = -40 + 2 \times 60 = -40 + 120 = 80$

$$\begin{aligned}
 \text{Elasticity, } \varepsilon_d &= -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2} \\
 &= \frac{1}{0,5} \times \frac{40 + 60}{40 + 80} \\
 &= \frac{2}{1} \times \frac{100}{120} \\
 \varepsilon_d &= \frac{5}{3}
 \end{aligned}$$

(b)

(i) If  $P = 40$ , then  $Q = 40$ ,  $\varepsilon_d = \frac{1}{0,5} \times \frac{40}{40} = \frac{1}{0,5} = 2$ .

If  $\% \Delta P = 10\%$ , then  $\% \Delta Q = \% \Delta P \times \varepsilon_d = 10\% \times 2 = 20\%$ .

(ii) If  $P = 40$ , then  $Q = 40$

If  $P$  increases by 10%, then new  $P = 40 + 0,1 \times 40 = 40 + 4 = 44$ .

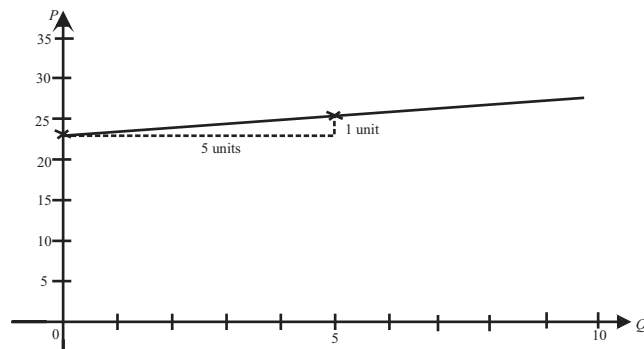
If  $P = 44$ , then  $Q = 2(44) - 40 = 88 - 40 = 48$

$$\% \Delta Q = \frac{48 - 40}{40} = \frac{8}{40} = 20\%.$$

The answers in (i) and (ii) are the same.

### 19. Test Exercise 2, Question 6, page 99 / page 99

(a) If  $Q = 0$ ,  $P = 24$  (vertical intercept).



(b)

$$\text{Slope} = \frac{\text{change in } P}{\text{change in } Q} = \frac{1}{5} = 0,2.$$

From

$$y - y_1 = m(x - x_1)$$

$$P - 24 = 0,2(Q - 0)$$

$$P = 0,2Q + 24.$$

(c) If  $P = 45$ , then

$$0,2Q + 24 = 45$$

$$0,2Q = 45 - 24$$

$$0,2Q = 21$$

$$Q = \frac{21}{0,2}$$

$$Q = 105$$

## 20. Test Exercise 2, Question 7, page 99 / page 99

$$20P = 5Q + 80$$

If a tax of £1,50 is introduced per unit, then

$$20(P + 1,50) = 5Q + 80$$

$$20P + 30 = 5Q + 80$$

$$20P = 5Q + 80 - 30$$

$$20P = 5Q + 50.$$

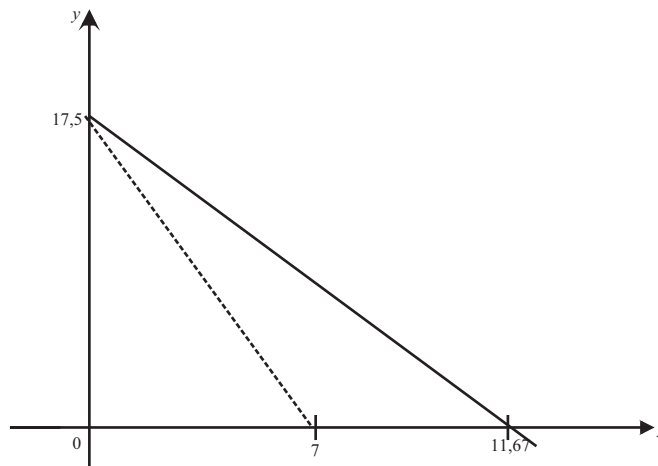
## 21. Test Exercise 2, Question 8, page 99 / page 99

Let  $x$  = number of times a student attends a football match.  
 $y$  = number of times a student goes to the cinema.

Then  $12x + 8y = 140$  (budget constraint).

$$\text{If } x = 0, y = 17,5$$

$$\text{If } y = 0, x = 11,67.$$



If the price of watching football increases to £20, then the budget constraint becomes

$$20x + 8y = 140 \quad (\text{Shown by dotted line on graph}).$$

## 15.3 Self-Evaluation Exercise 3 : Unit 3

## 1. Progress exercise 3.1, Question 3, page 110 / page 111

$$x + y = 19 \quad (1)$$

$$x - 8y = 10 \quad (2)$$

$$(1) - (2)$$

$$y - (-8y) = 19 - 10$$

$$9y = 9$$

$$y = 1$$

Substitute for  $y$  in (1):

$$x + 1 = 19$$

$$x = 19 - 1$$

$$x = 18$$

Therefore  $x = 18, y = 1$ .

## 2. Progress Exercises 3.1, Question 4, page 110 / page 111

$$3y + 2x = 5 \quad (1)$$

$$4y - x = 3 \quad (2)$$

$$1 \times (1)$$

$$3y + 2x = 5 \quad (1)$$

$$2 \times (2)$$

$$8y - 2x = 6 \quad (3)$$

$$(1) + (3)$$

$$\begin{aligned} 11y &= 11 \\ y &= 1 \end{aligned}$$

Substitute for  $y$  in (2)

$$\begin{aligned} 4 - x &= 3 \\ 4 - 3 &= x \\ 1 &= x \\ \text{or } x &= 1 \end{aligned}$$

Therefore  $x = 1$  and  $y = 1$

**3. Progress Exercises 3.1, Question 6, page 110 / page 111**

$$y = 2x + 3 \quad (1)$$

$$y = 7 - 2x \quad (2)$$

Substitute for  $y$  in (2) using  $y$  in (1)

$$\begin{aligned} 2x + 3 &= 7 - 2x \\ 2x + 2x &= 7 - 3 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

Substitute for  $x$  in (1)

$$\begin{aligned} y &= 2 \times 1 + 3 \\ y &= 2 + 3 \\ y &= 5 \end{aligned}$$

Therefore  $x = 1, y = 5$

**4. Progress Exercises 3.1, Question 9, page 110 / page 111**

$$4x - y = 12 \quad (1)$$

$$2y - 3x = 11,2 \quad (2)$$

Re-writing, gives

$$\begin{aligned} 4x - y &= 12 \quad (1) \\ -3x + 2y &= 11,2 \quad (3) \end{aligned}$$

To eliminate  $y$  :

$$2 \times (1)$$

$$8x - 2y = 24 \quad (4)$$

$$1 \times (3)$$

$$-3x + 2y = 11,2 \quad (3)$$

(4) + (3)

$$\begin{aligned} 5x &= 35,2 \\ x &= \frac{35,2}{5} \\ x &= 7,04 \end{aligned}$$

Substitute for  $x$  in (2)

$$\begin{aligned} 2y - 3(7,04) &= 11,2 \\ 2y &= 11,2 + 21,12 \\ 2y &= 32,32 \\ y &= \frac{32,32}{2} \\ y &= 16,16 \end{aligned}$$

Therefore  $x = 7,04$  and  $y = 16,16$ **5. Progress Exercises 3.1, Question 10, page 110 / page 111**

$$5x - 2y = 15 \quad (1)$$

$$15x - 45 = 6y \quad (2)$$

Re-writing (2) and keeping (1) unchanged

$$5x - 2y = 15 \quad (1)$$

$$15x - 6y = 45 \quad (3)$$

Comparing (1) and (3) shows that

$$3 \times (1) \quad 15x - 6y = 45 \quad (4)$$

which is identical to (3) .

Therefore, there is only one equation and 2 unknowns. We can only express  $y$  in terms of  $x$ .

From (1):

$$\begin{aligned} 5x - 15 &= 2y \\ \frac{5x - 15}{2} &= y \\ \text{or } y &= \frac{5x - 15}{2} \quad (5) \end{aligned}$$

For each  $x$ , there is a corresponding value of  $y$ , and there are infinitely many such combinations.**6. Progress Exercises 3.1, page 110 / page 111**(a) **Question 12**

$$4P - 3Q = 4 \quad (1)$$

$$1,5P + 2Q = 20 \quad (2)$$

To eliminate  $Q$ :

$$2 \times (1)$$

$$8P - 6Q = 8 \quad (3)$$

$$3 \times (2)$$

$$4,5P + 6Q = 60 \quad (4)$$

$$(3) + (4)$$

$$\begin{aligned} 12,5P &= 68 \\ \frac{12,5P}{12,5} &= \frac{68}{12,5} \\ P &= 5,44 \end{aligned}$$

Substitute for  $P$  in (2)

$$\begin{aligned} 1,5(5,44) + 2Q &= 20 \\ 8,16 + 2Q &= 20 \\ 2Q &= 20 - 8,16 \\ 2Q &= 11,84 \\ Q &= 5,92 \quad (\text{dividing both sides by } 2) \end{aligned}$$

Therefore  $P = 5,44$  and  $Q = 5,92$

(b) **Question 13**

$$5 + 2P = 6Q \text{ or } 2P - 6Q = -5 \quad (1)$$

$$5P + 8Q = 25 \text{ or } 5P + 8Q = 25 \quad (2)$$



To eliminate  $P$ :

$$(1) \times (5) \qquad 10P - 30Q = -25 \qquad (3)$$

$$(2) \times (2) \qquad 10P + 16Q = 50 \qquad (4)$$

$$(4) - (3)$$

$$46Q = 75$$

$$Q = 1,63$$

$$Q = 1,63$$

Substitute for  $Q$  in (1)

$$2P - 6(1,63) = -5$$

$$2P = 9,78 - 5$$

$$2P = 4,78$$

$$P = 2,39$$

Therefore  $P = 2,39$  and  $Q = 1,63$

(c) **Question 14**

$$x - y + z = 0 \qquad (1)$$

$$2y - 2z = 2 \qquad (2)$$

$$-x + 2y + 2z = 29 \qquad (3)$$

$$2 \times (1) \qquad 2x - 2y + 2z = 0 \qquad (4)$$

$$1 \times (2) \qquad 2y - 2z = 2 \qquad (2)$$

$$(4) + (2)$$

$$2x = 2$$

$$x = 1$$

From (2)

$$2y = 2 + 2z$$

$$y = \frac{2}{2} + \frac{2z}{2}$$

$$y = 1 + z \qquad (5)$$

Substitute for  $x$  and  $y$  in (3)

$$\begin{aligned} -1 + 2z + 2 + 2z &= 29 \\ 4z &= 29 - 1 \\ 4z &= 28 \\ z &= \frac{28}{4} \\ z &= 7 \end{aligned}$$

Substitute for  $z$  in (5):

$$\begin{aligned} y &= 1 + 7 \\ y &= 8 \end{aligned}$$

Therefore  $x = 1$ ,  $y = 8$  and  $z = 7$ .

(d) **Question 15**

$$P_1 - 3P_2 = 0 \quad (1)$$

$$5P_2 - P_3 = 10 \quad (2)$$

$$P_1 + P_2 + P_3 = 8 \quad (3)$$

$$\text{From (1) } P_1 = 3P_2 \quad (4)$$

$$\text{From (2) } 5P_2 - 10 = P_3 \quad (5)$$

Substitute for  $P_1$  and  $P_3$  in (3)

$$\begin{aligned} 3P_2 + P_2 + 5P_2 - 10 &= 8 \\ 9P_2 &= 8 + 10 \\ 9P_2 &= 18 \\ P_2 &= 2 \end{aligned}$$

Substitute for  $P_2$  in (4) and (5)

$$\begin{aligned} P_1 &= 3 \times 2 \quad \text{and} \quad P_3 = 5 \times 2 - 10 \\ P_1 &= 6 \quad \quad \quad = 10 - 10 \\ P_3 &= 0 \end{aligned}$$

Therefore  $P_1 = 6$ ,  $P_2 = 2$ ,  $P_3 = 0$ .

## 7. Progress Exercises 3.2, Question 5, page 117 / page 118

$$\text{Demand function : } P_d = 50 - 3Q_d \quad (1)$$

$$\text{Supply function : } P_s = 14 + 1,5Q_s \quad (2)$$

(a) At equilibrium (1) = (2) or supply = demand .

Therefore

$$\begin{aligned}
 14 + 1,5Q &= 50 - 3Q \quad (\text{Removing the subscripts } s \text{ and } d) \\
 1,5Q + 3Q &= 50 - 14 \\
 4,5Q &= 36 \\
 Q &= \frac{36}{4,5} \\
 Q &= 8
 \end{aligned}$$

You can substitute for  $Q$  either in (1) or (2). The answer for price,  $P$ , will be the same either way.

Substituting in (1)

$$\begin{aligned}
 P &= 50 - 3 \times 8 \\
 &= 50 - 24 \\
 P &= 26
 \end{aligned}$$

At equilibrium, the price is 26 and the quantity is 8 pairs.

(b) Let  $P = 38$ . From (1)

$$\begin{aligned}
 38 &= 50 - 3Q_d \\
 \text{or } 3Q_d &= 50 - 38 \\
 3Q_d &= 12 \\
 Q_d &= 4
 \end{aligned}$$

From (2)

$$\begin{aligned}
 14 + 1,5Q_s &= 38 \\
 \text{or } 1,5Q_s &= 38 - 14 \\
 1,5Q_s &= 24 \\
 Q_s &= 16
 \end{aligned}$$

$$\begin{aligned}
 \text{Excess supply} &= Q_s - Q_d \\
 &= 16 - 4 \\
 &= 12
 \end{aligned}$$

### 8. Progress Exercises 3.2, Question 7, page 117 / page 118

(a) Let  $P = 20$ . From (1)

$$\begin{aligned}
 20 &= 50 - 3Q_d \\
 3Q_d &= 50 - 20 \\
 3Q_d &= 30 \\
 Q_d &= 10
 \end{aligned}$$

From (2)

$$\begin{aligned} 14 + 1,5Q_s &= 20 \\ 1,5Q_s &= 20 - 14 \\ 1,5Q_s &= 6 \\ Q_s &= 4 \end{aligned}$$

$$\begin{aligned} \text{Demand Excess} &= Q_d - Q_s \\ &= 10 - 4 \\ &= 6 \end{aligned}$$

(b) At a price of £20,  $Q_s = 4$ .

If  $Q = 4$ , from the demand function (1), the black-market price is

$$\begin{aligned} P &= 50 - 3 \times 4 \\ &= 50 - 12 \\ P &= 38 \end{aligned}$$

$$\begin{aligned} \text{Profit} &= \text{Income} - \text{Cost} \\ &= \text{price} \times \text{quantity} - \text{cost} \times \text{quantity} \\ &= 38 \times 4 - 20 \times 4 \\ &= 152 - 80 \\ &= 72 \end{aligned}$$

### 9. Progress Exercises 3.2, Question 7, page 117/page 118

$$\text{Labour demand function : } W_d = 70 - 4L \quad (1)$$

$$\text{Labour supply function : } W_s = 10 + 2L \quad (2)$$

(a) At equilibrium, (1) = (2)

$$\begin{aligned} 10 + 2L &= 70 - 4L \\ 4L + 2L &= 70 - 10 \\ 6L &= 60 \\ L &= 10 \end{aligned}$$

Substitute for  $L$  in (2)

$$\begin{aligned} W &= 10 + 2 \times 10 \\ &= 10 + 20 \\ W &= 30 \end{aligned}$$

(b) If  $W = 20$ , then from (1)

$$\begin{aligned} 20 &= 70 - 4L_d \\ 4L_d &= 70 - 20 \\ 4L_d &= 50 \\ L_d &= 12,5 \end{aligned}$$

and from (2)

$$\begin{aligned} 10 + 2L_s &= 20 \\ 2L_s &= 20 - 10 \\ 2L_s &= 10 \\ L_s &= 5 \\ L_d - L_s &= 12,5 - 5 \\ &= 7,5 \end{aligned}$$

(c) If  $W = 40$ , then from (1)

$$\begin{aligned} 40 &= 70 - 4L_d \\ 4L_d &= 70 - 40 \\ 4L_d &= 30 \\ L_d &= 7,5 \end{aligned}$$

and from (2)

$$\begin{aligned} 10 + 2L_s &= 40 \\ 2L_s &= 40 - 10 \\ 2L_s &= 30 \\ L_s &= 15 \\ L_s - L_d &= 15 - 7,5 \\ &= 7,5 \end{aligned}$$

#### 10. Progress Exercises 3.3, Question 1, page 125 / page 126

(a)

$$\text{Demand function : } Q = 81 - 0,05P \quad (1)$$

$$\text{Supply function : } Q = -24 + 0,025P \quad (2)$$

At equilibrium:

$$\begin{aligned} -24 + 0,025P &= 81 - 0,05P \\ 0,05P + 0,025P &= 81 + 24 \\ 0,075P &= 105 \\ P &= 1400 \end{aligned}$$

$$\begin{aligned}
 \text{Corresponding equilibrium quantity} &= 81 - 0,05(1\,400) \\
 &= 81 - 70 \\
 &= 11
 \end{aligned}$$

(b) Re-arranging the equations (1) and (2): From (1)

$$\begin{aligned}
 Q &= 81 - 0,05P \\
 0,05P &= 81 - Q \\
 P &= 1\,620 - 20Q
 \end{aligned}$$

From (2)

$$\begin{aligned}
 Q &= -24 + 0,025P \\
 0,025P &= Q + 24 \\
 P &= 40Q + 960
 \end{aligned}$$

Therefore

$$\text{Demand function : } P = 1\,620 - 20Q \quad (3)$$

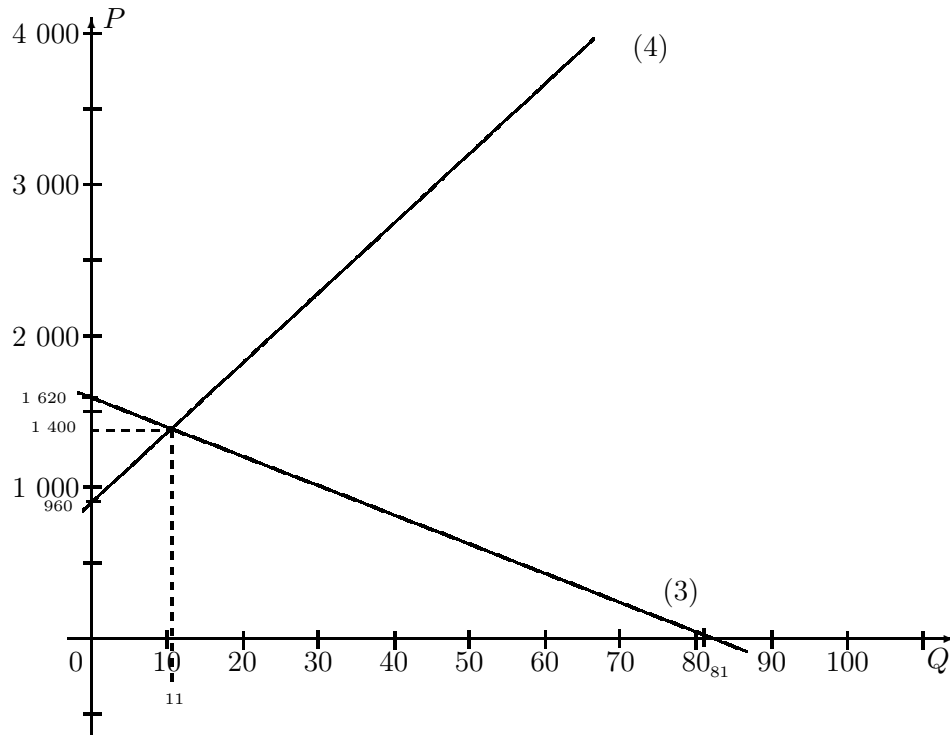
$$\text{Supply function : } P = 40Q + 960 \quad (4)$$

Consider (3)

$$\begin{aligned}
 \text{If } Q = 0, P &= 1\,620 \\
 \text{If } P = 0, 0 &= 1\,620 - 20Q \\
 20Q &= 1\,620 \\
 Q &= 81
 \end{aligned}$$

Consider (4)

$$\begin{aligned}
 \text{If } Q = 0, P &= 960 \\
 \text{If } Q = 81, P &= 40 \times 81 + 960 \\
 &= 3\,240 + 960 \\
 P &= 4\,200
 \end{aligned}$$



11. Progress Exercises 3.3, Question 7, page 126 / Question 7, page 127

$$\text{Demand function : } P = 200 - 5Q \quad (3)$$

$$\text{Supply function : } P = 92 + 4Q \quad (4)$$

(a) At equilibrium

$$\begin{aligned} \text{Demand} &= \text{Supply} \\ 200 - 5Q &= 92 + 4Q \\ 200 - 92 &= 4Q + 5Q \\ 108 &= 9Q \\ 12 &= Q \\ \text{or } Q &= 12 \end{aligned}$$

$$\begin{aligned} \text{Equilibrium price: } P &= 200 - 5 \times 12 \\ &= 200 - 60 \\ P &= 140 \end{aligned}$$

(b) (i) The new supply function is

$$\begin{aligned} P_s - 9 &= 92 + 4Q \\ P_s &= 92 + 9 + 4Q \\ P_s &= 101 + 4Q \end{aligned}$$

(ii) At new equilibrium

$$\begin{aligned} 101 + 4Q &= 200 - 5Q \\ 4Q + 5Q &= 200 - 101 \\ 9Q &= 99 \\ Q &= 11 \end{aligned}$$

$$\begin{aligned} \text{The corresponding price } P &= 101 + 4 \times 11 \\ &= 101 + 44 \\ P &= 145 \end{aligned}$$

(iii) The consumer always pays the equilibrium price.

$$\begin{aligned} \text{Therefore, tax paid by customer} &= 145 - 140 = 5, \\ \text{Tax paid by the club} &= 9 - 5 = 4. \end{aligned}$$

## 12. Progress Exercises 3.3, Question 9, page 127 / Question 9 page 127

$$\text{Demand function : } P_d = 80 - 0,4Q_d \quad (1)$$

$$\text{Supply function : } P_s = 20 + 0,4Q_s \quad (2)$$

(a) At equilibrium

$$\begin{aligned} \text{supply} &= \text{demand} \\ 20 + 0,4Q &= 80 - 0,4Q \\ 0,4Q + 0,4Q &= 80 - 20 \\ 0,8Q &= 60 \\ Q &= 75 \end{aligned}$$

The equilibrium price is

$$\begin{aligned} P &= 20 + 0,4 \times 75 \\ &= 20 + 30 \\ P &= 50 \end{aligned}$$

(b) (i) With subsidy, the equation of the supply function is:

$$\begin{aligned} P_s + 4 &= 20 + 0,4Q_s \\ P_s &= 20 - 4 + 0,4Q_s \\ P_s &= 16 + 0,4Q_s \end{aligned}$$

(ii)

$$\begin{aligned} \text{At equilibrium } P_d &= P_s \\ 16 + 0,4Q &= 80 - 0,4Q \text{ (removing subscripts } d \text{ and } s) \\ 0,4Q + 0,4Q &= 80 - 16 \\ 0,8Q &= 64 \\ Q &= 80 \text{ (dividing by } 0,8 \text{ both sides)} \end{aligned}$$



(iii) The consumer always pays the equilibrium price, which is

$$\begin{aligned} P &= 80 - 0,4 \times 80 \\ &= 80 - 32 \\ P &= 48. \end{aligned}$$

In this case both consumer and producer receive a subsidy of  $50 - 48 = 2$ .

13. **Progress Exercises 3.3, Question 10, page 127 / Question 10, page 127**

$$TC = 800 + 0,2Q$$

(a) Total revenue is given by

$$\begin{aligned} TR &= \text{price} \times \text{quantity} \\ &= 6,6 \times Q \\ TR &= 6,6Q \end{aligned}$$

$$\text{To break-even } TR = TC$$

$$6,6Q = 800 + 0,2Q$$

$$6,6Q - 0,2Q = 800$$

$$6,4Q = 800$$

$$Q = 125 \text{ (dividing both sides by } 6,4)$$

Therefore, 125 clocks should be sold to break-even.

(b) If the charge per clock is  $P$ , then  $TR = PQ$ .

$$\text{To break-even } TR = TC$$

$$PQ = 800 + 0,2Q$$

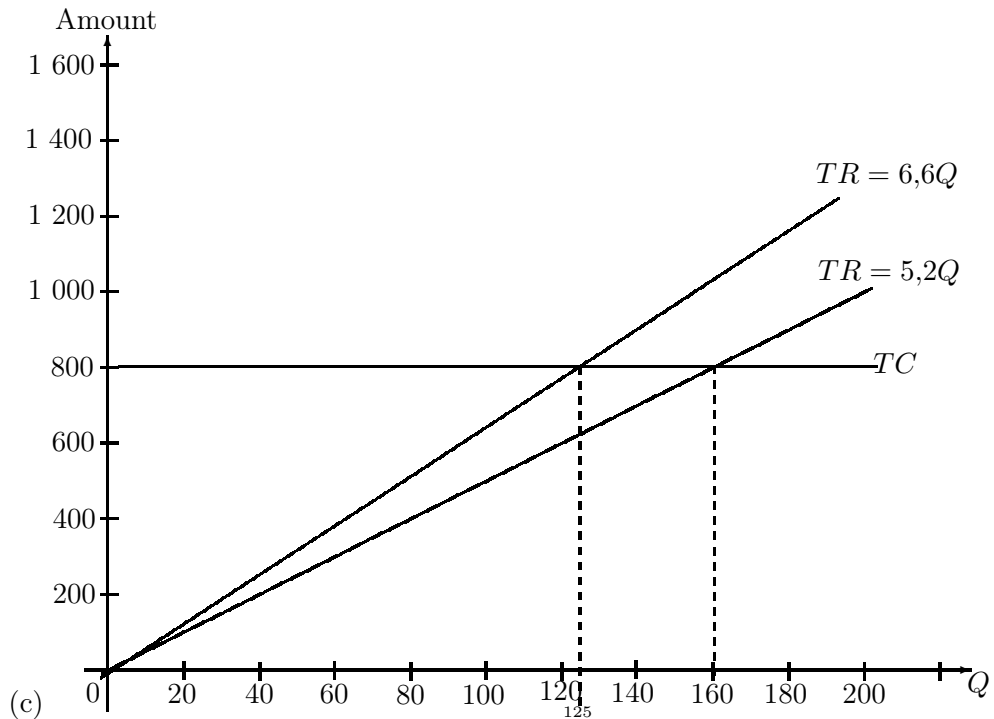
If  $Q = 160$  then

$$160P = 800 + 0,2 \times 160$$

$$160P = 832$$

$$P = 5,20 \text{ (dividing by } 160 \text{ both sides)}$$

Therefore the new equation (on total revenue is  $TR = 5,2Q$ ).



14. Progress Exercises 3.4, Question 2, page 131 / page 131

$$\text{Demand function : } P = 58 - 0,2Q$$

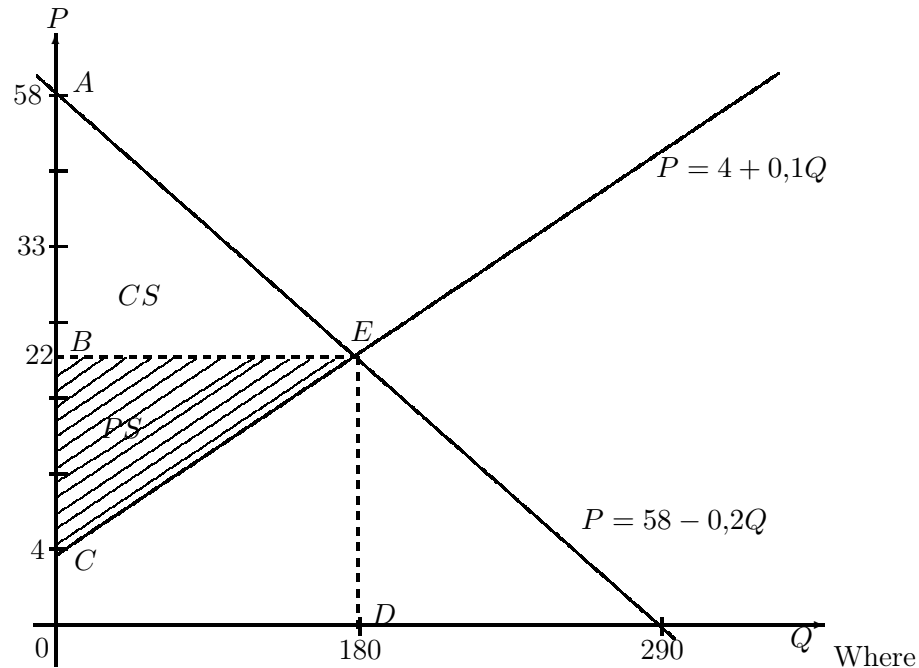
$$\text{Supply function : } P = 4 + 0,1Q$$

(a) At equilibrium

$$\begin{aligned} 4 + 0,1Q &= 58 - 0,2Q \\ 0,2Q + 0,1Q &= 58 - 4 \\ 0,3Q &= 54 \\ Q &= 180 \text{ (dividing by } 0,3 \text{ on both sides)} \end{aligned}$$

Equilibrium price is

$$\begin{aligned} P &= 58 - 0,2 \times 180 \\ &= 58 - 36 \\ P &= 22 \end{aligned}$$



CS = consumer surplus

PS = producer surplus

- (b) (i) At equilibrium, consumers pay  $180 \times 22 = 3\,960$   
(ii) Consumers are willing to pay for bus journeys up to equilibrium the amount equivalent to the area of the trapezium AODE.

$$\begin{aligned} \text{Amount} &= \frac{1}{2}(\text{sum of lengths}) \times \text{width} \\ &= 0,5(58 + 22) \times 180 \\ &= 7\,200 \end{aligned}$$

(iii)

$$\begin{aligned} \text{Consumer Surplus } CS &= \text{Area of triangle } ABE \\ &= \frac{1}{2}(58 - 22) \times 180 \\ &= 0,5 \times 36 \times 180 \\ &= 3\,240 \end{aligned}$$

Note:

$$7\,200 - 3\,960 = 3\,240$$

Therefore,  $CS = \text{(ii)} - \text{(i)}$ .

- (c) (i) At equilibrium, producer receives  $22 \times 180 = 3\,960$   
(ii) Amount the producer is willing to accept for bus journeys up to equilibrium is given by the area under the supply function, which is the trapezium CODE.

$$\begin{aligned} \text{Amount} &= 0,5(4 + 22) \times 180 \\ &= 0,5 \times 26 \times 180 \\ &= 2\,340 \end{aligned}$$

(iii) The producer surplus is given by

$$\begin{aligned}
 PS &= \text{area of triangle BCE} \\
 &= 0,5(22 - 4) \times 180 \\
 &= 0,5 \times 18 \times 180 \\
 PS &= 1620
 \end{aligned}$$

Note:

$$3960 - 2340 = 1620$$

Therefore,  $PS = (i) - (ii)$ .

15. Progress Exercises 3.4, Question 3, page 131 / page 132

$$\text{Demand function : } Q = 50 - 0,1P \quad (1)$$

$$\text{Supply function : } Q = -10 + 0,1P \quad (2)$$

(a) At equilibrium

$$-10 + 0,1P = 50 - 0,1P$$

$$0,1P + 0,1P = 50 + 10$$

$$0,2P = 60$$

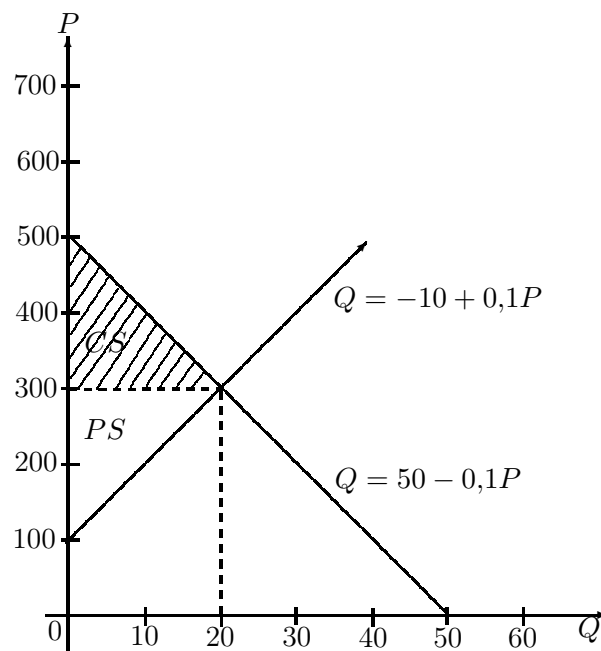
$$P = 300 \quad (\text{Dividing by } 0,2 \text{ both sides})$$

Equilibrium quantity is given by

$$Q = 50 - 0,1 \times 300$$

$$Q = 50 - 30$$

$$Q = 20$$



(b)

$$\text{Consumer surplus} = \left( \begin{array}{c} \text{What the consumer} \\ \text{is willing to pay} \\ \text{up to equilibrium point} \end{array} \right) - \left( \begin{array}{c} \text{What the consumer} \\ \text{pays at} \\ \text{equilibrium} \end{array} \right)$$

$$\begin{aligned} CS &= [0,5(500 + 300) \times 20] - 300 \times 20 \\ &= 8\,000 - 6\,000 \\ CS &= 2\,000 \end{aligned}$$

(c)

$$\text{Producer surplus} = \left( \begin{array}{c} \text{What the producer} \\ \text{receives at} \\ \text{equilibrium} \end{array} \right) - \left( \begin{array}{c} \text{What the producer is} \\ \text{willing to accept up to} \\ \text{equilibrium point} \end{array} \right)$$

$$\begin{aligned} PS &= [300 \times 20] - [0,5(100 + 300) \times 20] \\ &= 6\,000 - 4\,000 \\ &= 2\,000 \end{aligned}$$

(d)

$$\text{Total surplus} = \left( \begin{array}{c} \text{Consumer} \\ \text{surplus} \end{array} \right) + \left( \begin{array}{c} \text{Producer} \\ \text{surplus} \end{array} \right)$$

$$\begin{aligned} TS &= 2\,000 + 2\,000 \\ TS &= 4\,000 \end{aligned}$$

## 16. Progress Exercises 9.1, page 485 / page 487

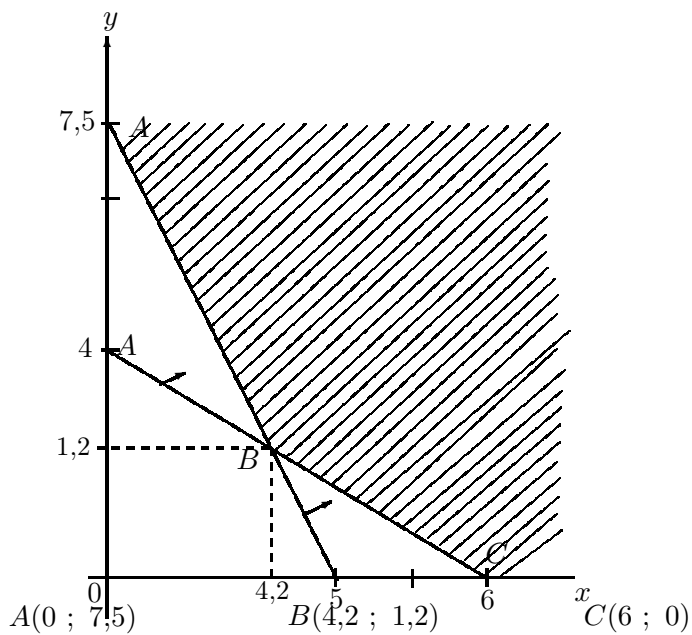
(a) Question 1

(i)

$$\begin{aligned} 3x + 2y &\geq 15 & (1) \\ 6x + 9y &\geq 36 & (2) \\ x \geq 0 & \quad y \geq 0 \end{aligned}$$

Consider (1): If  $x = 0, y = 7,5$  and if  $y = 0, x = 5$ Consider (2): If  $x = 0, y = 4$  and if  $y = 0, x = 6$ 

Since (=) is included for both inequalities, the lines are solid.



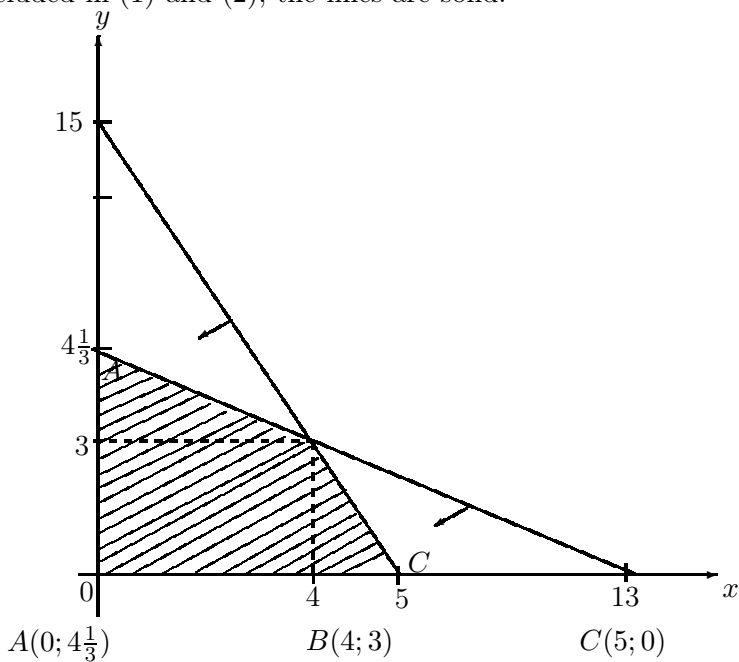
(ii)

$$\begin{aligned} 6x + 2y &\leq 30 & (1) \\ 2x + 6y &\leq 26 & (2) \\ x \geq 0 & \quad y \geq 0 \end{aligned}$$

Consider (1):  $6x + 2y = 30$   
 If  $x = 0$ ,  $y = 15$  and if  $y = 0$ ,  $x = 5$

Consider (2) :  $2x + 6y = 26$   
 If  $x = 0$ ,  $y = 4\frac{1}{3}$  and if  $y = 0$ ,  $x = 13$

Since (=) is included in (1) and (2), the lines are solid.

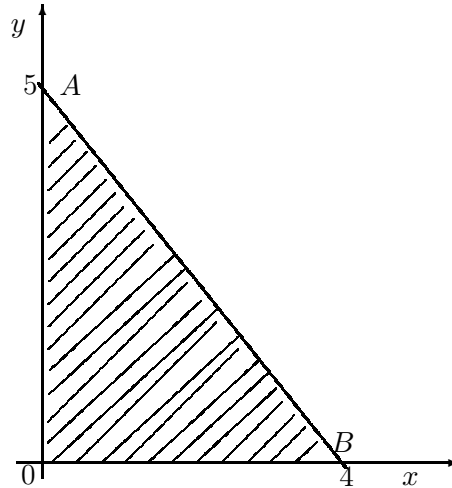


**(b) Question 3**

(i)

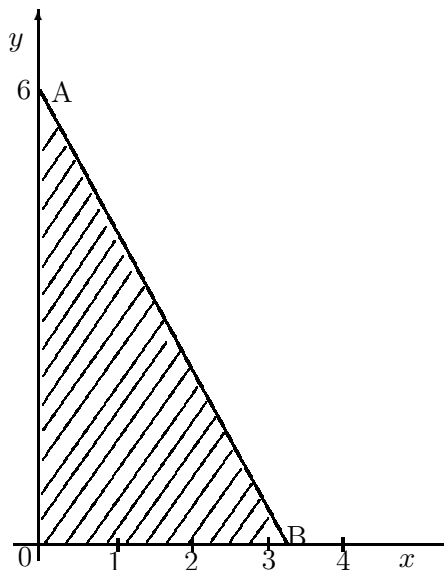
$$\begin{aligned} 4y + 5x &\leq 20 & (1) \\ x &\geq 0, \quad y &\geq 0 \end{aligned}$$

Consider (1)

If  $x = 0, y = 5$ If  $y = 0, x = 4$ (= $\geq$ ) is included, line is solid $A(0; 5), B(4; 0),$  origin

(ii)

$$\begin{aligned} 8y + 15x &\leq 48 & (2) \\ x &= 0, \quad y &\geq 0 \end{aligned}$$

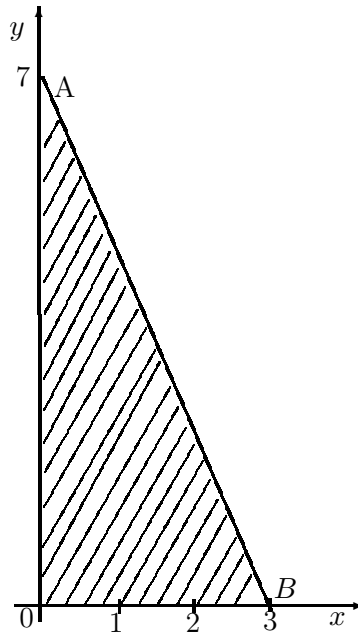
Consider (2): If  $x = 0, y = 6$ If  $y = 0, x = 3\frac{1}{5}$ (= $\leq$ ) is included in the inequality, hence line is solid.

$A(0; 6)$ ,  $B(3\frac{1}{5}; 0)$ , origin  
(iii)

$$\begin{aligned} 3y + 7x &\leq 21 & (3) \\ x &\geq 0, \quad y &\geq 0 \end{aligned}$$

Consider (3): If  $y = 0, x = 3$  and if  $x = 0, y = 7$ .

(=) is included in the inequality, hence line is solid.



17. Progress Exercises 9.1, page 485 / page 487

(a) Question 6

$$\begin{aligned} W &= 3x + 2y \\ &\text{subject to} \end{aligned}$$

$$4y + 5x \leq 20 \quad (1)$$

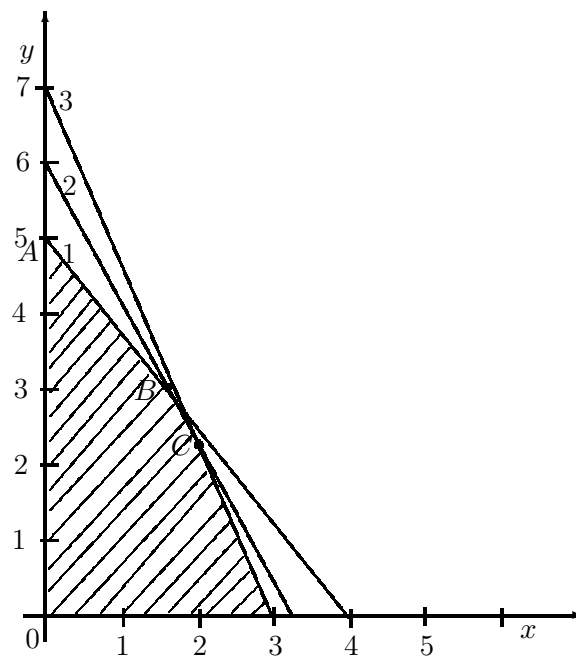
$$8y + 15x \leq 48 \quad (2)$$

$$3y + 7x \leq 21 \quad (3)$$

$$x \geq 0, \quad y \geq 0$$

The lines have been determined in the previous question. They are drawn on the same graph.





| Point                   | Value of $W$  |
|-------------------------|---|
| A: $x = 0; y = 5$       | $W = 3 \times 0 + 5 \times 2 = 10$                        |
| B: $x = 1,6; y = 3$     | $W = 3 \times 1,6 + 2 \times 3 = 10,8 \leftarrow$ maximum |
| C: $x = 2,18; y = 1,91$ | $W = 3 \times 2,18 + 2 \times 1,91 = 10,36$               |
| D: $x = 3; y = 0$       | $W = 3 \times 3 + 2 \times 0 = 9$                         |
| origin : $x = 0, y = 0$ | $W = 3 \times 0 + 2 \times 0 = 0$                         |

Maximum  $W$  is at  $B$  where  $x = 1,6$  and  $y = 3$ .

(b) **Question 7**

$$C = 3x + 2y$$

subject to

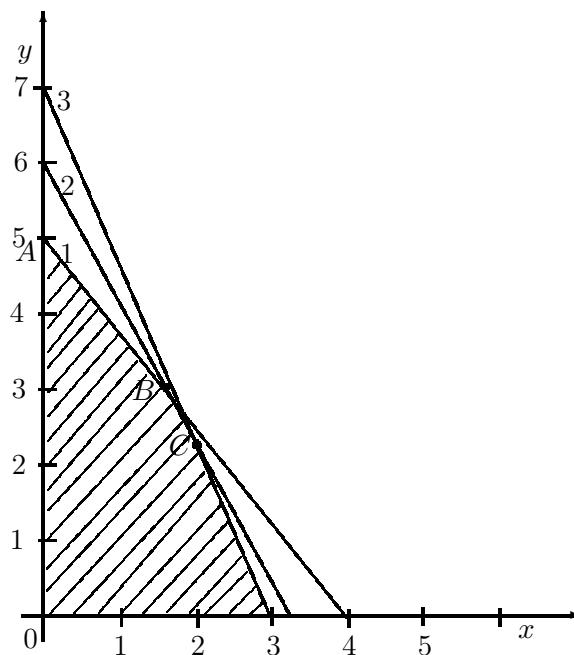
$$4y + 5x \leq 20 \quad (1)$$

$$8y + 15x \leq 48 \quad (2)$$

$$3y + 7x \leq 21 \quad (3)$$

$$x \geq 0 \quad y \geq 0$$

The lines have been drawn in the previous question. They are drawn here on the same diagram.



| Point                   | Value of $C$   |
|-------------------------|--|
| A: $x = 0; y = 5$       | $C = 3 \times 0 + 5 \times 2 = 10$                   |
| B: $x = 1,6; y = 3$     | $C = 3 \times 1,6 + 2 \times 3 = 10,8$               |
| C: $x = 2,18; y = 1,91$ | $C = 3 \times 2,18 + 2 \times 1,91 = 10,36$          |
| D: $x = 3; y = 0$       | $C = 3 \times 3 + 2 \times 0 = 9$                    |
| origin : $x = 0, y = 0$ | $C = 3 \times 0 + 2 \times 0 = 0 \leftarrow$ minimum |

Minimum  $C$  is at the origin.

18. **Progress Exercises 9.1, Question 10, page 485 / page 487**

Let

$x$  = number of Machine A used

$y$  = number of Machine B used

- (a) • Consider cost per day:

$$6x + 3y \leq 360 \quad (1)$$

- Available operators:

$$2x + 4y \leq 280 \quad (2)$$

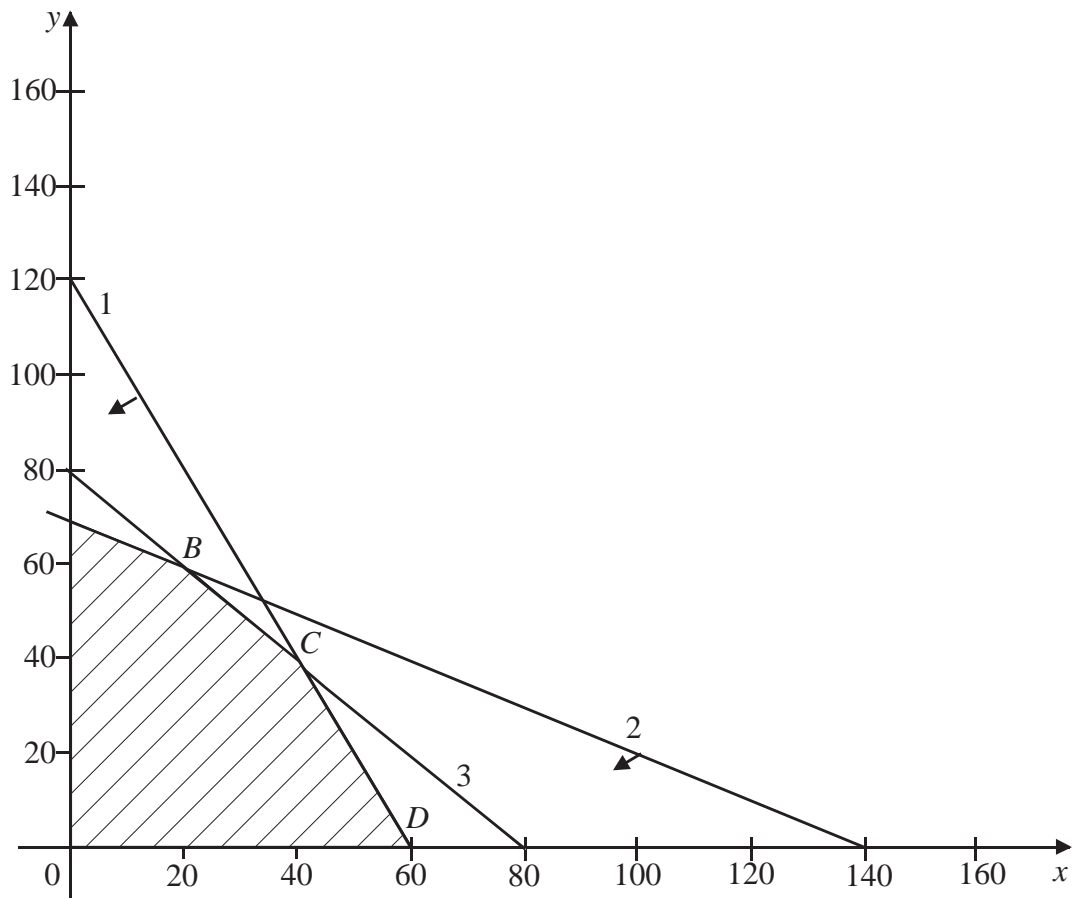
- Floor area ( $\text{m}^2$ ):

$$2x + 2y \leq 160 \quad (3)$$

- Profit:

$$P = 20x + 30y$$

(b)



Maximum profit occurs at B. Therefore 20 Machine A and 60 Machine B should be used to maximise profit.

## 15.4 Self-Evaluation Exercise 4: Unit 4

## 1. Progress Exercises 4.1, page 152 / page 152

## (i) Question 1

$$\begin{aligned}
 x^2 - 6x + 5 &= 0 \\
 x^2 - 5x - x + 5 &= 0 \\
 x(x - 5) - 1(x - 5) &= 0 \\
 (x - 5)(x - 1) &= 0
 \end{aligned}$$

Either  $x - 5 = 0$  or  $x - 1 = 0$   
 $x = 5$  or  $x = 1$

## (ii) Question 2

$$\begin{aligned}
 2Q^2 - 7Q + 5 &= 0 \\
 a = 2, b = -7; c = 5
 \end{aligned}$$

From the quadratic formula

$$\begin{aligned}
 Q &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(5)}}{2 \times 2} \\
 &= \frac{7 \pm \sqrt{49 - 40}}{4} \\
 &= \frac{7 \pm \sqrt{9}}{4} \\
 &= \frac{7 \pm 3}{4} \\
 &= \frac{10}{4} \quad \text{or} \quad \frac{4}{4} \\
 Q &= 2,5 \quad \text{or} \quad 1
 \end{aligned}$$

## (iii) Question 3

$$\begin{aligned}
 -Q^2 + 6Q - 5 &= 0 \\
 a &= -1, b = 6, c = -5 \\
 Q &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-6 \pm \sqrt{(6)^2 - 4(-1)(-5)}}{2 \times -1} \\
 &= \frac{-6 \pm \sqrt{36 - 20}}{-2} \\
 &= \frac{-6 \pm \sqrt{16}}{-2} \\
 &= \frac{-6 \pm 4}{-2} \\
 &= \frac{-2}{-2} \quad \text{or} \quad \frac{-10}{-2} \\
 &= 1 \quad \text{or} \quad 5
 \end{aligned}$$

(iv) **Question 4**

$$\begin{aligned}
 Q^2 + 6Q + 5 &= 0 \\
 Q^2 + 5Q + Q + 5 &= 0 \\
 Q(Q + 5) + 1(Q + 5) &= 0 \\
 (Q + 5)(Q + 1) &= 0
 \end{aligned}$$

Either  $Q + 5 = 0$  or  $Q + 1 = 0$   
 $Q = -5$  or  $Q = -1$ .

(v) **Question 5**

$$\begin{aligned}
 P^2 - 7 &= 0 \\
 P^2 &= 7 \\
 P &= \pm\sqrt{7}
 \end{aligned}$$

Therefore  $P = \sqrt{7} = 2,65$  or  $P = -\sqrt{7} = -2,65$

(vi) **Question 6**

$$\begin{aligned}
 Q^2 - 6Q + 9 &= 0 \\
 Q^2 - 3Q - 3Q + 9 &= 0 \\
 Q(Q - 3) - 3(Q - 3) &= 0 \\
 (Q - 3)(Q - 3) &= 0
 \end{aligned}$$

$Q - 3 = 0$  or  $Q - 3 = 0$   
 $Q = 3$

(vii) **Question 7**

$$\begin{aligned}
 Q^2 - 6Q - 9 &= 0 \\
 a = 1, b = -6, c = -9 \\
 Q &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-9)}}{2 \times 1} \\
 &= \frac{6 \pm \sqrt{36 + 36}}{2} \\
 &= \frac{6 \pm \sqrt{72}}{2} \\
 &= \frac{6 \pm 8,49}{2} \\
 &= 7,245 \text{ or } -1,245
 \end{aligned}$$

(viii) **Question 8**

$$\begin{aligned}
 Q^2 &= 6Q \\
 Q^2 - 6Q &= 0 \\
 Q(Q - 6) &= 0
 \end{aligned}$$

Either  $Q = 0$  or  $Q - 6 = 0$  implying that  $Q = 6$ .

(ix) **Question 9**

$$\begin{aligned}
 x^2 - 6x &= 7 + 3x \\
 x^2 - 9x - 7 &= 0 \\
 a &= 1; b = -9; c = -7 \\
 x &= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-7)}}{2(1)} \\
 &= \frac{9 \pm \sqrt{81 + 28}}{2} \\
 &= \frac{9 \pm \sqrt{109}}{2} \\
 &= \frac{9 \pm 10,44}{2} \\
 &= 9,72 \quad \text{or} \quad -0,72
 \end{aligned}$$

(x) **Question 10**

$$\begin{aligned}
 P^2 + 12 &= 3 \\
 P^2 &= 3 - 12 \\
 P^2 &= -9
 \end{aligned}$$

Therefore  $P = \sqrt{-9}$  or  $P = -\sqrt{-9}$ .

These are complex numbers beyond the scope of this module.

(xi) **Question 11**

$$\begin{aligned}
 P + 10 &= 11P^2 - P + 1 \\
 0 &= 11P^2 - P + 1 - P - 10 \\
 0 &= 11P^2 - 2P - 9 \\
 P &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(11)(-9)}}{2 \times 11} \\
 &= \frac{2 \pm \sqrt{400}}{22} \\
 &= \frac{2 \pm 20}{22} \\
 &= \frac{22}{22} \quad \text{or} \quad -\frac{18}{22} \\
 &= 1 \quad \text{or} \quad -\frac{9}{11}
 \end{aligned}$$

(xii) **Question 12**

$$\begin{aligned}
 Q^2 - 8Q &= Q^2 - 2 \\
 Q^2 - 8Q - Q^2 &= -2 \\
 -8Q &= -2 \\
 Q &= \frac{1}{4}
 \end{aligned}$$

(xiii) **Question 13**

$$12 = P^2 - 2P + 12$$

$$0 = P^2 - 2P + 12 - 12$$

$$0 = P^2 - 2P$$

$$0 = P(P - 2)$$

$$\text{Either } P = 0 \text{ or } P - 2 = 0$$

$$P = 0 \text{ or } P = 2$$

(xiv) **Question 14**

$$5 + P = 4P^2 - 4 + P$$

$$0 = 4P^2 - 4 + P - 5 - P$$

$$0 = 4P^2 - 9$$

$$9 = 4P^2$$

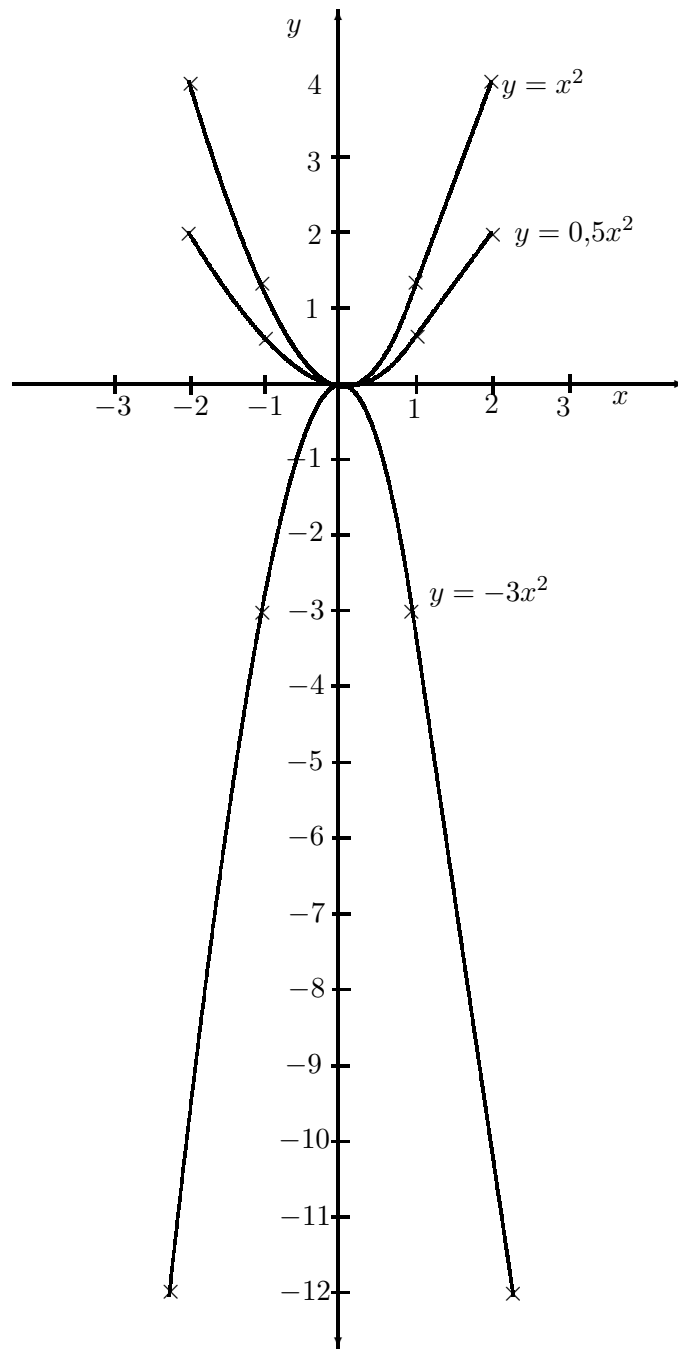
$$\frac{9}{4} = P^2$$

$$\text{Either } P = \sqrt{\frac{9}{4}} \text{ or } P = -\sqrt{\frac{9}{4}}$$

$$P = \frac{3}{2} \text{ or } P = -\frac{3}{2}$$

2. **Progress Exercises 4.2, Question 1, page 158 / page 158**

|              |     |     |   |     |     |
|--------------|-----|-----|---|-----|-----|
| $x$          | -2  | -1  | 0 | 1   | 2   |
| $y = x^2$    | 4   | 1   | 0 | 1   | 4   |
| $y = -3x^2$  | -12 | -3  | 0 | -3  | -12 |
| $y = 0,5x^2$ | 2   | 0,5 | 0 | 0,5 | 2   |



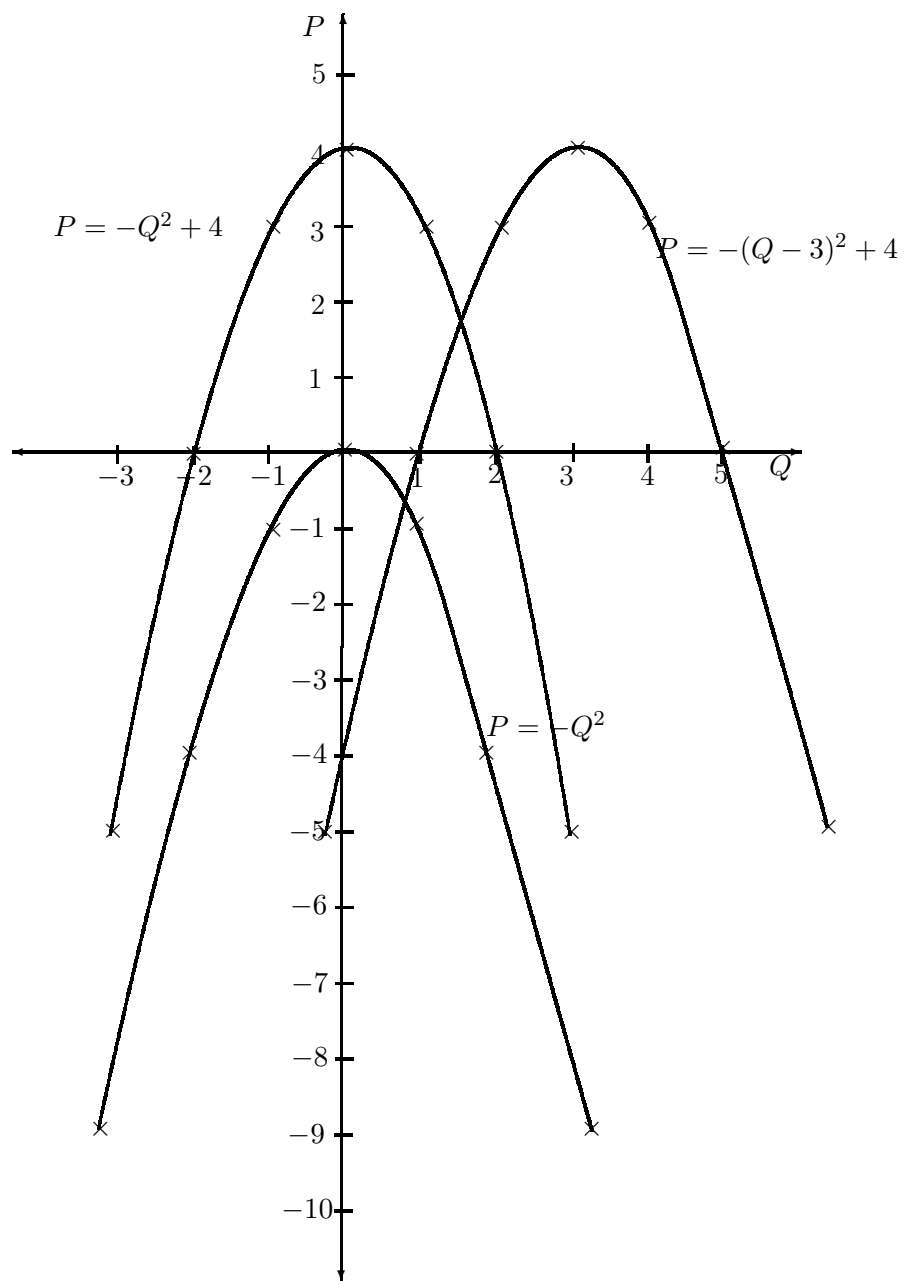


3. Progress Exercises 4.2, Question 6, page 158 / page 158

- (a)  $P = -Q^2$       (b)  $P = -Q^2 + 4$       (c)  $P = -(Q - 3)^2 + 4$

| $Q$     | -3  | -2  | -1  | 0  | 1  | 2  | 3  | 4   | 5   | 6   |
|---------|-----|-----|-----|----|----|----|----|-----|-----|-----|
| (a) $P$ | -9  | -4  | -1  | 0  | -1 | -4 | -9 | -16 | -25 | -36 |
| (b) $P$ | -5  | 0   | 3   | 4  | 3  | 0  | -5 | -12 | -21 | -32 |
| (c) $P$ | -32 | -21 | -12 | -5 | 0  | 3  | 4  | 3   | 0   | -5  |

(ii)



- (iii) • Graph (b) is a shift of graph (a) 4 steps up.  
 • Graph (c) is a shift of graph (b) 3 steps to the right.

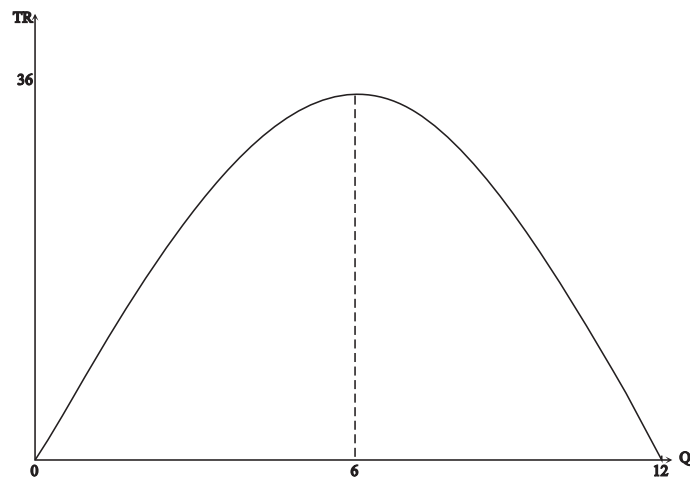
4. Progress Exercises 4.3, Question 2, page 163 / page 163

$$P = 12 - Q$$

(a)

$$\begin{aligned} \text{Total revenue} &= \text{price} \times \text{quantity} \\ TR &= PQ \\ &= (12 - Q)Q \text{ (substituting for } P) \\ TR &= 12Q - Q^2 \end{aligned}$$

(b)



(c)

$$\begin{aligned} 12Q - Q^2 &= 0 \\ Q(12 - Q) &= 0 \text{ (factoring } Q \text{ out)} \\ \text{Either } Q &= 0 \text{ or } 12 - Q = 0 \\ &Q = 0 \text{ or } 12 = Q. \\ &Q = 0 \text{ or } Q = 12. \end{aligned}$$

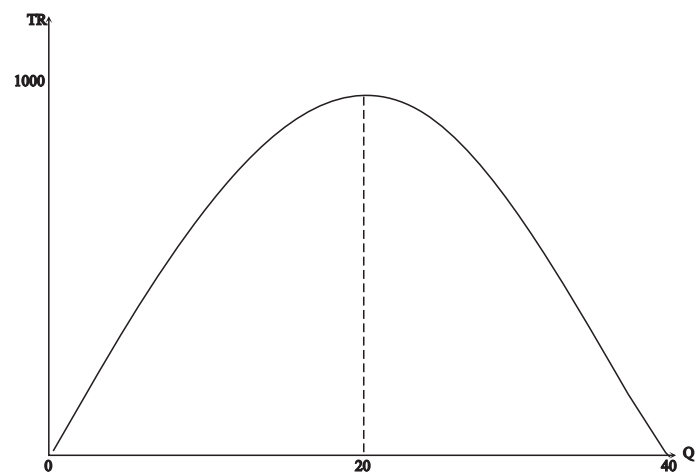
5. Progress Exercises 4.3, Question 3, page 164 / page 164

Let

$$\begin{aligned} TR &= Q(a + bQ) \\ &= aQ + bQ^2 \end{aligned}$$

$$\text{If } Q = 40 \text{ then } 40a + 1600b = 0 \quad (1)$$

$$\text{If } Q = 20 \text{ then } 20a + 400b = 1000 \quad (2)$$



$$\begin{aligned}
 2 \times (2) - (1) : 800b - 1600b &= 2000 \\
 -800b &= 2000 \\
 b &= -2,5
 \end{aligned}$$

Substitute for  $b$  in (1)

$$\begin{aligned}
 40a - 4000 &= 0 \\
 40a &= 4000 \\
 a &= 100
 \end{aligned}$$

Therefore

$$TR = 100Q - 2,5Q^2.$$

**6. Progress Exercises 4.3, Question 4, page 164 / page 164**

Demand function:  $P = 100 - 2Q$

(a)

$$\begin{aligned}
 \text{Total Revenue} &= \text{price} \times \text{quantity} \\
 TR &= PQ \\
 &= (100 - 2Q)Q \\
 TR &= 100Q - 2Q^2
 \end{aligned}$$

$$\begin{aligned}
 \text{If } Q = 10, TR &= 100 \times 10 - 2(10)^2 \\
 &= 1000 - 200 \\
 &= 800
 \end{aligned}$$

(b)

$$\begin{aligned}
 \text{If } P = 100 - 2Q, \text{ then } 2Q &= 100 - P \\
 Q &= 50 - 0,5P
 \end{aligned}$$

Total Revenue = price  $\times$  quantity

$$\begin{aligned} TR &= P \times Q \\ &= P(50 - 0,5P) \\ TR &= 50P - 0,5P^2 \end{aligned}$$

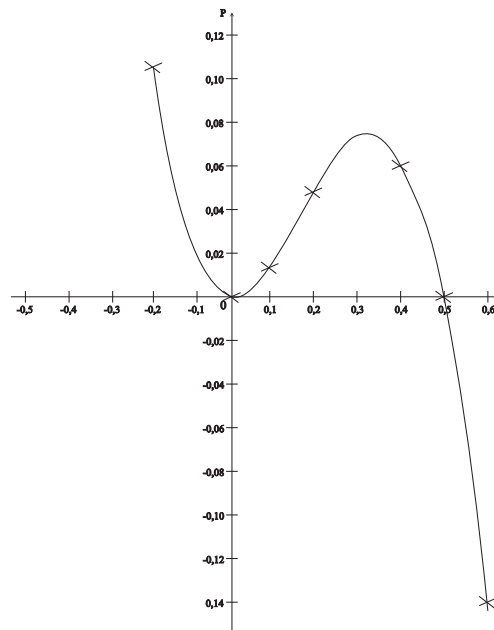
If  $P = 10$ , then

$$\begin{aligned} TR &= 50 \times 10 - 0,5 \times 10^2 \\ &= 500 - 50 \\ TR &= 450 \end{aligned}$$

7. Progress Exercises 4.4, Question 3, page 170 / page 170

$$P = -4Q^3 + 2Q^2$$

| Q    | P  |
|------|--|
| -0,4 | $P = -4 \times (-0,4)^3 + 2 \times (-0,4)^2 = 0,576$ |
| -0,2 | $P = -4 \times (-0,2)^3 + 2 \times (-0,2)^2 = 0,112$ |
| 0    | $P = -4 \times (0)^3 + 2 \times (0)^2 = 0$           |
| 0,2  | $P = -4 \times (0,2)^3 + 2 \times (0,2)^2 = 0,048$   |
| 0,4  | $P = -4 \times (0,4)^3 + 2 \times (0,4)^2 = 0,064$   |
| 0,6  | $P = -4 \times (0,6)^3 + 2 \times (0,6)^2 = -0,144$  |



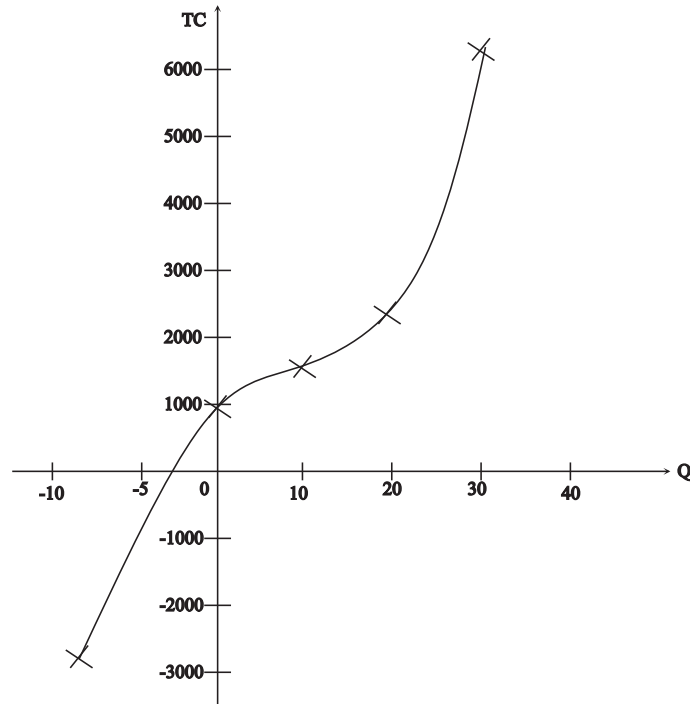
**Roots** :  $Q = 0$  and  $Q = 0,5$

**Turning Points** Maximum  $P$  :  $Q \approx 0,3$  and  $P \approx 0,07$   
 Minimum  $P$  :  $Q = 0$  and  $P = 0$

## 8. Progress Exercises 4.4, Question 4, page 170 / page 170

$$TC = 0,5Q^3 - 15Q^2 + 175Q + 1000$$

|    |       |      |      |      |      |
|----|-------|------|------|------|------|
| Q  | -10   | 0    | 10   | 20   | 30   |
| TC | -2750 | 1000 | 1750 | 2500 | 6250 |



**Roots :**  $Q \approx -4$ .

**Turning Points :** No maximum and minimum turning points.

## 9. Progress Exercises 4.5, Question 1, page 176 / page 177

(a)  $6^2 = 6 \times 6 = 36$

(b)  $3^3 = 3 \times 3 \times 3 = 27$

(c)  $5^1 = 5$

(d)  $5^3 = 5 \times 5 \times 5 = 125$

(e)  $(-3)^2 = -3 \times -3 = 9$

(f)  $(-4)^2 = -4 \times -4 = 16$

(g)  $25^0 = 1$

(h)  $5^{-1} = \frac{1}{5^1} = \frac{1}{5}$

(i)  $6^{-2} = \frac{1}{6^2} = \frac{1}{6 \times 6} = \frac{1}{36}$

(j)  $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$

(k)  $(2,5)^{0,5} = 1,58113883 \approx 1,58$  to 2 decimal places

(l)  $(1,5)^{-5} = 0,131687242 \approx 0,132$  to 3 decimal places

10. Progress Exercises 4.5, Question 7, page 177 / page 177

$$\begin{aligned}
 \left( \frac{4(0,6)K^{0,4}L^{-0,4}}{L} \right) &= \frac{4(0,6)K^{0,4}L^{-0,4}}{L} \times \frac{K}{4(0,4)K^{-0,6}L^{0,6}} \\
 &= \frac{4(0,6)K^{0,4+1}L^{-0,4}}{4(0,4)K^{-0,6}L^{1+0,6}} \\
 &= \frac{0,6K^{1,4}L^{-0,4}}{0,4K^{-0,6}L^{1,6}} \times \frac{10}{10} \\
 &= \frac{6K^{1,4}L^{-0,4}}{4K^{-0,6}L^{1,6}} \quad \text{divided by 2 above and below the line} \\
 &= \frac{3K^{1,4+0,6}}{2L^{1,6+0,4}} \\
 &= \frac{3K^2}{2L^2} \\
 &= \frac{3}{2} \left( \frac{K}{L} \right)^2
 \end{aligned}$$

11. Progress Exercises 4.5, Question 13, page 177 / page 177

$$\begin{aligned}
 \frac{e^{2x+3}}{e^{5x-3}} &= e^{2x+3-(5x-3)} \\
 &= e^{2x+3-5x+3} \\
 &= e^{6-3x}
 \end{aligned}$$

12. Progress Exercises 4.6, Question 1, page 179 / page 179

$$\begin{aligned}
 2^x &= \frac{1}{\sqrt{16}} \\
 &= \frac{1}{4} \\
 &= \frac{1}{2^2} \\
 &= 2^{-2} \quad \text{Equating powers,} \\
 x &= -2
 \end{aligned}$$

## 13. Progress Exercises 4.6, Question 4, page 179 / page 179

$$\begin{aligned}\frac{2^x}{4} &= 2 \\ \frac{2^x}{2^2} &= 2^1 \\ 2^{x-2} &= 2^1 \\ x-2 &= 1 \quad (\text{equating powers}) \\ x &= 1+2 \\ x &= 3\end{aligned}$$

## 14. Progress Exercises 4.6, Question 5, page 179 / page 179

$$\begin{aligned}3^{Q+2} &= 9 \\ 3^{Q+2} &= 3^2 \quad (\text{writing 9 as an index number}) \\ Q+2 &= 2 \quad \text{equating powers} \\ Q &= 2-2 \\ Q &= 0\end{aligned}$$

## 15. Progress Exercises 4.6, Question 7, page 179 / page 179

$$\begin{aligned}\frac{1}{K} &= 8 \\ 1 &= 8K \quad (\text{multiply by } K \text{ both sides}) \\ \frac{1}{8} &= K \quad (\text{dividing by 8 both sides}) \\ \text{or } K &= \frac{1}{8} = 0,125\end{aligned}$$

## 16. Progress Exercises 4.6, Question 8, page 179 / page 179

$$\begin{aligned}\frac{4}{K^{0,5}} &= 8 \\ \frac{K^{0,5}}{4} &= \frac{1}{8} \quad (\text{inverting both sides}) \\ K^{0,5} &= \frac{4}{8} \\ K^{0,5} &= 0,5 \\ K &= (0,5)^2 \quad (\text{squaring both sides}) \\ K &= 0,25\end{aligned}$$

## 17. Progress Exercises 4.6, Question 20, page 179 / page 179

$$\begin{aligned} \frac{4}{L} + L &= -4 \\ 4 + L^2 &= -4L \text{ (multiplying both sides by } L \text{ )} \\ L^2 + 4L + 4 &= 0 \text{ (forming a quadratic equation )} \\ (L + 2)(L + 2) &= 0 \text{ (factorising )} \\ (L + 2)^2 &= 0 \\ L + 2 &= 0 \text{ (finding square-root of both sides )} \\ L &= -2 \text{ (Solving for } L \text{ )} \end{aligned}$$

## 18. Progress Exercises 4.8, Question 2, page 184 / page 184

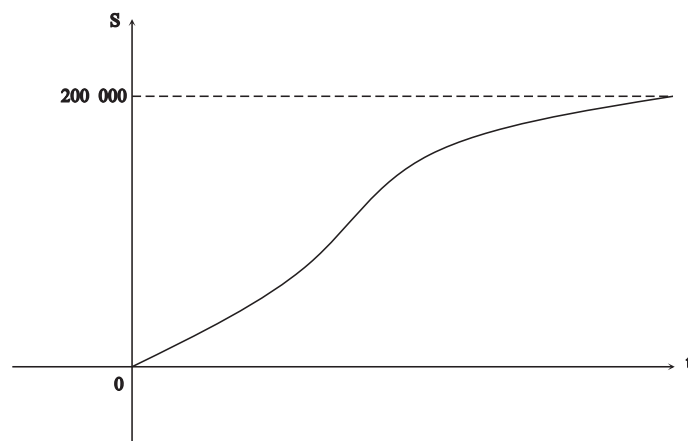
$$S = 200\,000(1 - e^{-0,05t})$$

(a) If  $t = 1$  week,

$$\begin{aligned} S &= 200\,000(1 - e^{-0,05}) \\ &= 200\,000 \times (1 - 0,951229424) \\ S &= 9\,754,115\,098 \\ S &\approx 9\,754 \end{aligned}$$

(b)

|   |        |         |         |         |         |         |
|---|--------|---------|---------|---------|---------|---------|
| t | 5      | 20      | 35      | 45      | 50      | 52      |
| s | 44 240 | 126 424 | 165 245 | 178 920 | 183 583 | 185 145 |





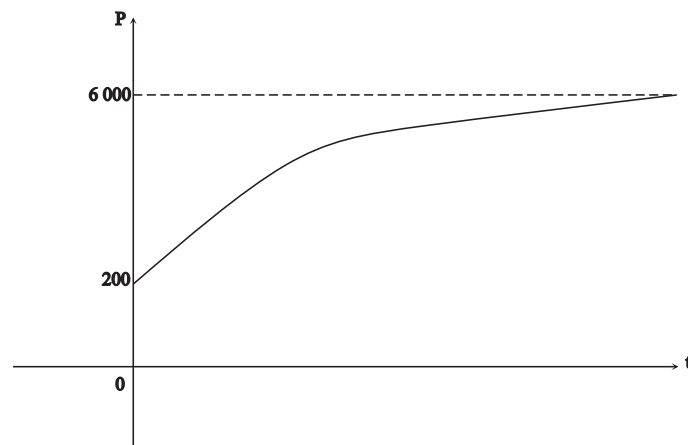
## 19. Progress Exercises 4.10, Question 11, page 188 / page 188

$$P(t) = \frac{6000}{1 + 29e^{-0,4t}}$$

(a)

| $t$ | $P(t)$  |
|-----|---|
| 0   | $P = \frac{6000}{1+29} = \frac{6000}{30} = 200$           |
| 4   | $P = \frac{6000}{1+29e^{-1,6}} = 875,2736479 \approx 875$ |
| 10  | $P = \frac{6000}{1+29e^{-4}} = 3918,614229 \approx 3919$  |

(b)



(c)

$$\begin{aligned} \text{If } P &= 1000, \text{ then} \\ 1000 &= \frac{6000}{1 + 29e^{-0,4t}} \\ 1 + 29e^{-0,4t} &= 6 \\ 29e^{-0,4t} &= 5 \\ e^{-0,4t} &= \frac{5}{29} \\ \ln(e^{-0,4t}) &= \ln\left(\frac{5}{29}\right) \\ -0,4t \ln e &= \ln\left(\frac{5}{29}\right) \\ t &= \frac{\ln \frac{5}{29}}{-0,4} \\ t &= 4,394644794 \\ t &\approx 4,39 \text{ years} \end{aligned}$$

$$\begin{aligned}
 \text{If } P &= 3\,000, \text{ then} \\
 1 + 29e^{-0,4t} &= \frac{6\,000}{3\,000} \\
 1 + 29e^{-0,4t} &= 2 \\
 e^{-0,4t} &= \frac{1}{29} \\
 -0,4t \ln e &= \ln\left(\frac{1}{29}\right) \\
 t &= \frac{-\ln(29)}{-0,4} \\
 t &= 8,418239575 \\
 t &\approx 8,42 \text{ years}
 \end{aligned}$$

$$\begin{aligned}
 \text{If } P &= 4\,000, \text{ then} \\
 1 + 29e^{-0,4t} &= \frac{6\,000}{4\,000} \\
 29e^{-0,4t} &= 0,5 \\
 e^{-0,4t} &= \frac{1}{58} \\
 -0,4t \ln e &= -\ln 58 \\
 t &= \frac{-\ln 58}{0,4} \\
 t &= 10,15110753 \\
 t &\approx 10,15 \text{ years}
 \end{aligned}$$

20. Progress Exercises 4.13, Question 8, page 201 / page 202

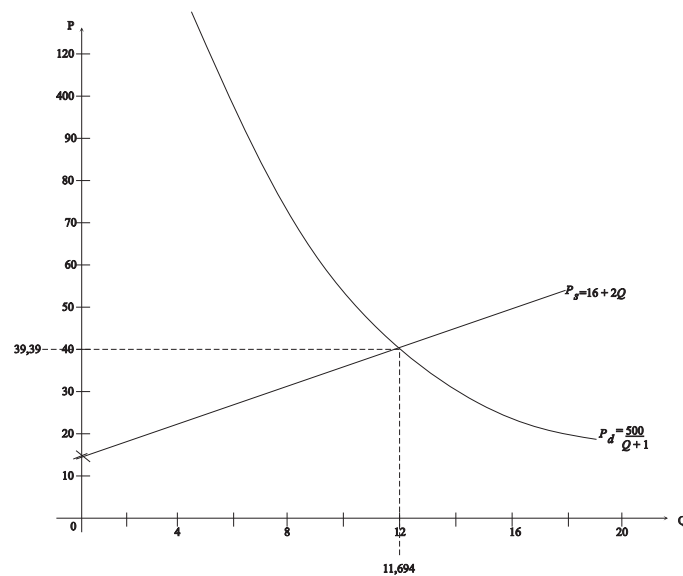
$$\text{Demand function : } P_d = \frac{500}{Q+1}; \quad \text{Supply function : } P_s = 16 + 2Q$$

(a) At equilibrium

$$\begin{aligned}
 P_s &= P_d \\
 16 + 2Q &= \frac{500}{Q + 1} \\
 (16 + 2Q)(Q + 1) &= 500 \text{ (Cross-multiplying)} \\
 16Q + 16 + 2Q^2 + 2Q - 500 &= 0 \\
 2Q^2 + 18Q - 484 &= 0 \\
 Q &= \frac{-18 \pm \sqrt{18^2 - 4(2)(-484)}}{2 \times 2} \\
 &= \frac{-18 \pm \sqrt{324 + 3872}}{4} \\
 Q &= \frac{-18 \pm 64,777}{4} \\
 Q &= \frac{-82,777}{4} \text{ or } \frac{+46,777}{4} \\
 Q &= -20,694 \text{ or } +11,694
 \end{aligned}$$

Therefore equilibrium quantity is  $Q = 11,694$  and equilibrium price is

$$\begin{aligned}
 P &= 16 + 2(11,694) \\
 &= 16 + 23,388 \\
 &= 39,388 \\
 P &\approx 39,39.
 \end{aligned}$$



## 15.5 Self-Evaluation Exercise 5: Unit 5

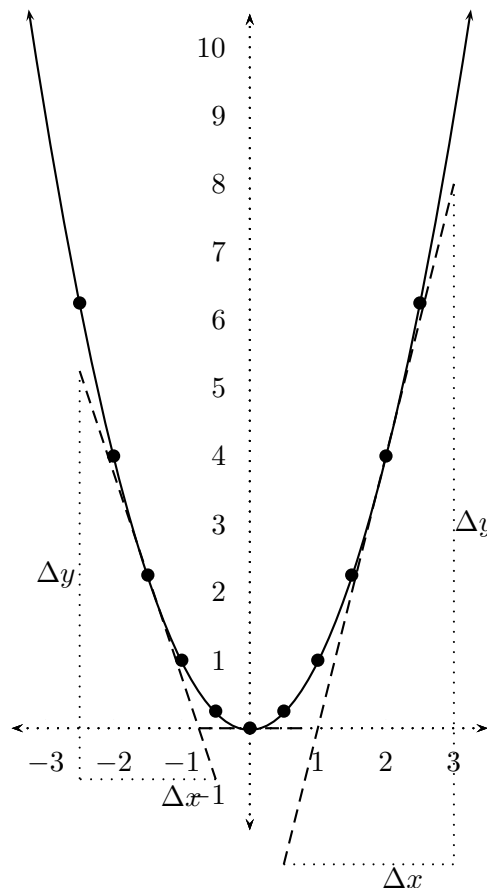
## 1. Progress Exercises 6.1, Question 1, page 266 / page 268

(a) Tabulate the values,

|           |      |    |      |    |      |   |      |   |      |   |      |
|-----------|------|----|------|----|------|---|------|---|------|---|------|
| $x$       | -2,5 | -2 | -1,5 | -1 | -0,5 | 0 | 0,5  | 1 | 1,5  | 2 | 2,5  |
| $y = x^2$ | 6,25 | 4  | 2,25 | 1  | 0,25 | 0 | 0,25 | 1 | 2,25 | 4 | 6,25 |

and plot the points on a coordinate system. Then connect these points and continue the curve (solid line) to the left and right in order to indicate that we are attempting to draw the entire  $y = x^2$ .

(b) The tangents (dashed lines) can be estimated using a ruler. Recall that the tangent line at a point is that line which intersects the curve only at that specific point. Clearly the tangent line at  $(0; 0)$  is horizontal and therefore has slope 0. The slopes of the other two tangent lines are estimated by using  $\Delta x$  and  $\Delta y$  for each line as below. Your  $\Delta x$  and  $\Delta y$  for each line will depend on how long you drew the tangent but the *ratio*  $\frac{\Delta y}{\Delta x}$  does not depend on the length of the line and will be equal to (except for measurement error) the derivative computed in the next part. Note that it is important to use the correct signs, so if  $\Delta x$  is taken to be positive in each case (imagining that one moves from left to right on the tangent) then  $\Delta y$  has to be negative for the tangent sloping down (as we go from left to right) and positive for the tangent with an upward slope.



(c)  $y' = \frac{dy}{dx} = 2x$  so  $\frac{x}{y'} \mid \begin{array}{c|c|c} -1,5 & 0 & 2 \\ \hline -3 & 0 & 4 \end{array}$ .

2. **Progress Exercises 6.1, Question 3(c), page 267 / page 269**

$y = 10 + 5x + \frac{1}{x^2} = 10 + 5x + x^{-2}$  and therefore

$$\frac{dy}{dx} = 5 + (-2)x^{-3} = 5 - 2\frac{1}{x^3}.$$

Note: we write  $\frac{1}{x^3}$  instead of  $x^{-3}$  since the question stipulated we give the answer using *positive indices*.

3. **Progress Exercises 6.1, Question 3(e), page 267 / page 269**

Since  $P = \frac{1}{3}Q^3 + 70Q - 15Q^2$ ,

$$\frac{dP}{dQ} = \frac{1}{3}(3Q^2) + 70 - 15(2Q) = Q^2 - 30Q + 70.$$

4. **Progress Exercises 6.3, Question 1, page 278 / page 280**

(a) Recall that

$$TR = Q \cdot P = Q \left( \frac{120 - Q}{3} \right) = 40Q - \frac{1}{3}Q^2$$

where we have used the demand equation in the form

$$P = \frac{120 - Q}{3}$$

to obtain  $TR$  as a function of  $Q$ . Then

$$MR = \frac{d(TR)}{dQ} = 40 - \frac{1}{3}(2Q) = 40 - \frac{2}{3}Q$$

and

$$AR = \frac{TR}{Q} = \frac{120 - Q}{3}.$$

Therefore, when  $Q = 15$  :

**TR:**  $TR = 40(15) - \frac{1}{3}(15^2) = 325$

The total revenue from selling 15 items is 325.

**MR:**  $MR = 40 - \frac{2}{3}(15) = 30$

The marginal revenue from selling an addition item if 15 have already been sold, is 30.

**AR:**  $AR = \frac{120-15}{3} = 35$

The average revenue per item (i.e. the price per item) when 15 items are sold, is 35.

(b)  $AR = 0$  where  $\frac{120 - Q}{3} = 0$  which is simply where  $Q = 120$ . If  $Q = 120$  then  $MR = 40 - \frac{2}{3}(120) = -40$ . It makes no sense to sell this quantity as marginal revenue is negative.

## 5. Progress Exercises 6.3, Question 2, page 278 / page 280

(a) Recall that

$$TR = Q \cdot P = Q(125 - Q^{1,5}) = 125Q - Q^{2,5}$$

where we have used the demand equation in the form given. Then

$$MR = \frac{d(TR)}{dQ} = 125 - 2,5Q^{1,5} \quad \text{and} \quad AR = \frac{TR}{Q} = 125 - Q^{1,5}.$$

The slope of the MR curve is not twice the slope of the AR curve, except possibly at certain specific values of  $Q$ .

(b) Therefore, when  $Q = 10$  :

**TR:**  $TR = 125(10) - (10^{2,5}) \approx 933,72$

The total revenue from selling 10 items is approximately 933,72.

**MR:**  $MR = 125 - 2,5(10)^{1,5} \approx 45,94$

The marginal revenue from selling an addition item if 10 have already been sold, is approximately 45,84.

**AR:**  $AR = 125 - 10^{1,5} \approx 93,38$

The average revenue per item (i.e. the price per item) when 10 items are sold, is approximately 93,38.

And when  $Q = 25$  :

**TR:**  $TR = 125(25) - (25^{2,5}) = 0$

The total revenue from selling 25 items is 0.

**MR:**  $MR = 125 - 2,5(25)^{1,5} = -187,5$

The marginal revenue from selling an addition item if 25 have already been sold, is  $-187,5$ .

**AR:**  $AR = 125 - 25^{1,5} = 0$

The average revenue per item (i.e. the price per item) when 25 items are sold, is 0.

(c)  $MR = 0$  where  $125 - 2,5Q^{1,5} = 0$ , i.e. where

$$Q^{1,5} = \frac{125}{2,5} = 50$$

so

$$Q = 50^{\frac{1}{1,5}} = 50^{\frac{2}{3}} \approx 13,572.$$

$AR = 0$  where

$$125 - Q^{1,5} = 0$$

i.e. where

$$Q = 125^{\frac{1}{1,5}} = 125^{\frac{2}{3}} = \left(125^{\frac{1}{3}}\right)^2 = 25.$$

The sale of further units starts to reduce total revenue where the marginal revenue becomes negative, which will be from 14 units on.

## 6. Progress Exercises 6.5, Question 1, page 287 / page 289

$\frac{dy}{dx} = 2x - 6$  so the only turning point is at  $x = 3$ .

7. **Progress Exercises 6.5, Question 6, page 287 / page 289**

$\frac{dy}{dx} = 3x^2 - 6x - 9$  so the turning points are where  $3x^2 - 6x - 9 = 0$  which can be written as

$$(3x - 9)(x + 1) = 0$$

with solutions  $x = 3$  or  $x = -1$  which indicate the turning points. Note: you may also have used the quadratic formula to solve the quadratic equation.

8. **Progress Exercises 6.5, Question 7, page 287 / page 289**

$\frac{d(TC)}{dQ} = 144 - Q^{-2} = 144 - \frac{1}{Q^2}$  which is zero when  $Q = \frac{1}{12}$  or when  $Q = -\frac{1}{12}$ , giving the two turning points of the function TC.

9. **Progress Exercises 6.5, Question 10, page 287 / page 289**

$\frac{dy}{dx} = 4x^3 - 4x$  so we try to solve  $4x^3 - 4x = 0$ . Fortunately  $4x^3 - 4x = 4x(x^2 - 1)$  and therefore the turning points are at  $x = 0$ ,  $x = 1$  and  $x = -1$ .

10. **Progress Exercises 6.9, Question 3, page 315 / page 318**

(a)  $TR = Q \cdot P = Q(240 - 10Q) = 240Q - 10Q^2$

$\pi = TR - TC = 240Q - 10Q^2 - (120 + 8Q) = -120 + 232Q - 10Q^2$

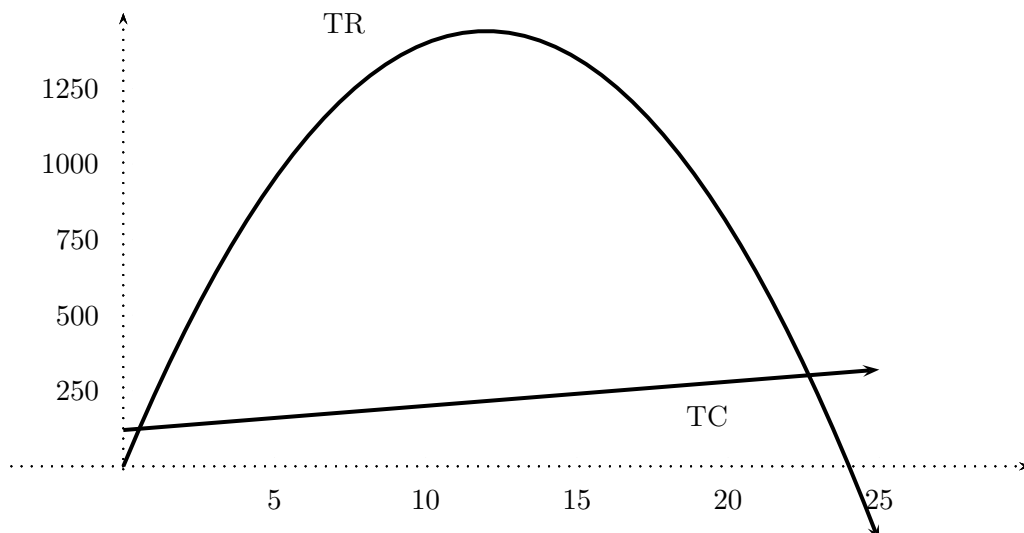
(b) (i)  $\frac{d\pi}{dQ} = 232 - 20Q$  and therefore profit is maximised for  $Q = 11,6$ .

(ii)  $\frac{d(TR)}{dQ} = 240 - 20Q$  and therefore TR is maximised for  $Q = 12$ .

(c)  $MR = \frac{d(TR)}{dQ} = 240 - 20Q$  and  $MC = \frac{d(TC)}{dQ} = 8$ . As long as  $MR > MC$  profit can be increased by producing more units. Therefore the maximum profit has been reached when  $MR = MC$ .

(d)

(i)



We estimate the break-even point from the graph to be below  $Q = 1$  (that is, where  $TR = TC$ ). This can be confirmed algebraically by noting that the profit function

$$\pi = TR - TC = -120 + 232Q - 10Q^2$$

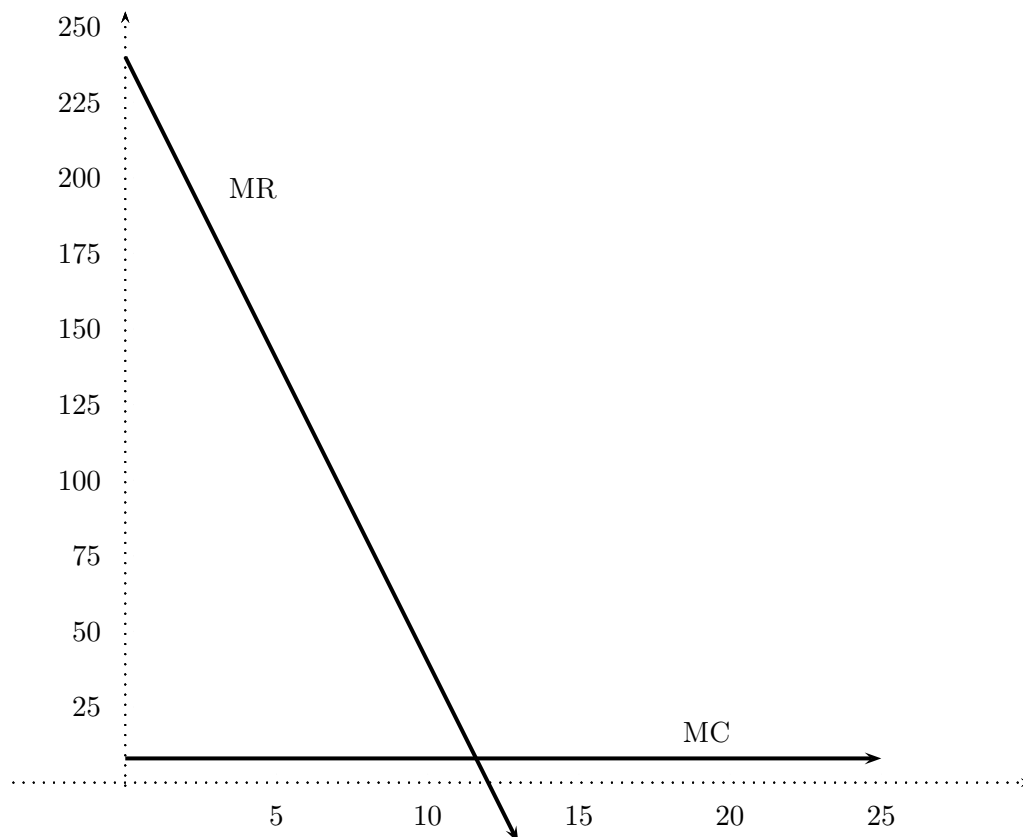
determined above, is negative for  $Q = 0$  but is positive for  $Q = 1$ . Since we are—naturally—dealing with integer numbers of T-shirts, it is not necessary to determine the solution

$$Q \approx 0,52932$$

using a formula. Of course there is a second break-even point (where the cost starts outstripping the revenue—for ever) which can be computed using the quadratic formula,

$$Q \approx 22,671.$$

(ii)



Where  $MR <$

$MC$  it is no longer worth manufacturing one more item. Therefore the intersection of the lines, where  $MR = MC$ , is at the production level  $Q$  where the profit is maximised.

11. **Progress Exercises 6.9, Question 4, page 315 / page 318**

(a) Given that  $AC = 15 + \frac{8000}{Q}$  and  $AR = 25$  we can compute

$$TR = Q \cdot AR = 25Q \quad TC = AC \cdot Q = 15Q + 8000$$

$$MR = \frac{d(TR)}{dQ} = 25 \quad \text{and} \quad MC = \frac{d(TC)}{dQ} = 15.$$

(b) Break-even point is reached where  $TR = TC$ , i.e.

$$25Q = 15Q + 8000$$

which is where  $Q = 800$ .

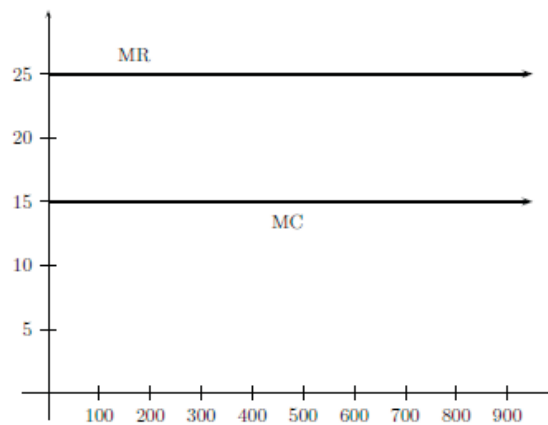
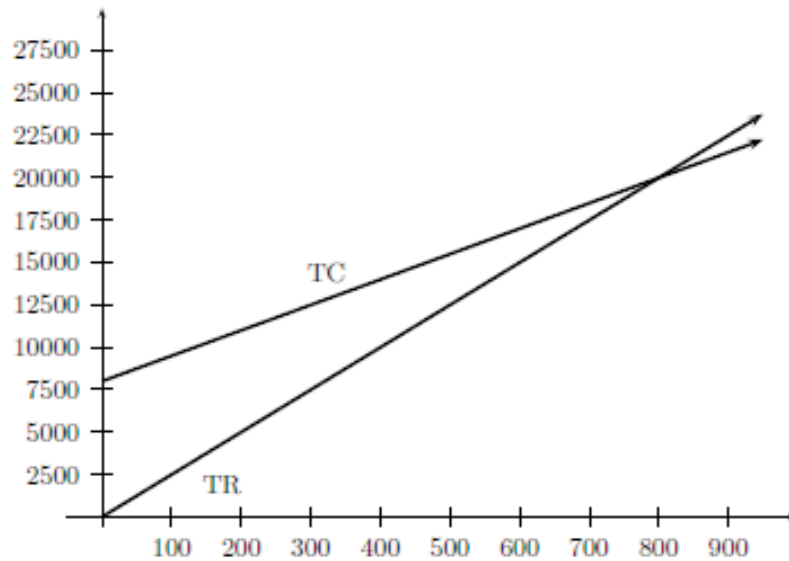


- (c) The profit function is  $\pi = TR - TC = 25Q - (15Q + 8000) = 10Q - 8000$  which is a straight line and therefore has no maximum. We can check this using differentiation by observing that

$$\frac{d\pi}{dQ} = 10Q \quad \text{and} \quad \frac{d(TR)}{dQ} = 25$$

neither of which can be zero. This is obvious since  $MR > MC$  for all values of  $Q$ .

- (d)



## 12. Progress Exercises 6.9, Question 5, page 315 / page 318

$$TC = 608\,580 + 120Q$$

(a)

| I  | II  |
|--|---|
| $P = 2\,374$                                       | $P = 5\,504 - 0,8Q$   |
| $TR = P \cdot Q = 2\,374Q$                         | $TR = P \cdot Q = (5\,504 - 0,8Q)Q = 5\,504Q - 0,8Q^2$      |
| $MR = \frac{d(TR)}{dQ} = 2\,374$                   | $MR = \frac{d(TR)}{dQ} = 5\,504 - 1,6Q$                     |
| $MC = \frac{d(TC)}{dQ} = 120$                      | $MC = \frac{d(TC)}{dQ} = 120$                               |
| $\pi = TR - TC$<br>$= 2\,374Q - (608\,580 + 120Q)$ | $\pi = TR - TC$<br>$= 5\,504Q - 0,8Q^2 - (608\,580 + 120Q)$ |
| $= 2\,254Q - 608\,580$                             | $= -0,8Q^2 + 5\,384Q - 608\,580$                            |

(i) I: solve  $2\,374 = 120$  which has no solution, so there is no maximum or minimum profit. It is clear from the profit function that the profit can increase indefinitely if  $Q$  is increasing.

II: solve  $5\,504 - 1,6Q = 120$  to find that  $Q = 3\,365$ .

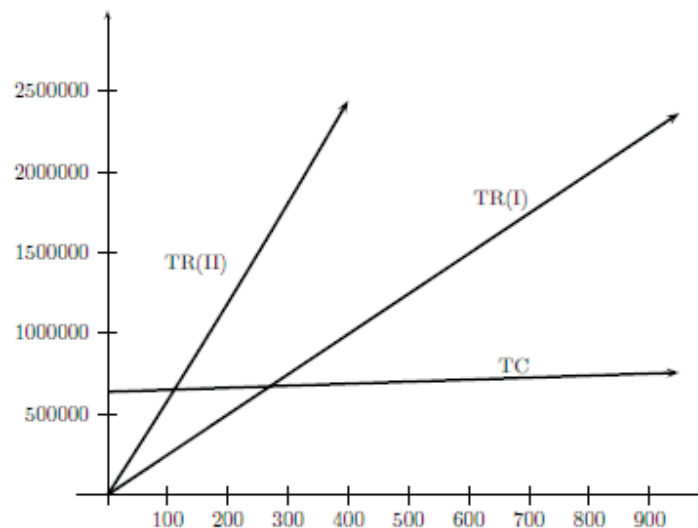
(ii) I:  $\frac{d\pi}{dQ} = 2\,254$  which is never zero, so there is no maximum or minimum profit.

II:  $\frac{d\pi}{dQ} = -1,6Q + 5\,384$  which is zero where  $Q = 3\,365$ .

(b) When  $Q = 3\,365$  in II,  $\pi = -0,8(3\,365)^2 + 5\,384(3\,365) - 608\,580 = 8\,450\,000$ .

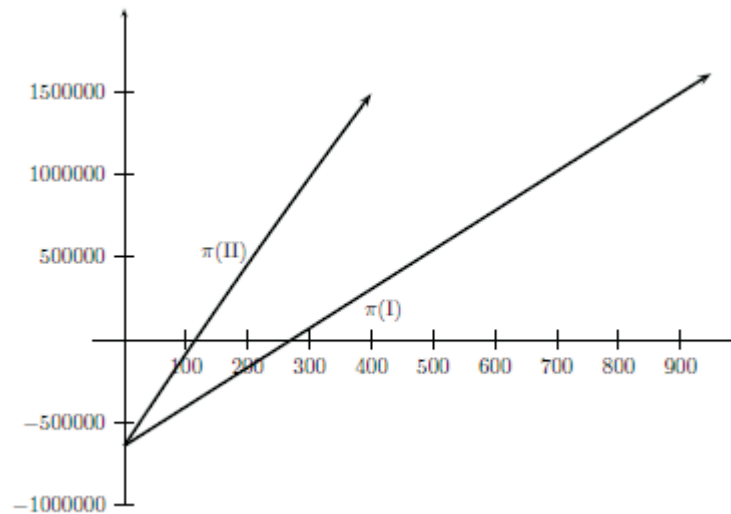
When  $Q = 3\,365$  in I,  $\pi = 2\,254(3\,365) - 608\,580 = 6\,976\,130$ .

(c)



The break-even point appears to be around  $Q = 100$  for II and around  $Q = 275$  for I. The total revenue is the same for both schemes for quite small  $Q$ .

(d)



We can read off the break-even points somewhat more easily from this graph. Scheme II gives a higher profit for the range of values of  $Q$  plotted but since the profit function for II is quadratic, it has a maximum and at some point the profit in II will start falling. For very large values of  $Q$ , then, the scheme I will result in a higher profit.

### 13. Progress Exercises 6.17, Question 1, page 352 / page 355

(a)  $\varepsilon_d = \frac{dQ}{dP} \cdot \frac{P}{Q}$  in the general case but for small discrete changes we also use

$$\varepsilon_d = \frac{\% \text{ change in } Q}{\% \text{ change in } P}$$

and therefore

$$\% \text{ change in } Q = \varepsilon_d (\% \text{ change in } P)$$

and

$$\% \text{ change in } Q = -0,8 (\% \text{ change in } P)$$

if  $\varepsilon_d = -0,8$  is constant. In other words, if the price increases by 5%, the quantity demanded will fall by 4%.

## 14. Progress Exercises 6.17, Question 2, page 352/ page 355

(a)

| Demand function  | $\varepsilon_d(P)$  | $\varepsilon_d(Q)$   |
|--|---|--|
| $P = 80 - 2Q \quad \left(Q = 80 - \frac{1}{2}P\right)$         | $\frac{dQ}{dP} \cdot PQ = -\frac{1}{2} \cdot \frac{P}{80 - \frac{1}{2}P}$<br>$= \frac{P}{P - 160}$            | $\frac{80 - 2Q}{80 - 2Q - 160} = \frac{Q - 40}{40 + Q}$                |
| $Q = 120 - 4P \quad \left(P = 30 - \frac{1}{4}Q\right)$        | $\frac{dQ}{dP} \frac{P}{Q} = -4 \cdot P / (120 - 4P) = \frac{P}{P - 30}$                                      | $\frac{30 - \frac{1}{4}Q}{30 - \frac{1}{4}Q - 30} = \frac{Q - 120}{Q}$ |
| $P = 432$ ( $Q$ independent of $P$ )                           | undefined   | undefined  |
| $P = a - bQ \quad \left(Q = \frac{a}{b} - \frac{1}{b}P\right)$ | $\frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{1}{b} \cdot \frac{P}{\frac{a}{b} - \frac{1}{b}P} = \frac{P}{P - a}$ | $\frac{a - bQ}{a - bQ - a} = \frac{bQ - a}{bQ}$                        |

(b)

| Demand function  | $\varepsilon_d(P = 50)$                | $\varepsilon_d(Q = 30)$                    |
|--|--|--|
| $P = 80 - 2Q \quad \left(Q = 80 - \frac{1}{2}P\right)$         | $\frac{50}{50 - 160} \approx -0,45455$ | $\frac{30 - 40}{40 + 30} \approx -0,14286$ |
| $Q = 120 - 4P \quad \left(P = 30 - \frac{1}{4}Q\right)$        | $\frac{50}{50 - 30} = 2,5$             | $\frac{30 - 120}{30} = -3$                 |
| $P = 432$ ( $Q$ independent of $P$ )                           | undefined                              | undefined                                  |
| $P = a - bQ \quad \left(Q = \frac{a}{b} - \frac{1}{b}P\right)$ | $\frac{50}{50 - a}$                    | $\frac{30b - a}{30b}$                      |

## 15. Progress Exercises 6.17, Question 7, page 352 / page 355

(a) Rewrite the demand equation first, as  $P = 50 - \frac{1}{2}Q$ .

$$TR = P \cdot Q = \left(50 - \frac{1}{2}Q\right) \cdot Q = 50Q - \frac{1}{2}Q^2$$

$$MR = \frac{d}{dQ}(TR) = 50 - Q$$

$$AR = \frac{1}{Q}(TR) = 50 - \frac{1}{2}Q$$

(b) Revenue is maximised where  $MR = 0$ , i.e. where  $Q = 50$  with corresponding price  $P = 50 - \frac{1}{2}(50) = 25$ .(c) (i)  $\varepsilon_d = \frac{dQ}{dP} \cdot PQ = -2 \cdot \frac{P}{100 - 2P} = \frac{P}{P - 50}$ (ii)  $\varepsilon_d = \frac{50 - \frac{1}{2}Q}{50 - \frac{1}{2}Q - 50} = \frac{Q - 100}{Q}$

(d) TR is a maximum where  $MR = 0$  anyway, i.e. where  $Q = 50$ . For  $Q = 50$

$$\varepsilon_d = \frac{50 - 100}{50} = -1.$$

16. **Progress Exercises 8.1, Question 1, page 433 / page 435**

$$\int (x + x^3 + x^{3,5}) dx = \frac{1}{4,5}x^{4,5} + \frac{1}{4}x^4 + \frac{1}{2}x^2 + c = \frac{2}{9}x^{4,5} + \frac{1}{4}x^4 + \frac{1}{2}x^2 + c$$

17. **Progress Exercises 8.1, Question 9, page 433 / page 435**

$$\begin{aligned} \int x(x-3)^2 dx &= \int ((x^2 - 6x + 9) dx = \int (x^3 - 6x^2 + 9x) dx = \frac{1}{4}x^4 - \frac{6}{3}x^3 + \frac{9}{2}x^2 + c \\ &= \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 + c \end{aligned}$$

18. **Progress Exercises 8.1, Question 11, page 433 / page 435**

$$\int x^2 \left(1 + \frac{1}{x^2}\right) dx = \int (x^2 + 1) dx = \frac{1}{3}x^3 + x + c$$

19. **Progress Exercises 8.1, Question 17, page 433 / page 435**

$$\int Q(20 - 0,5Q) dQ = \int (20Q - 0,5Q^2) dQ = \frac{20}{2}Q^2 - \frac{0,5}{3}Q^3 + c = 10Q^2 - \frac{1}{6}Q^3 + c$$

20. **Progress Exercises 8.3, Question 1, page 445 / page 447**

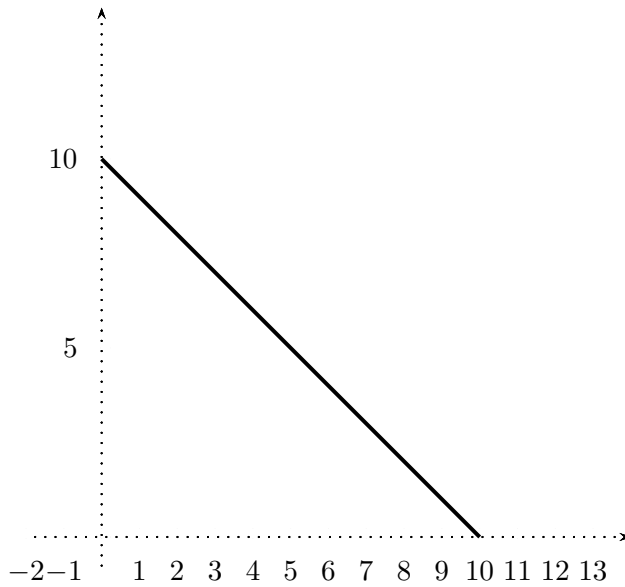
$$\int_{x=1}^{x=3} (x+5) dx = \left(\frac{1}{2}x^2 + 5x\right)\Big|_{x=1}^{x=3} = \left(\frac{1}{2}3^2 + 5(3)\right) - \left(\frac{1}{2}1^2 + 5(1)\right) = \frac{9}{2} + 15 - \left(\frac{1}{2} + 5\right) = 14$$

21. **Progress Exercises 8.3, Question 4, page 445 / page 447**

$$\begin{aligned} \int_{x=-2}^{x=2} (x^2 - 3) dx &= \left(\frac{1}{3}x^3 - 3x\right)\Big|_{x=-2}^{x=2} \\ &= \left(\frac{1}{3}2^3 - 3(2)\right) - \left(\frac{1}{3}(-2)^3 - 3(-2)\right) \\ &= \frac{8}{3} - 6 - \left(-\frac{8}{3} + 6\right) = \frac{16}{3} - 12 = -\frac{20}{3} = -6\frac{2}{3} \end{aligned}$$

## 22. Progress Exercises 8.3, Question 20, page 446 / page 448

(a)



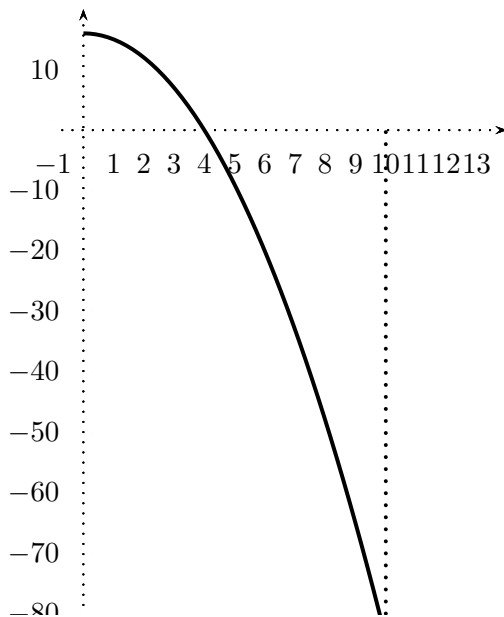
(b) The net area is given by

$$\int_{Q=0}^{Q=10} (10 - Q)dQ = \left(10Q - \frac{1}{2}Q^2\right) \Big|_{Q=0}^{Q=10} = 100 - \frac{1}{2}(100) - 0 = 50.$$

(c) The area below the horizontal axis is 0 and the area above the axis is 50.

## 23. Progress Exercises 8.3, Question 22, page 446 / page 448

(a)



Note: the graph of the function is the solid line. The vertical axis is not to the same scale as the horizontal axis.

(b) The net area is given by  $\int_{Q=0}^{Q=10} (16 - Q^2) dQ = (16Q - \frac{1}{3}Q^3) \Big|_{Q=0}^{Q=10} = 16(10) - \frac{1}{3}(10^3) - 0 = 160 - \frac{1000}{3} = -\frac{520}{3} = -173\frac{1}{3}$

(c) The area above the horizontal axis is given by

$$\int_{Q=0}^{Q=4} (16 - Q^2) dQ = \left(16Q - \frac{1}{3}Q^3\right) \Big|_{Q=0}^{Q=4} = 16(4) - \frac{1}{3}(4^3) - 0 = 64 - \frac{64}{3} = \frac{128}{3} = 42\frac{2}{3}$$

and the area below the horizontal axis is given by

$$\begin{aligned} \int_{Q=4}^{Q=10} (16 - Q^2) dQ &= \left(16Q - \frac{1}{3}Q^3\right) \Big|_{Q=4}^{Q=10} = 16(10) - \frac{1}{3}(10^3) - \left(16(4) - \frac{1}{3}(4^3)\right) \\ &= -173\frac{1}{3} - 42\frac{2}{3} \\ &= -216. \end{aligned}$$

The negative sign simply denotes that the area lies below the horizontal axis. Properly speaking, an area is always a positive quantity.