Pooled Variance t Test

- Tests means of 2 independent populations having *equal* variances
- Parametric test procedure
- Assumptions
  - Both populations are normally distributed
  - If not normal, can be approximated by normal distribution ($n_1 \geq 30$ & $n_2 \geq 30$)
  - Population variances are unknown but assumed equal

Two Independent Populations Examples

- An economist wishes to determine whether there is a difference in mean family income for households in 2 socioeconomic groups.
- An admissions officer of a small liberal arts college wants to compare the mean SAT scores of applicants educated in rural high schools & in urban high schools.

Pooled Variance t Test Example

You're a financial analyst for Charles Schwab. You want to see if there a difference in dividend yield between stocks listed on the NYSE & NASDAQ.

	NYSE	NASDAQ
Number	21	25
Mean	3.27	2.53
Std Dev	1.30	1.16

Assuming equal variances, is there a difference in average yield ($\alpha = .05$)?

Pooled Variance t Test Solution

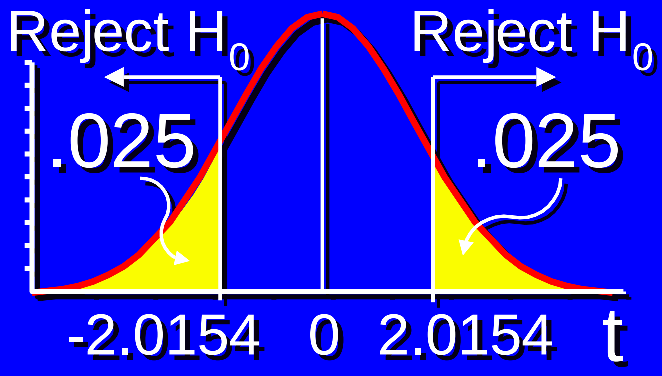
H_0: $\mu_1 - \mu_2 = 0$ ($\mu_1 = \mu_2$)

H_1: $\mu_1 - \mu_2 \neq 0$ ($\mu_1 \neq \mu_2$)

$\alpha = .05$

df = 21 + 25 - 2 = 44

Critical Value(s):



Test Statistic:

$$t = \frac{3.27 - 2.53}{\sqrt{1.510 \cdot \left(\frac{1}{21} + \frac{1}{25}\right)}} = +2.03$$

Decision:

Reject at α = .05

Conclusion:

There is evidence of a difference in means

Test Statistic Solution

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_P^{\,2} \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(3.27 - 2.53) - (0)}{\sqrt{1.510 \cdot \left(\frac{1}{21} + \frac{1}{25}\right)}} = +2.03$$

$$S_P^{\,2} = \frac{(n_1 - 1) \cdot s_1^{\,2} + (n_2 - 1) \cdot s_2^{\,2}}{(n_1 - 1) + (n_2 - 1)}$$

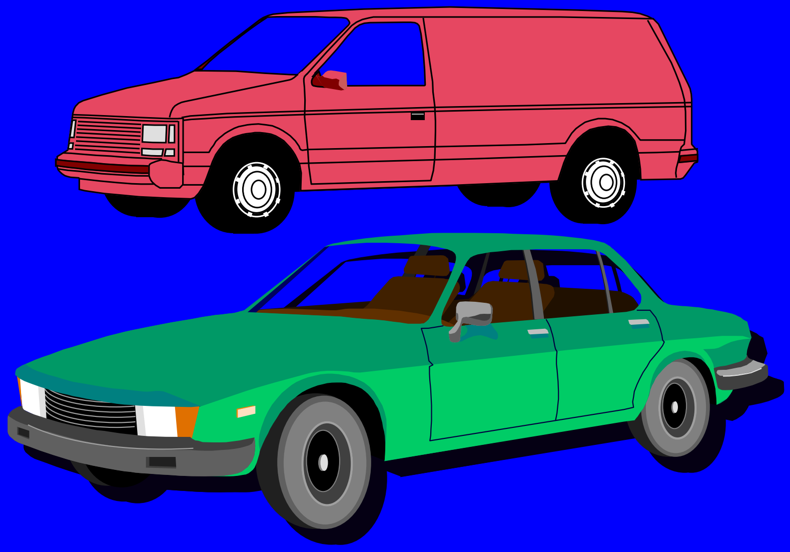
$$= \frac{(21 - 1) \cdot (1.30)^2 + (25 - 1) \cdot (1.16)^2}{(21 - 1) + (25 - 1)} = 1.510$$

Pooled Variance t Test Thinking Challenge

You're a research analyst for General Motors. Assuming equal variances, is there a difference in the average miles per gallon (mpg) of two car models ($\alpha = .05$)?

You collect the following:

	Sedan	Van
Number	15	11
Mean	22.00	20.27
Std Dev	4.77	3.64



Alone Group Class

Test Statistic Solution*

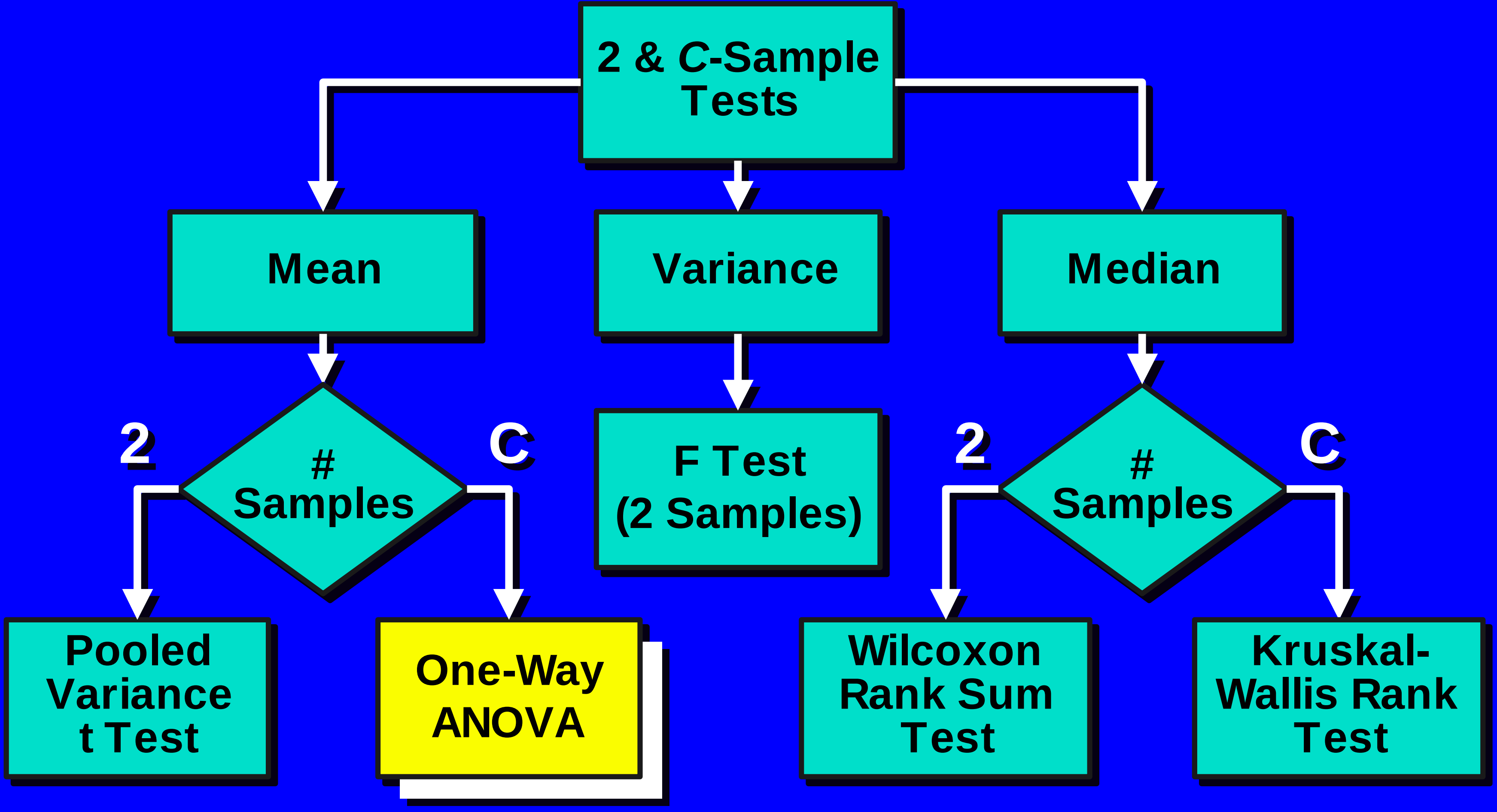
$$t = \frac{\left(\bar{x}_1 - \bar{x}_2\right) - \left(\mu_1 - \mu_2\right)}{\sqrt{S_P^{\,2} \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\left(22.00 - 20.27\right) - \left(0\right)}{\sqrt{18.793 \cdot \left(\frac{1}{15} + \frac{1}{11}\right)}} = +1.00$$

$$S_P^{\,2} = \frac{\left(n_1 - 1\right) \cdot s_1^{\,2} + \left(n_2 - 1\right) \cdot s_2^{\,2}}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}$$

$$= \frac{\left(15 - 1\right) \cdot \left(4.77\right)^2 + \left(11 - 1\right) \cdot \left(3.64\right)^2}{\left(15 - 1\right) + \left(11 - 1\right)} = 18.793$$

One-Way ANOVA F-Test

2 & c-Sample Tests with Numerical Data



Experiment

- Investigator controls one or more independent variables
  - Called treatment variables or factors
  - Contain two or more levels (subcategories)
- Observes effect on dependent variable
  - Response to levels of independent variable
- Experimental design: Plan used to test hypotheses

Completely Randomized Design

- Experimental units (subjects) are assigned randomly to treatments
  - Subjects are assumed homogeneous
- One factor or independent variable
  - 2 or more treatment levels or classifications
- Analyzed by:
  - One-Way ANOVA
  - Kruskal-Wallis rank test

Randomized Design Example

	Factor (Training Method)		
Factor levels (Treatments)	Level 1 	Level 2 	Level 3 
Experimental units	😐 😐 😐	😐 😐 😐	😐 😐 😐
Dependent variable (Response)	21 hrs.	17 hrs.	31 hrs.
	27 hrs.	25 hrs.	28 hrs.
	29 hrs.	20 hrs.	22 hrs.

One-Way ANOVA F-Test

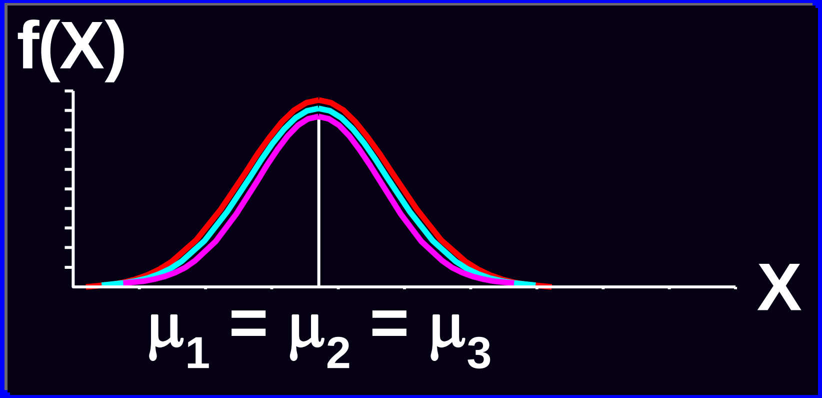
- Tests the equality of 2 or more (c) population means
- Variables
  - One nominal scaled independent variable
    - 2 or more (c) treatment levels or classifications
  - One interval or ratio scaled dependent variable
- Used to analyze completely randomized experimental designs

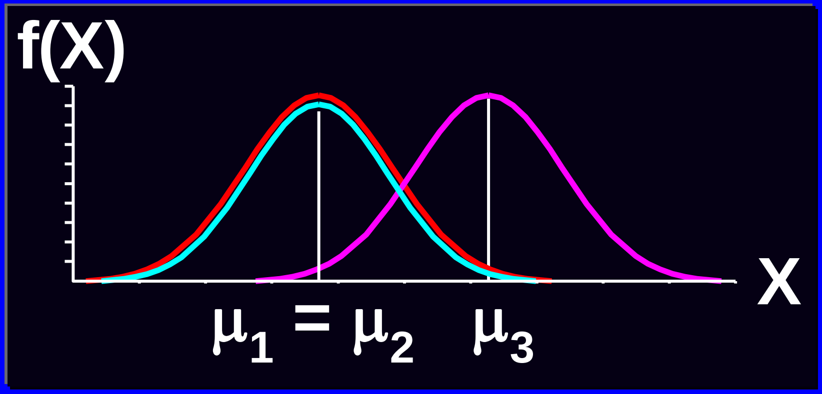
One-Way ANOVA
F-Test Assumptions

- Randomness & independence of errors
  - Independent random samples are drawn
- Normality
  - Populations are normally distributed
- Homogeneity of variance
  - Populations have equal variances

One-Way ANOVA F-Test Hypotheses

- $H_0: \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_c$
  - All population means are equal
  - No treatment effect
- H_1: Not all μ_j are equal
  - At least 1 population mean is different
  - Treatment effect
  - $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_c$ is wrong

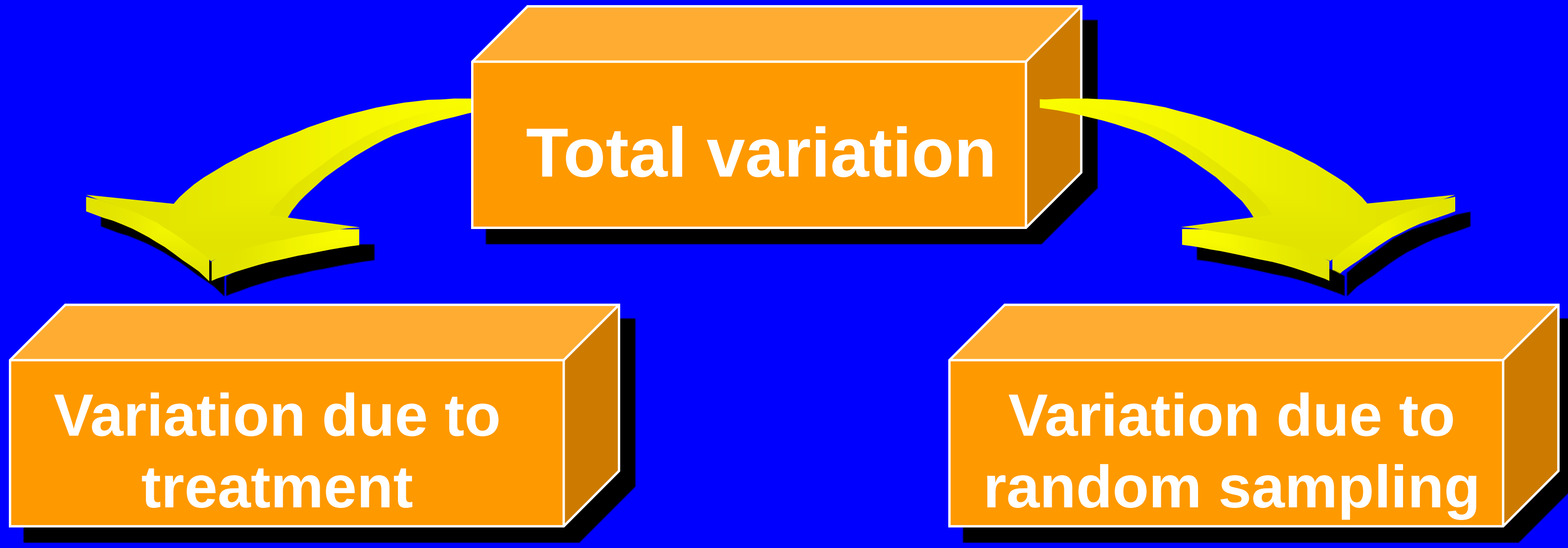




One-Way ANOVA
Basic Idea

- Compares 2 types of variation to test equality of means
- Ratio of variances is comparison basis
- If treatment variation is significantly greater than random variation then means are not equal
- Variation measures are obtained by 'partitioning' total variation

ANOVA Partitions Total Variation

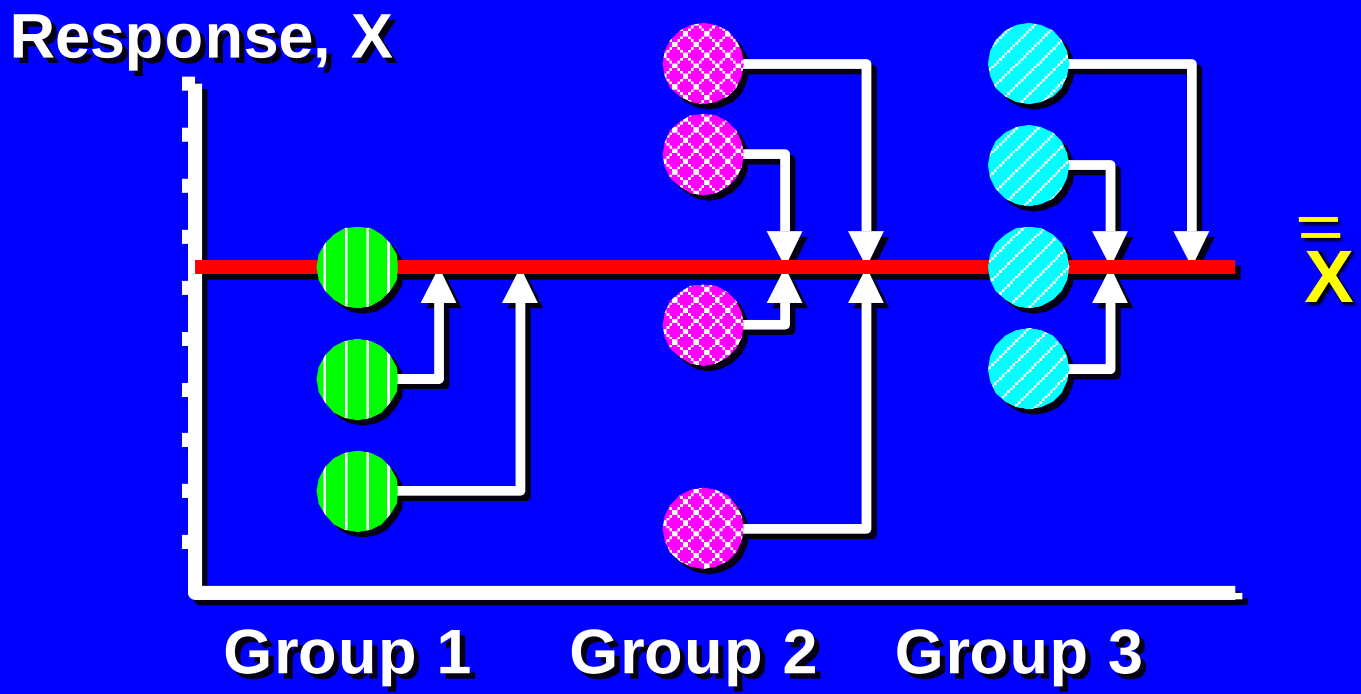


- Sum of squares among
- Sum of squares between
- Sum of squares model
- Among groups variation

- Sum of squares within
- Sum of squares error
- Within groups variation

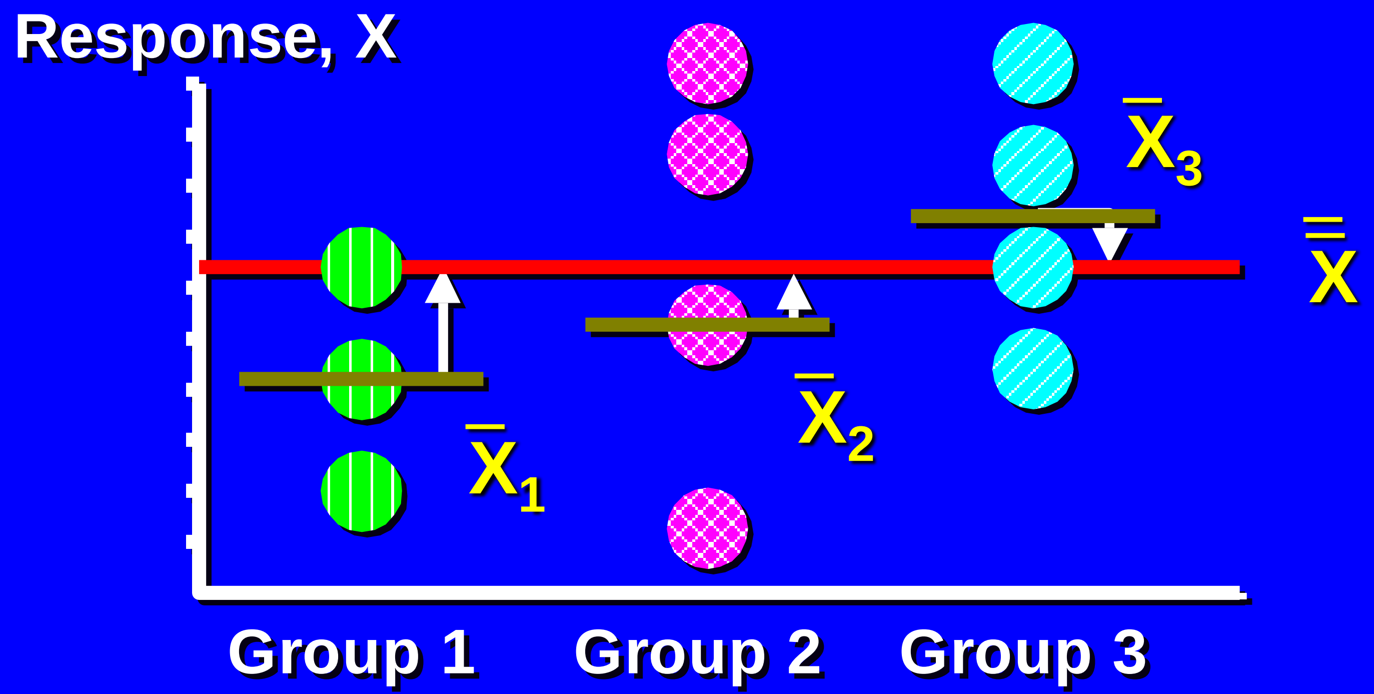
Total Variation

$$SST = \left(X_{11} - \bar{\bar{X}}\right)^2 + \left(X_{21} - \bar{\bar{X}}\right)^2 + \ldots + \left(X_{n_c c} - \bar{\bar{X}}\right)^2$$



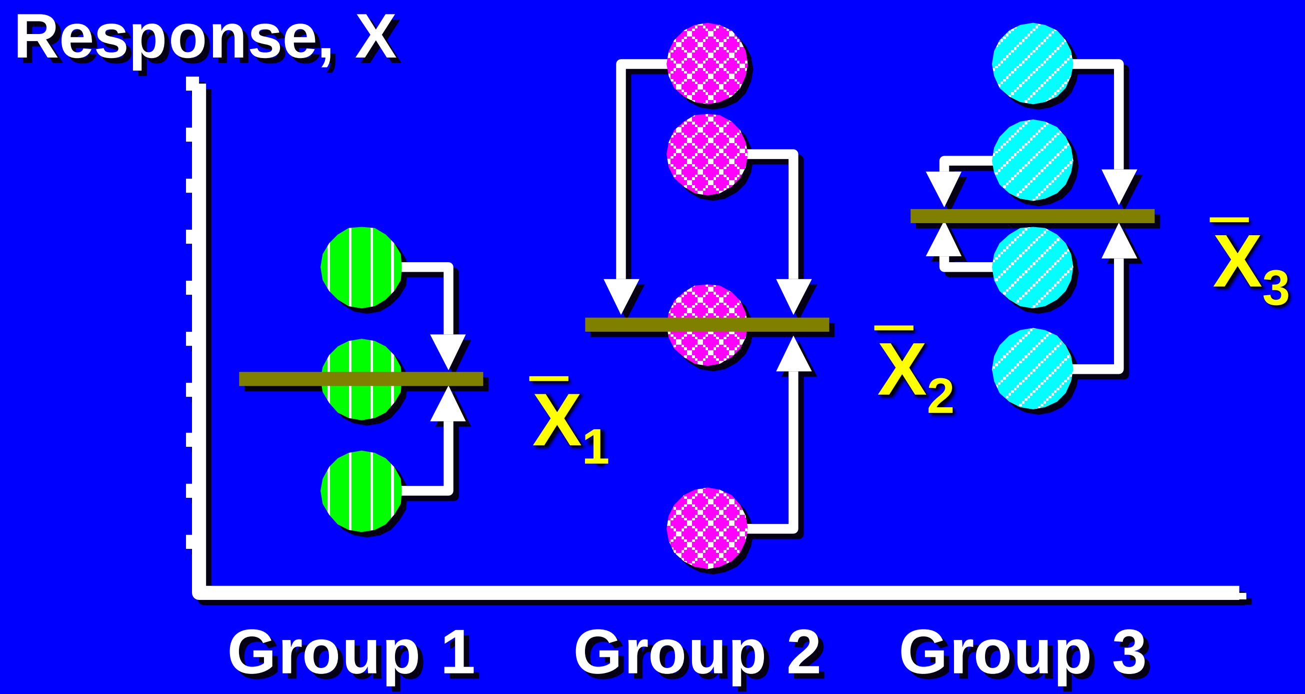
Among-Groups Variation

$$SSA = n_1\left(\bar{X}_1 - \bar{\bar{X}}\right)^2 + n_2\left(\bar{X}_2 - \bar{\bar{X}}\right)^2 + \ldots + n_c\left(\bar{X}_c - \bar{\bar{X}}\right)^2$$



Within-Groups Variation

$$SSW = \left(x_{11} - \bar{x}_1\right)^2 + \left(x_{21} - \bar{x}_1\right)^2 + \ldots + \left(x_{n_c c} - \bar{x}_c\right)^2$$



One-Way ANOVA Test Statistic

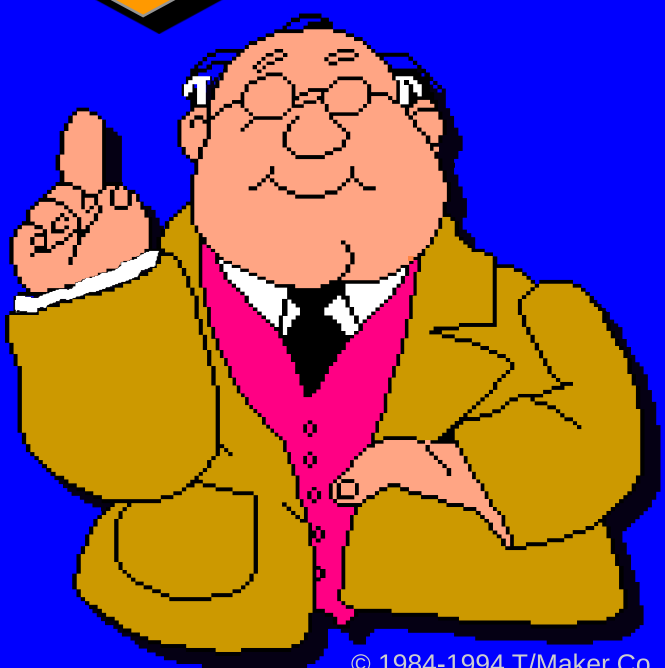
- Test statistic
  - $F = MSA / MSW$
    - MSA is Mean Square Among
    - MSW is Mean Square Within
- Degrees of freedom
  - $df_1 = c - 1$
  - $df_2 = n - c$
    - c = # Columns (populations, groups, or levels)
    - n = Total sample size

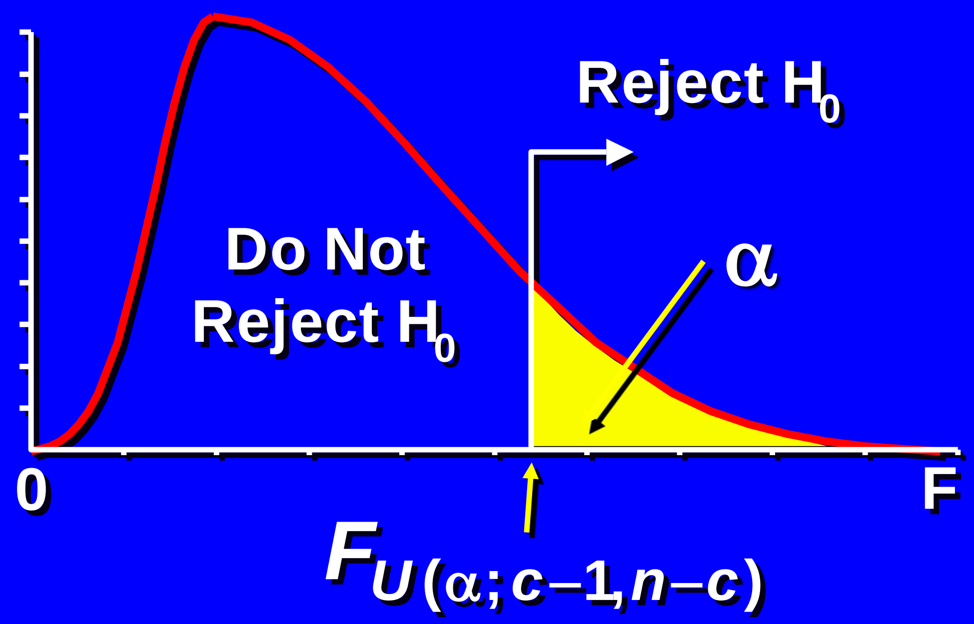
One-Way ANOVA Summary Table

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	F
Among (Factor)	c - 1	SSA	MSA = SSA/(c - 1)	$\frac{MSA}{MSW}$
Within (Error)	n - c	SSW	MSW = SSW/(n - c)	
Total	n - 1	SST = SSA+SSW		

One-Way ANOVA Critical Value

If means are equal,
***F* = *MSA* / *MSW* ≈ 1.**
Only reject large *F*!



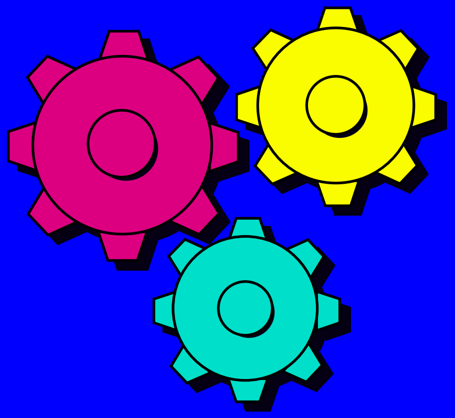


Always One-Tail!

One-Way ANOVA F-Test Example

As production manager, you want to see if 3 filling machines have different mean filling times. You assign 15 similarly trained & experienced workers, 5 per machine, to the machines. At the .05 level, is there a difference in mean filling times?

Mach1	Mach2	Mach3
25.40	23.40	20.00
26.31	21.80	22.20
24.10	23.50	19.75
23.74	22.75	20.60
25.10	21.60	20.40



One-Way ANOVA F-Test Solution

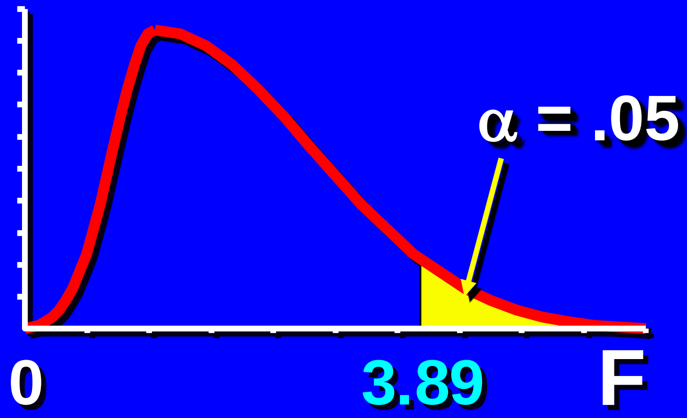
H_0: $\mu_1 = \mu_2 = \mu_3$

H_1: Not all equal

$\alpha = .05$

$df_1 = 2$ $df_2 = 12$

Critical Value(s):



Test Statistic:

$$F = \frac{MSA}{MSW} = \frac{23.5820}{.9211} = 25.6$$

Decision:

Reject at $\alpha = .05$

Conclusion:

There is evidence pop. means are different

Summary Table Solution

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	F
Among (Machines)	3 - 1 = 2	47.1640	23.5820	25.60
Within (Error)	15 - 3 = 12	11.0532	.9211	
Total	15 - 1 = 14	58.2172		

Summary Table Excel Output

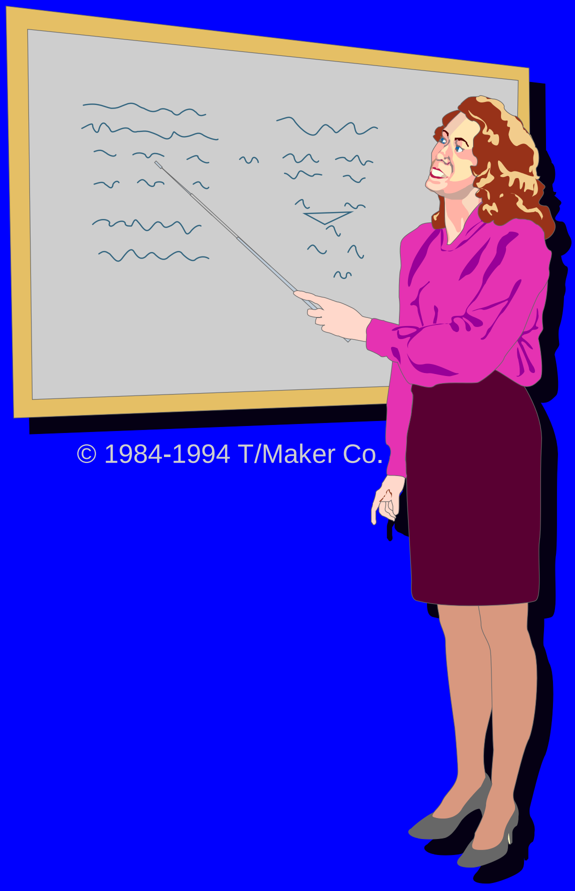
ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	47.1640	2	23.5820	25.60	0.000047	3.89
Within Groups	11.0532	12	0.9211			
Total	58.2172	14				

One-Way ANOVA Thinking Challenge

You're a trainer for Microsoft Corp. Is there a difference in mean learning times of 12 people using 4 different training methods (α =.05)?

M1	M2	M3	M4
10	11	13	18
9	16	8	23
5	9	9	25



Alone Group Class

One-Way ANOVA Solution*

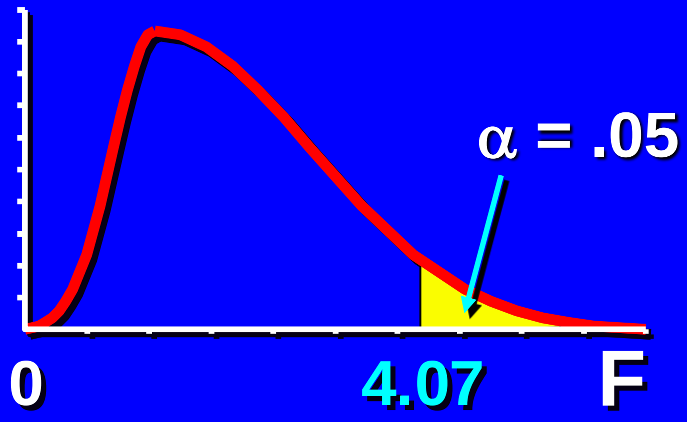
H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1: Not all equal

$\alpha = .05$

$df_1 = 3$ $df_2 = 8$

Critical Value(s):



Test Statistic:

$$F = \frac{MSA}{MSW} = \frac{116}{10} = 11.6$$

Decision:

Reject at $\alpha = .05$

Conclusion:

There is evidence pop. means are different

Summary Table Solution*

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	F
Among (Methods)	4 - 1 = 3	348	116	11.6
Within (Error)	12 - 4 = 8	80	10	
Total	12 - 1 = 11	428		