DSC1520 (496530) May/June 2009

QMG101P (455808)

QUANTITATIVE MODELLING I

Duration 2 Hours 100 Marks

EXAMINERS
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Use of a non-programmable pocket calculator is permissible

This examination paper remains the property of the University of South Africa and may not be removed from the examination room.

This paper consists of 20 pages, comprises 25 questions and counts a total of 100 marks

USE OF A SCIENTIFIC CALCULATOR IS PERMITTED

The paper is divided into two parts Section A and Section B

SECTION A

Answer ALL the questions in this section on the mark-reading sheet supplied. Follow the instructions for completing the mark-reading sheet carefully.

Also pay attention to the following information. Suppose you are asked the following question.

$$3 + 2 \times -1 + 4 - 2 =$$

[1]   7

[2]   1

[3]   3

[4]   4

[5]   None of the above

The correct answer is [3]. Only one option (indicated as [1]   [2]   [3]   [4]   [5]) per question is correct. If you mark more than one option, you will not receive any marks for the question. If your answer is correct you will receive 3 MARKS. Marks WILL NOT be deducted for incorrect answers.

Section A consists of 20 questions and counts 60 marks. Hand in the completed mark-reading sheet with your answers for Section B. DO NOT STAPLE IT!

SECTION B

This section must be completed in the spaces provided below each question. Section B counts 40 marks.

Remember to include your MARK-READING SHEET.

[TURN OVER]

Question 1

Find the slope of the line $0 = 6 + 3x - 2y$

[1] $\dfrac{2}{3}$

[2] $\dfrac{3}{2}$

[3]   3

[4]   2

[5]   None of the above

Question 2

$\log_3\left(\dfrac{3}{\sqrt{3}}\right)$ to four decimal places equals approximately

[1] $-0\,0795$

[2] $0,0795$

[3] $2\,0000$

[4] $0\,5000$

[5]   None of the above

Question 3

Solve the inequality

$$x^2 - 3x \geq 6 - 2x$$

[1] $-2 \leq x \leq 3$

[2] $-6 \leq x \leq 1$

[3] $x \leq -2 \quad x \geq 3$

[4] $x \leq -3 \quad x \geq 2$

[5] $-3 \leq x \leq 2$

Question 4

Find the equation of the straight line passing through the points (4 2) and (2 4)

[1] $y = -1x + 6$

[2] $y = -1x$

[3] $y = 2x + 4$

[4] $y = 1x + 2$

[5]   None of the above

Question 5

Find the value of quantity Q for the demand function $P = 60 - 4Q$ when the market price is $P = 24$

[1] 8
[2] 9
[3] 10
[4] 11
[5] 12

Question 6

Calculate the consumer surplus for the demand function $P = 60 - 4Q$ when the market price is $P = 16$

[1] 242
[2] 484
[3] 88
[4] 32
[5] 352

Question 7

If the demand function is $P = 90 - 0.05Q$ where P and Q are the price and quantity respectively determine the expression for price elasticity of demand in terms of P

[1] $\dfrac{P}{P - 90}$

[2] $\dfrac{P - 90}{P}$

[3] $\dfrac{P}{P - 1\,800}$

[4] $\dfrac{P - 1\,800}{P}$

[5] None of the above

Question 8

The supply and demand functions are given by

$$P = 50 - 3Q \quad \text{(supply function)}$$
$$P = 14 + 1.5Q \quad \text{(demand function)}$$

where P and Q are the price and quantity respectively. Calculate the level of excess supply if price $P = 20$.

[1]   10

[2]   4

[3]   14

[4]   6

[5]   None of the above

Question 9

What is the value of maximum revenue if total revenue is given by

$$R(z) = -\frac{1}{5}z^2 + 30z + 81$$

where z is the quantity?

[1]   75

[2]   1206

[3]   152.65

[4]   81

[5]   None of the above

Question 10

Solve the following system of linear equations

$$x + y + z = 8$$
$$x - 3y = 0$$
$$5y - z = 10$$

[1] $x = 6 \ y = 2 \ z = 0$

[2] $x = 0 \ y = 6 \ z = 2$

[3] $x = 2 \ y = 0, z = 6$

[4] $x = -6 \ y = 2 \ z = 6$

[5]   None of the above

Question 11

Determine the roots of $4x^2 + 3x - 1$

[1] $x = \dfrac{1}{4}$ $x = -1$

[2] $x = \dfrac{1}{4}$ $x = 1$

[3] $x = -\dfrac{1}{4}, x = 1$

[4] $x = -\dfrac{1}{4}$ $x = -1$

[5] None of the above

Question 12

If $y = 2^{-x}$ find x if $y = 0,0625$

[1] $x = -2$

[2] $x = 3$

[3] $x = 4$

[4] $x = 5$

[5] None of the above

Question 13

Evaluate the following definite integral

$$\int_{-2}^{2} (x^2 - 3)dx$$

[1] $6\dfrac{2}{3}$

[2] $-6\dfrac{2}{3}$

[3] $3\dfrac{1}{3}$

[4] $-3\dfrac{1}{3}$

[5] None of the above

Question 14

Evaluate

$$\int \imath^2 \left(1 + \frac{1}{\imath^2}\right) d\imath$$

[1] $\imath^3 + x + c$

[2] $\frac{1}{3}x^3 + \imath + c$

[3] $x^2 + 1$

[4] $\frac{1}{2}x^2 + x + c$

[5] None of the above

Question 15

Simplify

$$\frac{d}{dx}\left[\frac{\imath - \imath^2}{\sqrt{\imath}}\right]$$

[1] $\frac{3}{2}\sqrt{\imath} + \frac{1}{2\sqrt{x}}$

[2] $\frac{1}{2\sqrt{\imath}} - \frac{3}{2}\sqrt{\imath}$

[3] $\frac{3}{2}\sqrt{\imath} - \frac{1}{2\sqrt{\imath}}$

[4] $\frac{3}{2}\sqrt{\imath} + \frac{1}{2\sqrt{\imath}}$

[5] None of the above

Question 16

The demand function of a firm is $Q = 150 - 0.5P$ where P and Q represent the quantity and price respectively. At what value of Q is marginal revenue equal to zero?

[1] 150

[2] 75

[3] 113

[4] 0

[5] None of the above

Question 17

Given the demand function $P = 60 - 0.2Q$. What is the arc price elasticity of demand when price decreases from R50 to R40?

[1] $-\dfrac{1}{3}$

[2] $\dfrac{1}{3}$

[3] -3

[4] 3

[5] None of the above

Question 18

Consider the market defined by the following functions

$$\text{demand function} \quad P = 60 - 0.6Q$$
$$\text{supply function} \quad P = 20 + 0.2Q$$

where P and Q are the price and quantity respectively. Calculate the equilibrium price and quantity

[1] $P = 300$, $Q = 20$

[2] $P = 200$, $Q = 30$

[3] $P = 20$, $Q = 300$

[4] $P = 30$, $Q = 200$

[5] None of the above

Question 19
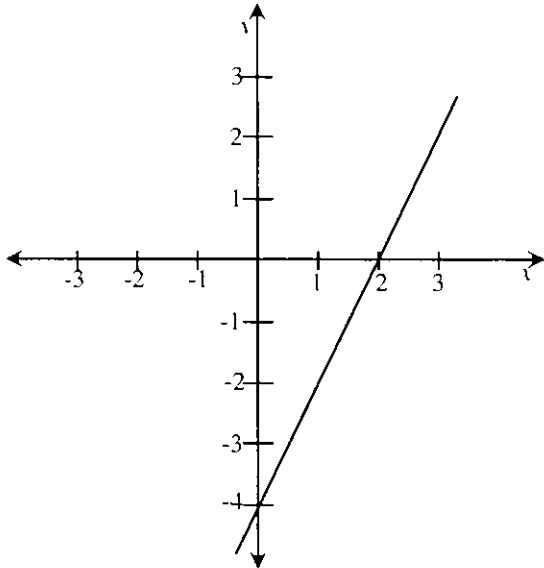
What is the point of intersection of the following lines
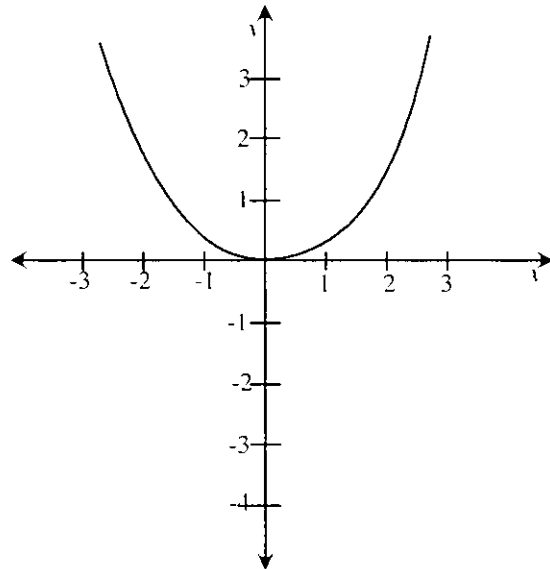
$$2x + y - 5 = 0$$
$$3x - 2y - 4 = 0$$

[1] $x = 3$, $y = 1$

[2] $x = 1$, $y = 2$

[3] $x = 2$, $y = 1$

[4] $x = 1$, $y = 3$

[5] None of the above
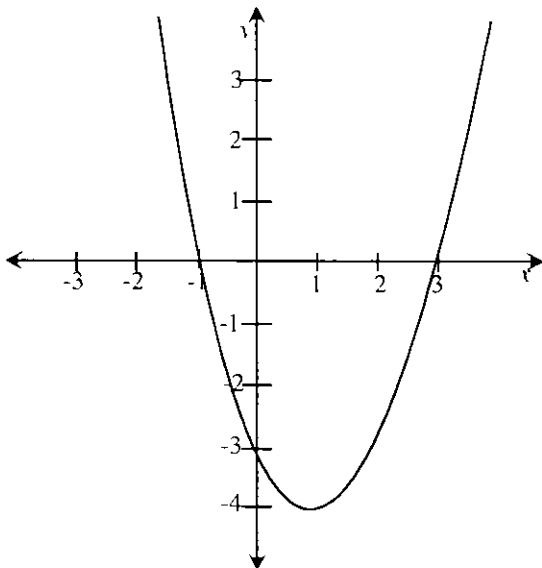
Question 20

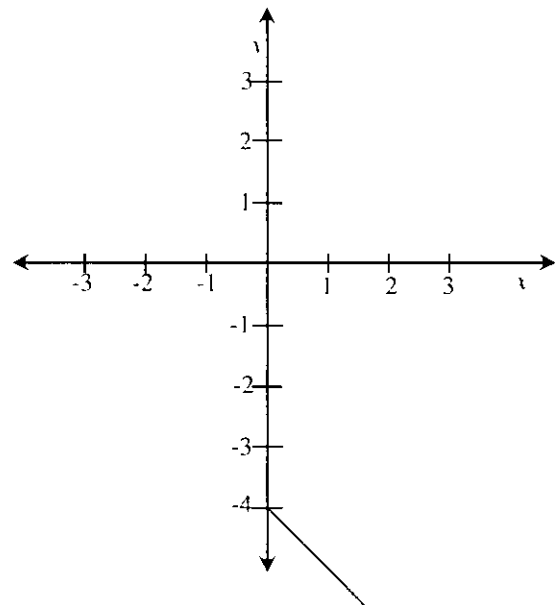The graph of $y = -2x + x^2 - 3$ is represented by

[1]

[2]

[3]

[4]

[5] None of the above

Please turn over for Section B.

Section B

Question 21

The monthly demand for a new line of computers t months after it has been introduced in the market is given by

$$D(t) = 2\,000 - 1\,500e^{-0.05t} \quad \text{for } t > 0$$

(a) Find demand two years after these computers were introduced   (2)

(b) Algebraically, determine the number of months after which demand will be $1\,000$ units   (3)

                                                                                                   [5]

Question 22

An electronics company manufactures radios and television sets. The time needed to manufacture a radio is 90 minutes and it takes 5 minutes to test a radio. The time needed to manufacture a television set is 150 minutes and it takes 15 minutes to test a television set. It costs R175 to make a radio and R850 to make a television set. The company has at most 95 hours of manufacturing time and at least 9 hours of testing time available. The production cost must not exceed R13 500.

Write down the inequalities that this production process must satisfy. [10]

Question 23

ABC intends manufacturing and marketing a new product. It has been determined that the cost of producing the product as a function of price is given by

$$C(P) = 432\,000 - 1\,800P$$

and the revenue generated when units are sold at price P rand each is given by

$$R(P) = 6\,000P - 30P^2$$

Plot the income and cost functions on the same graph using the grid below. Indicate clearly on the graph the break-even point(s) and profit area. [9]

Question 23 continued

Question 24

(a) Draw the lines representing the following constraints on the grid below (4)

$$
\begin{array}{llllll}
(1) & 6x & + & 2y & \leq & 840 \\
(2) & 2x & + & y & \leq & 300 \\
(3) & x & + & y & \leq & 250 \\
& & & x,\,y & \geq & 0
\end{array}
$$

(b) Show the feasible region (1)

(c) Determine the maximum value of $P = 120x + 95y$ subject to the constraints above (5)

[10]

Question 24 continued

Question 25

Let $f(x) = 3x^2 - 7$ Find the equation of the line tangent to the graph $y = f(x)$ at $x = 1$ [6]

TOTAL 100

ROUGH WORK

ROUGH WORK

ROUGH WORK

ROUGH WORK