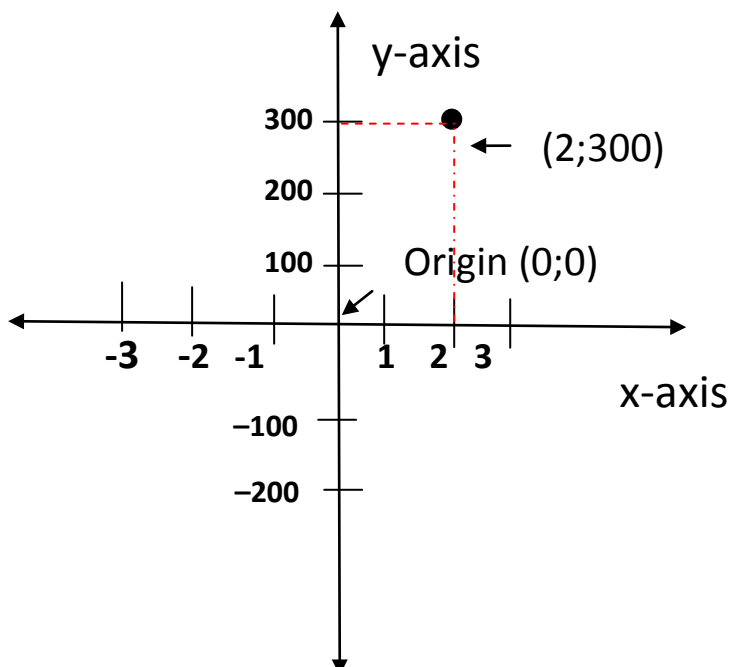


Study Unit 2 : Linear functions

Chapter 2 : Sections 2.1 – 2.4 and 2.6

1. Function

- Humans = relationships
- Function = mathematical form of a relationship
 Temperature and number of ice cream sold
- Independent variable – **if** variable : x
- Dependent variable – **then** variable : y
- Function of x : $f(x) = y$ $f(x) = 2x + 3$ or $y = 2x + 3$
- Relationship of x and y : ordered pair $(x;y)$
 If temp is 20° then number of ice creams sold is 400
 If temp is 30° then number of ice creams sold is 600
 $(x_1 ; y_1) = (20 ; 400)$ and $(x_2 ; y_2) = (30 ; 600)$
- To graph relationship use Cartesian plane
 - 2 number lines : x -axis : horizontal
 y -axis : vertical
 intersection : origin



2. Linear function

- Relationship between 2 variables is linear and graph is a straight line

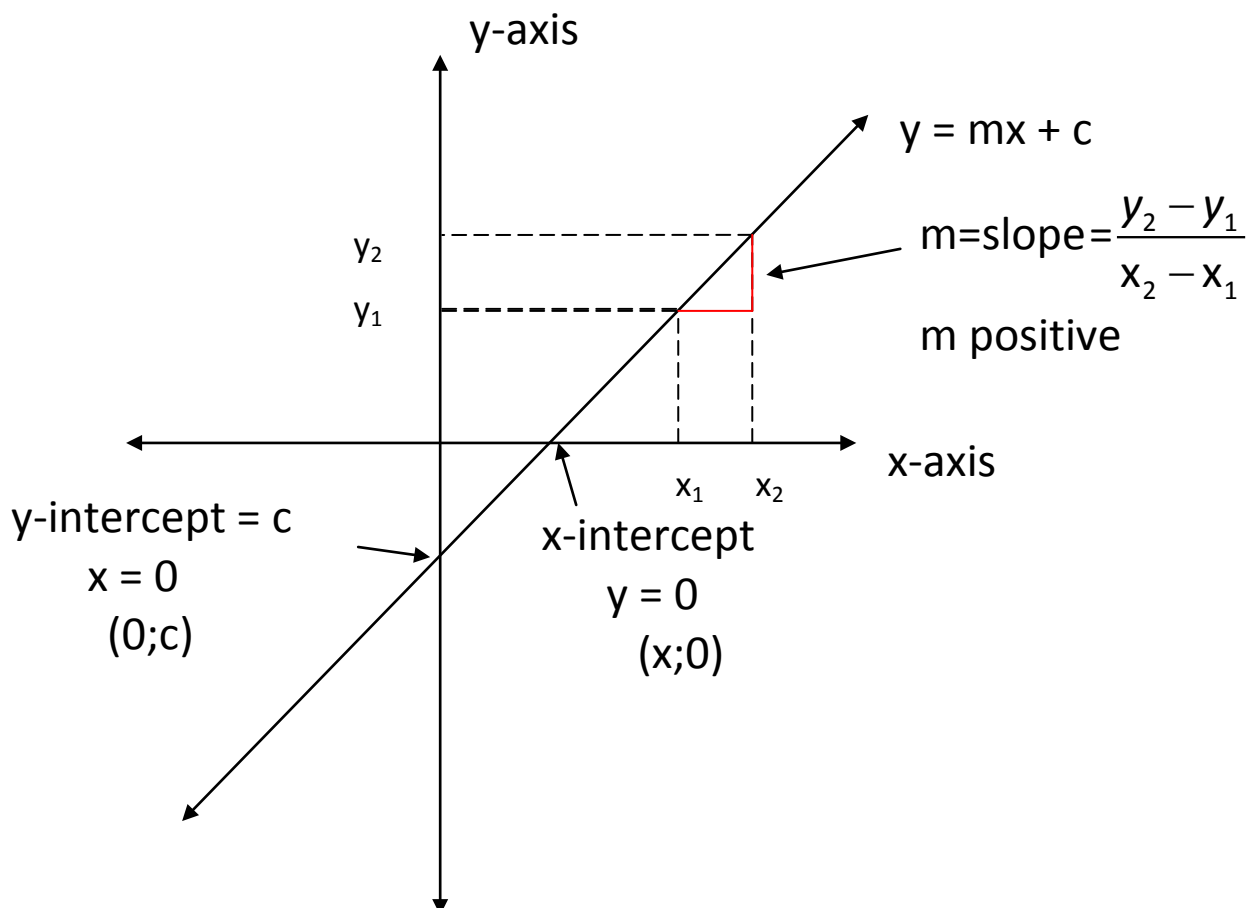
$$y = mx + c \quad \text{or} \quad y = ax + b$$

with y and x variables and m and c values and $(x_1; y_1)$ and $(x_2; y_2)$ are 2 points on the line.

- m = slope or how steep is line or how does y -values change if x values change

$$\blacksquare \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

- c = y -intercept : cut y -axis : where $x = 0$

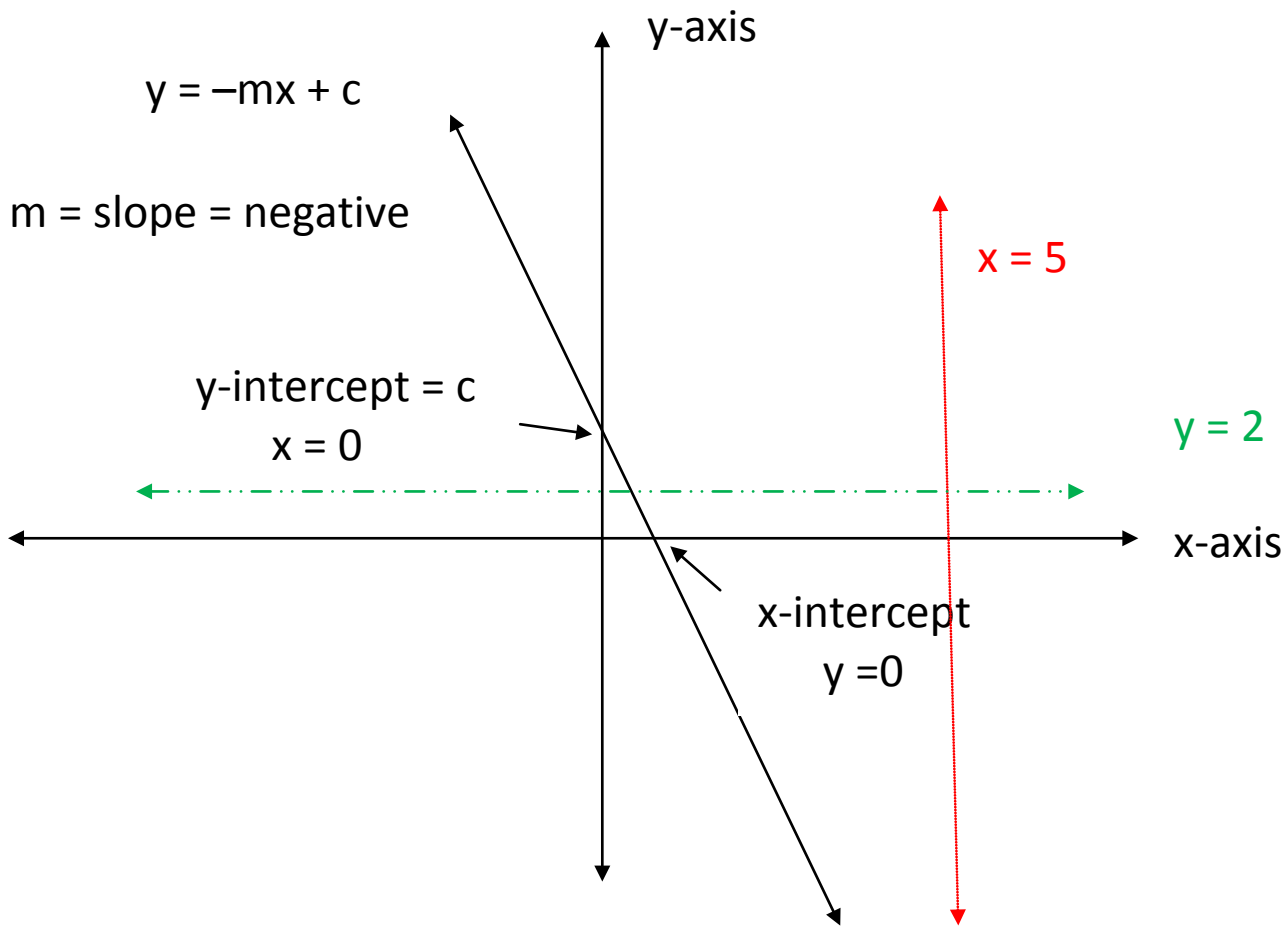


Other lines:

$$y = -mx + c$$

$m = \text{slope} = \text{negative}$

y-intercept = c
 $x = 0$



- If two lines are parallel they have the same slope

- **How to determine equation of line: $y = mx + c$**

- **Need 2 points on line $(x_1; y_1)$ and $(x_2; y_2)$**

1. Calculate $m = \frac{y_2 - y_1}{x_2 - x_1}$

- 2. Substitute m and any one of the 2 points into function $y = mx + c$ to determine c**

No 3a of discussion class

- **How to draw a line:**

- **Need two points on line or**
- **Equation of line**

- 1. Calculate any 2 points on line by choosing a x -value and calculate the y -value**

No 3b of discussion class

Question 3a

Find the equation of the line passing through the points (1; 20) and (5; 60).

Solution

$$y = mx + c.$$

Let $(x_1 ; y_1) = (1 ; 20)$ and $(x_2 ; y_2) = (5 ; 60)$

The slope m is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 20}{5 - 1} = \frac{40}{4} = 10$$

Therefore $y = 10x + c$.

Substitute any one of the points into the equation of the line to determine c . Let's choose the point (1 ; 20). Then

$$y = 10x + c$$

$$20 = 10 \times 1 + c$$

$$20 = 10 + c$$

$$-c = 10 - 20$$

$$-c = -10$$

$$c = 10$$

The equation of the line is $y = 10x + 10$.

Question 3 b

Draw the graph of the line $y = 10x + 10$.

Solution

Need two points to draw line :

Choose any x or y value and calculate y or x:

Choose $x = 0$ then $y = 10(0) + 10$

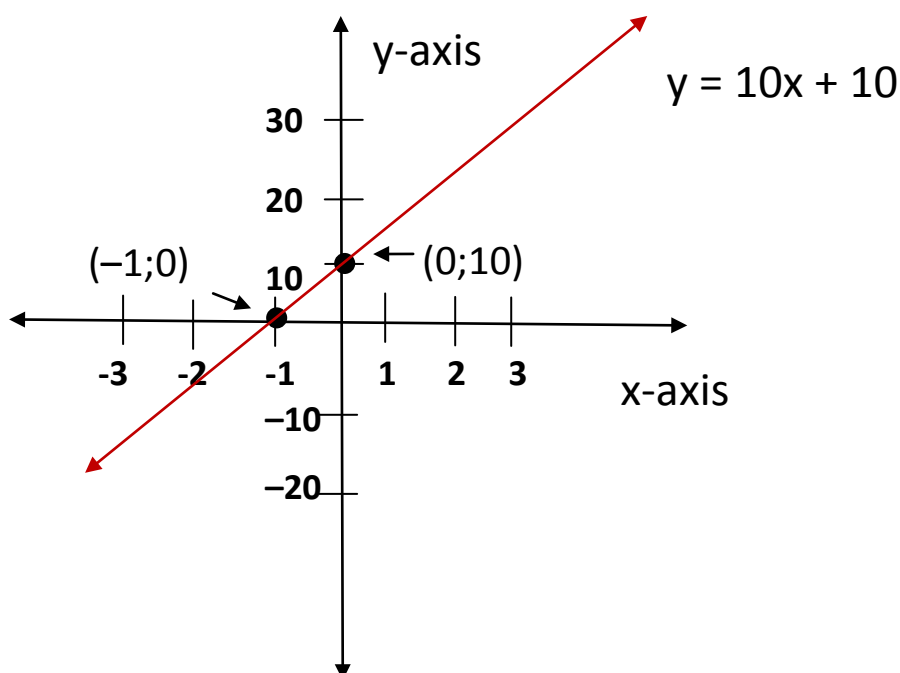
$$y = 10 \rightarrow \text{point 1} = (0 ; 10)$$

Choose $y = 0$ then $0 = 10x + 10$

$$-10x = 10$$

$$x = 10/-10$$

$$x = -1 \rightarrow \text{point2} = (-1 ; 0)$$



- How to determine a slope, y-intercept and x-intercept if given the equation of a line:

for example $3x + 4y - 8 = 4$ or $y = 4x + 20$

1. Write it in the format $y = mx + c$
2. Compare with standard form \Rightarrow slope is m , y-intercept is c
3. To calculate the x-intercept make $y = 0$ and solve for x

1. Write in format $y = mx + c$

$$3x + 4y - 8 = 4$$

$$4y = 4 + 8 - 3x$$

$$4y = 12 - 3x$$

$$y = 12/4 - 3/4x$$

$$y = 3 - 3/4x$$

2. slope = $m = -3/4$ y-intercept = $c = 3$

x-intercept is where $y = 0$ but $y = 3 - 3/4x$

$$0 = 3 - 3/4x$$

$$3/4x = 3$$

$$x = 3 \times 4/3$$

$$x = 12/3$$

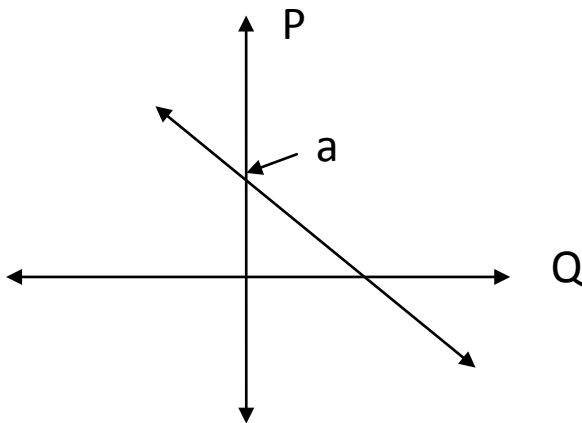
$$x = 4$$

3. Application in economics

Relationship between price P and quantity Q of a product

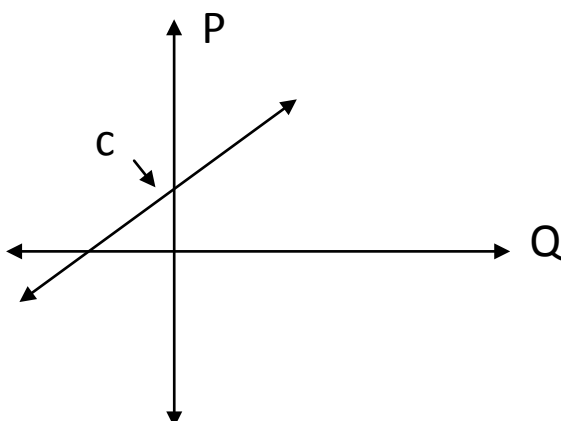
- Demand function

- If the price of a product \uparrow then the demand \downarrow
- $P = a - bQ$ with
 - $a = y\text{-intercept (c)}$
 - $b = \text{slope} = \text{negative}$



- Supply function

- If the price of a product \uparrow then the supply \uparrow
- $P = c + dQ$ with
 - $c = y\text{-intercept}$
 - $d = \text{slope} = \text{positive}$



Cost function

- Cost
 - Fixed cost
 - Variable cost dependent on quantity Q
- $TC = FC + VC \times Q \Rightarrow y = c + mx$

A supermarket's fixed cost is R5000 per month and the salary per employees is R2000 per month. What is the supermarket's linear cost function if the number of employees is Q?

$$\text{Cost} = 5000 + 2000Q$$

- Revenue
 - What you earn
 - $R = \text{Price} \times \text{Quantity}$
 - $R = P \times Q \Rightarrow px$
- Profit
 - Revenue – cost

- Depreciation

A R200 000 car depreciates linearly to R40 000 in 8 years' time. Derive a linear equation for the value of the car after x years with $0 \leq x \leq 8$.

Let y = value and x = time or years

$$y = mx + c$$

Need two points on graph:

Given (8; 40 000) and (0; 200 000)

$$\text{Now } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40000 - 200000}{8 - 0} = \frac{-160000}{8} = -20000$$

$$y = -20\,000x + c$$

Take any one of two points: Say point 2

$$200\,000 = -20\,000(0) + c$$

$$c = 200\,000$$

$$\text{Depreciation : } y = -20\,000x + 200\,000$$

- **Elasticity**
 - Important in economics
 - Think what happens with an elastic band: if you apply little pressure the band expand a little bit and if you apply a lot of pressure the band expand a lot.
 - How sensitive demand is for price change
 - If the price P and % change in price goes up or down what will happen to the % change in quantity
 - Ratio of % change
 - $\epsilon = \frac{\% \text{ change in demand}}{\% \text{ change in price}}$
 - Price elasticity of demand or supply
 - Point : At a point
 - Arc : Over an interval

1. Price elasticity of demand

- Point ($P_0; Q_0$)

Demand : $P = a - bQ$

$$\epsilon_d = -\frac{1}{b} \cdot \frac{P_0}{Q_0}$$

- In terms of P

$$\epsilon_d = -\frac{1}{b} \cdot \frac{P}{Q} = \frac{P}{P - a}$$

Discussion class 4a + 4b

Question 4a

If the demand function is $P = 80 - 2Q$, where P and Q are the price and quantity respectively, determine the expression for price elasticity of demand if the price $P = 20$.

Solution

Now

$$\varepsilon_d = -\frac{1}{b} \times \frac{P}{Q}$$

Given $P = 80 - 2Q$ and $P = 20$.

Comparing $P = 80 - 2Q$ with $P = a - bQ \rightarrow a = 80$ and $b = 2$.

To determine the value of Q we substitute $P = 20$ into the equation and solve for Q

$$\begin{aligned} 20 &= 80 - 2Q \\ 20 - 80 &= -2Q \\ \frac{-60}{-2} &= Q \\ Q &= 30 \end{aligned}$$

Now

$$\begin{aligned}
 \varepsilon_d &= -\frac{1}{b} \cdot \frac{P}{Q} \\
 &= -\frac{1}{2} \cdot \frac{20}{30} \\
 &= -\frac{1}{3} \\
 &= -0,33
 \end{aligned}$$

At $P = 20$ a 1% increase (decrease) in price will cause a 0,33% decrease (increase) in the quantity demanded

Question 4 b

If the demand function is $P = 80 - 2Q$, where P and Q are the price and quantity respectively, determine the expression for price elasticity of demand in terms of P only.

Solution

Now demand in terms of P

$$\varepsilon_d = \frac{P}{P - a}$$

Given $P = 80 - 2Q$ and $P = 20$.

Comparing $P = 80 - 2Q$ with $P = a - bQ$, $\rightarrow a = 80$ and $b = 2$.

Thus
$$\varepsilon_d = \frac{P}{P - 80}.$$

- Arc

Over an interval

Use the average P and Q at beginning and end of interval.

$P_1 \rightarrow P_2$ and $Q_1 \rightarrow Q_2$

$$\epsilon_d = -\frac{1}{b} \cdot \frac{P_1 + P_2}{Q_1 + Q_2}$$

Discussion class 5

Question 5

Given the demand function $P = 60 - 0,2Q$ where P and Q is the price and quantity respectively, calculate the arc price elasticity of demand when the price decreases from R50 to R40.

Solution

$$\text{Arc elasticity of demand} = -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2}$$

Given function $P = 60 - 0,2Q$, with $a = 60$ and $b = 0, 2$

Given $P_1 = 50$ and $P_2 = 40$.

Need to determine Q_1 and Q_2 .

$$P = 60 - 0,2Q$$

$$0,2Q = 60 - P$$

$$Q = 300 - 5P$$

Determine Q_1 and Q_2 by substituting $P_1 = 50$ and $P_2 = 40$ into the equation. Thus

$$\text{If } P_1 = 50 \text{ then } Q_1 = 300 - 5 \times 50 = 50$$

$$\text{If } P_2 = 40 \text{ then } Q_2 = 300 - 5 \times 40 = 100$$

Therefore

$$\begin{aligned} \text{Elasticity of demand} &= -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2} \\ &= -\frac{1}{0,2} \times \frac{50 + 40}{50 + 100} \\ &= -\frac{1}{0,2} \times \frac{90}{150} \\ &= \frac{-90}{30} \\ &= -3 \end{aligned}$$

2. Price elasticity of supply

Demand : $P = c + dQ$

$$\epsilon_s = \frac{1}{d} \bullet \frac{P_0}{Q_0}$$